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Bayesian Parameter Estimation of Non-stationary Collective Dynamics in Moving Animal Groups

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Abstract—Moving animal groups exhibit intriguing collective dynamics. To understand their mechanisms, mathematical models consisting of many interacting self-propelled particles, called swarm models, have been proposed. Statistical estimation of the parameters of the swarm models from observed data, in particular, from non-stationary data such as those observed when animals change their group formation, is an important issue. In this study, we propose a data assimilation method for estimating time-dependent parameters of the swarm model. We apply our method to the time series data of particle coordinates obtained by numerical simulations of the swarm model, and show that the proposed method can estimate the model parameters reasonably well even under relatively strong observation noise.

1. Introduction

Various animals move in groups, such as bird flocks and fish schools. They exhibit a wide variety of formations and intriguing collective dynamics that an individual animal cannot exhibit. It is known that individual animals in a group do not need to see the whole group and that simple interactions with other neighboring animals can make up the group dynamics. To explain their collective dynamics, simple mathematical models consisting of many interacting self-propelled particles, called swarm models, have been proposed [1, 2]. One of the well-known swarm models is the Boid model proposed by Reynolds in 1986 [3]. Reynolds introduced the following three basic interaction rules of flocks to the model:

1. Separation: each member steers to avoid crowding of local flockmates
2. Alignment: each member steers toward the average heading of local flockmates
3. Cohesion: each member steers toward the average position of local flockmates

These rules are commonly applicable to swarm models of various animals despite differences in their scales. However, to reproduce the dynamics of real animal groups, it is necessary to correctly set the parameters that characterize the interaction properties such as their ranges and intensities.

When the parameters are fixed, the swarm model exhibits steady collective motion. Couzin et al. showed that the same model can exhibit qualitatively different collective behaviors, which are similar to those observed in natural animal groups, depending on the model parameters [4]. Estimation of the interaction parameters from observed steady motion of swarms has been performed by Mann [5]. However, natural swarms can also exhibit unsteady motions such as formation changes. Estimation of time-varying interaction parameters from such non-stationary observation data is an unsolved, challenging problem.

In this study, we propose a recursive Bayesian method for estimating time-dependent parameters of the swarm model from non-stationary observed data. Our method estimates the probability density function (PDF) of the parameters at each time step and thereby enables us to track changes in the parameters. We apply our method to the data obtained by numerical simulations of the swarm model.

2. Swarm Model

The swarm model consists of N interacting self-propelled particles that represent individual members in the group. Each particle is characterized by the position $\boldsymbol{x}_{i,t}$ and the heading $\boldsymbol{v}(\theta_{i,t})$ in the 2-dimensional Euclidean space, where $t = 0, 1, ...$ is the discrete time, $i = 1, ..., N$ is the particle index, and $\theta_{i,t}$ represents the heading angle of the particle. We update the positions $\boldsymbol{x}_{i,t}$ using $\boldsymbol{v}(\theta_{i,t})$ according to

$$\boldsymbol{x}_{i,t+1} = \boldsymbol{x}_{i,t} + \alpha \boldsymbol{v}(\theta_{i,t+1}), \quad (1)$$

$$\boldsymbol{v}(\theta_{i,t}) = \begin{pmatrix} \cos\theta_{i,t} \\ \sin\theta_{i,t} \end{pmatrix}, \quad (2)$$

where α is the speed of the particle. The angle $\theta_{i,t+1}$ at time $t+1$ is given by

$$\theta_{i,t+1} = \arctan\left(\frac{\hat{v}_{2,i,t+1}}{\hat{v}_{1,i,t+1}}\right) + \gamma_{i,t}(\kappa_t), \quad (3)$$

where $\gamma_{i,t}(\kappa_t)$ is the angular noise of parameter κ_t (see Eq. (9)), and the angle vector at $t+1$ is determined by

$$\begin{pmatrix} \hat{v}_{1,i,t+1} \\ \hat{v}_{2,i,t+1} \end{pmatrix} = \boldsymbol{v}(\theta_{i,t}) + \omega_{a,t}\frac{\boldsymbol{a}_{i,t}}{|\boldsymbol{a}_{i,t}|} + \omega_{c,t}\frac{\boldsymbol{c}_{i,t}}{|\boldsymbol{c}_{i,t}|} + \omega_{s,t}\frac{\boldsymbol{s}_{i,t}}{|\boldsymbol{s}_{i,t}|}. \quad (4)$$

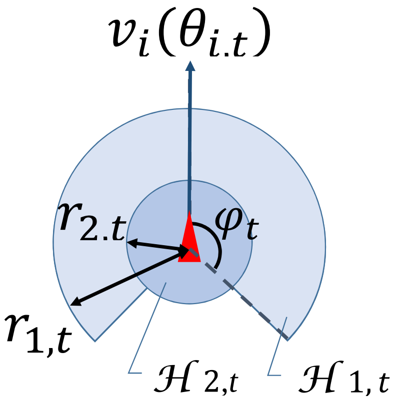


Fig. 1: Interaction zones of the swarm model. $\mathcal{H}_{1,t}$ represents the zone where alignment and cohesion forces work and is characterized by the radius $r_{1,t}$ and viewing angle ϕ_t. $\mathcal{H}_{2,t}$ represents the zone where separation force works and is characterized by the radius $r_{2,t}$.

The first term on the RHS of Eq. (4) represents the inertia, and the other terms represent the interaction forces of alignment, cohesion, and separation, respectively. The vectors $\boldsymbol{a}_{i,t}$, $\boldsymbol{c}_{i,t}$, and $\boldsymbol{s}_{i,t}$ represent the directions of the interaction forces, and the weight parameters $\omega_{a,t}$, $\omega_{b,t}$, and $\omega_{c,t}$ characterize their magnitudes.

Each particle interacts with other neighboring particles within the zone $\mathcal{H}_{1,t}$ or $\mathcal{H}_{2,t}$ shown in Fig. 1. The zone $\mathcal{H}_{2,t}$ represents where the separation force works, and is modeled as a circle of radius $r_{2,t}$ around the particle. The zone $\mathcal{H}_{1,t}$ represents where alignment and cohesion forces work, and is modeled as a circular sector of radius $r_{1,t}$ that does not overlap with $\mathcal{H}_{2,t}$. The parameter ϕ_t in Fig. 1 is the viewing angle of the particle. The vectors $\boldsymbol{a}_{i,t}$, $\boldsymbol{c}_{i,t}$, and $\boldsymbol{s}_{i,t}$ are calculated from the position and heading vectors of other neighboring particles in the zones $\mathcal{H}_{1,t}$ and $\mathcal{H}_{2,t}$ as

$$\boldsymbol{a}_{i,t} = \sum_{j\in\mathcal{H}_1} \boldsymbol{v}(\theta_{j,t}), \quad (5)$$

$$\boldsymbol{c}_{i,t} = \sum_{j\in\mathcal{H}_1} (\boldsymbol{x}_{j,t} - \boldsymbol{x}_{i,t}), \quad (6)$$

$$\boldsymbol{s}_{i,t} = \sum_{j\in\mathcal{H}_2} (-\boldsymbol{x}_{j,t} + \boldsymbol{x}_{i,t}). \quad (7)$$

The angular noise $\gamma_{i,t}$ represents unpredictable random variations in the movement and is given by

$$\gamma_{i,t} \sim f_{VM(0,\kappa_t)}(x), \quad (8)$$

$$f_{VM(\mu,\kappa)}(x) = \frac{e^{\kappa\cos(x-\mu)}}{2\pi I_0(\kappa)} \quad (-\pi \le x < \pi), \quad (9)$$

where $f_{VM(\mu,\kappa)}(x)$ denotes the von Mises distribution [6], which is a circular analog of the normal distribution. The distribution has two parameters, i.e., the mean μ $(-\pi \le \mu < \pi)$ and the concentration κ $(\kappa > 0)$, and can be approximated by the normal distribution $\mathcal{N}(\mu, 1/\kappa)$ when κ is large. The function $I_0(\kappa)$ is the modified Bessel function of order 0. Thus, the parameters to be estimated can be expressed as

$$\boldsymbol{\Omega} = (\omega_{a,t}\ \omega_{c,t}\ \omega_{s,t}\ r_{1,t}\ r_{2,t}\ \phi_t\ \kappa_t)^{\mathrm{T}}. \quad (10)$$

3. Parameter Estimation

In our method, we estimate the parameters of the swarm model by using a recursive Bayesian method [7]. We use the state space model to describe the probabilistic relation between the state variables and the observed data. The PDFs of the parameters is estimated at each time step by comparing the observed movements of the particles in the swarm with their predicted movements by the swarm model under observation noise. The PDFs are calculated by an algorithm called Merging Particle Filter (MPF) [8].

3.1. State Space Model

We consider the case that only the coordinates of the particles can be observed with some observation noise. We denote by a vector $\boldsymbol{X}_t$ the hidden states and parameters of the swarm, and by a vector $\boldsymbol{D}_t$ the noisy observed data $\boldsymbol{x}'_{i,t}$ of the particle coordinates as follows:

$$\boldsymbol{X}_t = \left(\boldsymbol{x}_{1,t}\ \ldots\ \boldsymbol{x}_{n,t}\ \theta_{1,t}\ \ldots\ \theta_{n,t}\ \boldsymbol{\Omega}_t^T \right)^T, \quad (11)$$

$$\boldsymbol{D}_t = \left(\boldsymbol{x}'_{1,t}\ \ldots\ \boldsymbol{x}'_{n,t} \right)^T. \quad (12)$$

The relation between $\boldsymbol{D}_t$ and $\boldsymbol{X}_t$ is assumed to be expressed by the following observation equation:

$$\boldsymbol{D}_t = \boldsymbol{H}\boldsymbol{X}_t + \boldsymbol{W}_t, \quad (13)$$

$$\boldsymbol{H} = \left(\boldsymbol{I}_n\ \ \boldsymbol{0}_{n,n+7}, \right), \quad (14)$$

where $\boldsymbol{I}_n$ is a $n \times n$ identity matrix and $\boldsymbol{0}_{n,n+7}$ is a $n \times (n+7)$ zero matrix. The vector $\boldsymbol{W}_t$ represents observation noise, and is given by

$$\boldsymbol{W}_t = \left(w_{1,t}\ \ldots\ w_{n,t} \right)^T, \quad w_{i,t} \sim \mathcal{N}(0, \sigma_w), \quad (15)$$

where σ_w is the variance.

The vector $\boldsymbol{X}_t$ is updated by the system equation,

$$\boldsymbol{X}_{t+1} = \boldsymbol{F}(\boldsymbol{X}_t, \boldsymbol{V}_t), \quad (16)$$

where $\boldsymbol{V}_t$ is the system noise representing probabilistic aspects in the updating process. In the present model, the system noise $\boldsymbol{V}_t$ is expressed as

$$\boldsymbol{V}_t = \left(\gamma_{1,t}\ \ldots\ \gamma_{n,t}\ \boldsymbol{B}_t^T \right)^T, \quad (17)$$

$$\boldsymbol{B}_t = \left(\beta_{1,t}\ \ldots\ \beta_{7,t} \right)^T, \quad \beta_{k,t} \sim \mathcal{N}(0, \sigma_k). \quad (18)$$

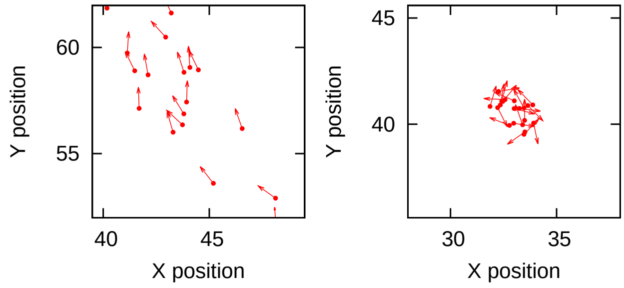
Here, $\gamma_{1,t} \ldots \gamma_{n,t}$ are the angular noise, and $\boldsymbol{B}_t$ represents probabilistic variations in the system parameters $\boldsymbol{\Omega}_t$ obeying normal distributions. Thus, the system equation can be written as

$$\boldsymbol{F}(\boldsymbol{X}_t, \boldsymbol{V}_t) = \begin{pmatrix} \boldsymbol{x}_{1,t} + \alpha \boldsymbol{v}(\theta_{1,t+1}) \\ \vdots \\ \boldsymbol{x}_{n,t} + \alpha \boldsymbol{v}(\theta_{n,t+1}) \\ f_1(\boldsymbol{X}_t, \gamma_{1,t}) \\ \vdots \\ f_n(\boldsymbol{X}_t, \gamma_{n,t}) \\ \boldsymbol{\Omega}_t + \boldsymbol{B}_t \end{pmatrix}, \qquad (19)$$

where the function $f_i(\boldsymbol{X}_t, \gamma_{i,t})$ represents the update rule of $\theta_{i,t+1}$ given by Eq. (3).

3.2. Merging Particle Filter

We use the MPF, which is a modification of the Particle Filter (PF) [9]. The MPF and PF are algorithms that estimate the PDFs of the state $\boldsymbol{X}_t(t = 0, 1, ...)$ from the observation data $\boldsymbol{D}_t(t = 0, 1, ...)$ by the sequential Bayesian method. These filtering methods make no restrictive assumptions on the dynamics of the state or the density function. The PF can preserve the shape of the filtered PDF, but it does not perform well when applied to high-dimensional systems. Although the MPF preserves only the first two moments of the PDF, it performs better than PF in high-dimensional systems. In this study, the dimension of $\boldsymbol{X}_t$ is $(2 \times N + 7)$ and increases linearly with N. Thus, we utilized the MPF to estimate the PDFs of $\boldsymbol{X}_t$.

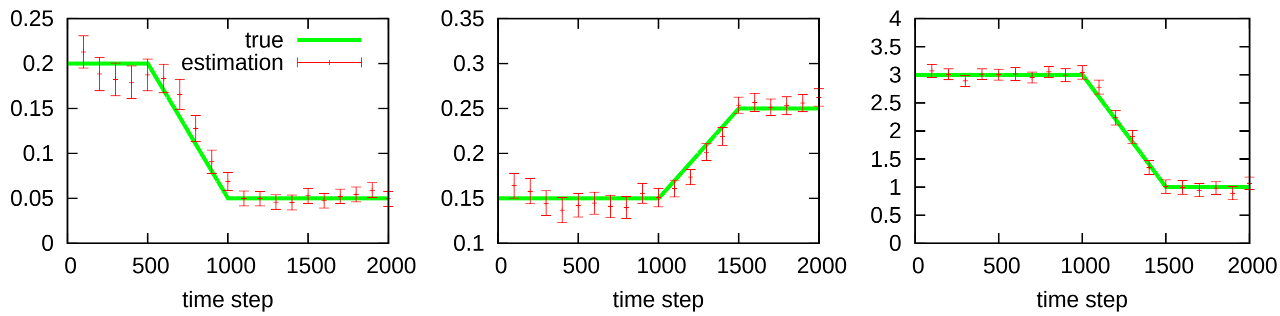


(a) parallel translation (b) rotating mill

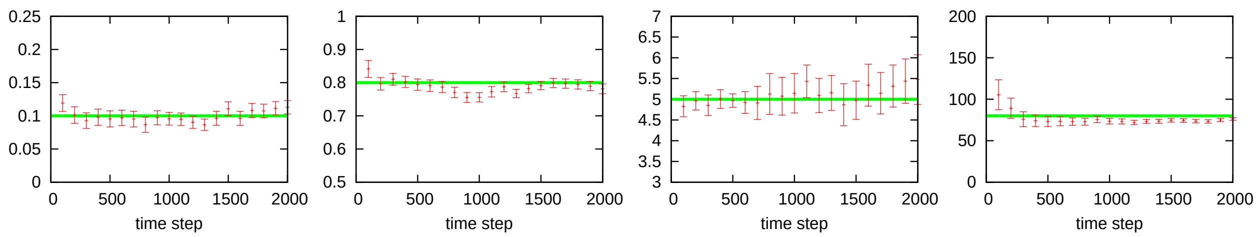
Fig. 2: Two types of collective motion. (a) Parallel translation. The particles move in the same direction. (b) Rotating mill. The particles rotate around an empty core and both clockwise and anti-clockwise moving particles exist simultaneously. Points represent particle positions $\boldsymbol{x}_{i,t}$ and vectors represent their headings $\boldsymbol{v}(\theta_{i,t})$ in the X-Y plane.

4. Numerical Simulation

We apply our method to the time series data obtained by numerical simulations of the swarm model. We generated time series data of the $N = 20$ particle coordinates from $t = 0$ to $t = 2000$ by simulating the swarm model, where Gaussian observation noise $\mathcal{N}(0.0, 0.1)$ is added to each coordinate. The particles started at random initial positions in a limited domain ($L \times L$ square) with ran-



(a) $\omega_{a,t}$ (b) $\omega_{c,t}$ (c) $r_{2,t}$



(d) $\omega_{s,t}$ (e) ϕ_t (f) $r_{1,t}$ (g) κ_t

Fig. 3: True and estimated values of the parameters. Time-dependent parameters are shown in (a) $\omega_{a,t}$, (b) $\omega_{c,t}$ and (c) $r_{2,t}$, and fixed parameters are shown in (d) $\omega_{s,t}$, (e) $r_{1,t}$, (f) ϕ_t and (g) κ_t. The solid lines represent true values of the parameters, the dots represent mean values of the estimated parameters, and the error bars represent standard deviations. The simulated particles exhibited parallel translation when $0 \leq t \leq 500$, and rotating mill when $1500 \leq t \leq 2000$. The speed of the particles is $\alpha = 0.1$. The initial domain size is $L \times L = 10 \times 10$.

dom orientations. As the interaction parameters were varied, the swarm changed the type of its collective motion from parallel translation ($0 \le t \le 500$) to rotating mill ($1500 \le t \le 2000$). Here, parallel translation means that the particles move in the same direction (Fig. 2a). Rotating mill means the case that the particles rotate around an empty core and both clockwise and anti-clockwise moving particles exist simultaneously (Fig. 2b).

We estimated three time-dependent parameters $\omega_{a,t}$, $\omega_{c,t}$ and $r_{2,t}$, and four fixed parameters $\omega_{s,t}$, $r_{1,t}$, ϕ_t and κ_t from the simulated data. The results are shown in Fig. 3, where the solid lines represent true values of the parameters, the dots represent mean values of the estimated parameters, and the error bars represent standard deviations. All PDFs start from uniform initial distributions at $t = 0$, and are updated using the observation data $\boldsymbol{D}_t$ at each time step. It can be seen that our method estimates the time-dependent parameters reasonably well.

The estimation of the alignment parameter $\omega_{a,t}$ was more precise in the case of the rotating mill than in the case of the parallel translation (Fig. 3a). This result implies that it is more difficult to estimate $\omega_{a,t}$ in the parallel translation than in the rotating mill because the inertia term of Eq. (4) and the direction vector $\boldsymbol{a}_{i,t}$ point to nearly the same directions when all particles move in the same direction. The radius r_2 was well estimated even with relatively large parameter variations. We also estimated the four fixed parameters. The mean values of the estimated PDFs agreed reasonably well with the true values (Fig. 3d-3g). When all particles gathered and exhibited high-density collective motion ($500 \le t \le 2000$), the distances between every pair of particles were smaller than r_1 and all particles interacted with all others. The variance of the estimated PDF of radius r_1 increases when $500 \le t \le 2000$, because observation data has less information about r_1 because of the high-density motion. In the initial stage ($t \le 200$), the estimation of some of the parameters is not precise because of the effect of the initial distributions.

5. Discussion

Systems of many interacting "self-propelled" individuals, such as birds, insects, and fish, exhibit intriguing collective dynamics. Various collective behaviors have been studied and many swarm models have been proposed [10]. For example, Couzin et al. showed that a small number of leaders can guide the swarm on the basis of a swarm model [11, 12]. As for fish schools, Sannomiya et al. estimated non-measurable parameters from observation data of a water tank experiment by applying the least squares algorithm [13]. They also investigated the relationship between a stable steady state of the system and its parameters [14, 15]. These studies focused on statistical steady state motion. Recent advances in the global positioning systems (GPS) and digital image analysis have made non-stationary observation data from natural groups available [16, 17, 18].

In this study, we proposed a method to estimate the parameters of swarm models from non-stationary noisy data. We demonstrated that the proposed method can estimate the time-dependent parameters reasonably well even with relatively strong observation noise. Though we used a relatively simple swarm model to verify the effectiveness of the proposed method, it can also be generalized to nonlinear system models. We will apply the present method to more realistic model that is capable of describing real observation data from natural animal groups.

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