On statistical analysis of object pattern formation by autonomous transporting agents

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Abstract—In this paper, we discuss pattern formation of objects generated by distributed autonomous agents capable of loading and unloading the objects. This work was inspired by social behavior of *termite* colonies which often build elaborate three-dimensional structures (nest towers). This paper challenges to clarify the mechanism of this excellent building ability of termite-like agents, by computational and *minimalistic* approach. We introduce a cellular automata (i.e., spatially discretized) model for the agents and objects, where each agent follows a simple 'rule' to choose its action from move/load/unload based on the state of its neighboring cells. An advantage of this approach is that each rule can be *encoded as an integer*, so that we can enumerate all the combinations of possible rules. After examining all the rules, we propose to evaluate and classify the resulting object patterns *quantitatively*, using a couple of statistical indices in image processing, Kolmogorov dimension and HLAC. Finally, some extensions to pattern formation in three-dimensional space are also presented.

1. Introduction

Quantity may overwhelm quality. A large number of tiny agents often outperform single smart agent, if they are appropriately organized. Social insects such as ants, bees or termites, which have physically tiny brains with limited memory and deduction capacity, often construct huge complicated structures (nest towers) [1][2]. In this study, we aim at understanding the mechanism behind these apparently intellectual behavior of swarms. In particular, we will focus our attention on *object pattern formation* generated by autonomous transporting agents.

The authors have proposed a basic model of object stacking by simple loading/unloading agents evolving in the vertical plane [3], based on cellular automata (spatial discretization) approach [4, 5]. This paper extends this result to the pattern formation evolving on the horizontal plane followed by quantitative statistic analyses. First, we define fundamental event rules in the 2-dimensional cellular world, and introduce a robot, and an object as in our previous work. In order to search the possibility of forming a wide variety of structure by simple agents with *"minimum"* ability; carrying an object and perceiving a local information. Since the searching range is limited in the cellular world, we can demonstrate the formed patterns for all possible combinations. After examining all the rules, we *quantitatively* classify the resulting object patterns using a couple of statistical indices in image processing: one is the *Kolmogorov dimension* to characterize the pattern complexity, while the other is the Higher Order Local Autocorrelation (HLAC) to characterize the pattern similarity. Finally, we expand the pattern formation in three-dimensional space.

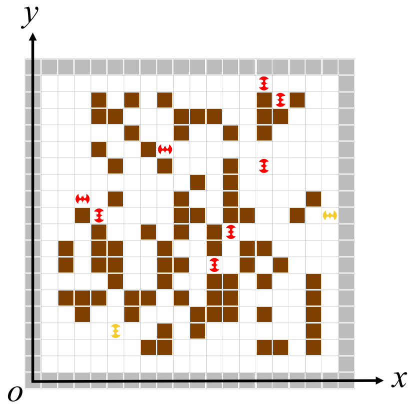
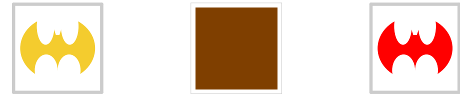


Figure 1: Overview of the horizontal cellular space

2. Cellular modeling of 2-D transporting agents

In this section, we begin with defining the primitive events occurying in this 2-dimensional cellular field, and introduce robots' action rules based on perception of neighboring cells. We suppose tessellation of the 2-dimensional Euclidean space with unit squares, as shown in Figure 1. Each robot or each block (Fig. 2) is placed in a cell. A block does not move autonomously. A robot would be either *empty* or *full* (carrying a block inside it).

In each 'tick' of discrete event, a robot takes one of four primitive actions, identified with the integers from 0 to 3, listed below:



(a) Robot(empty) (b) Block (c) Robot(full)

Figure 2: Agents and blocks

Table 1: Code252: indicates the action rules based on local configurations respectively

(6)	(5)	(4)	(3)	(2)	(1)
Load	Unload	Unload	Load	Unload	Move
1	0	0	1	0	0

Primitive actions of a robot:

0 : Unload Put down the carrying block and step backward.

1 : Load Pick up the object in front of it and step forward.

2 : Move Step forward.

3 : Turn Rotate 90 degree to the right or to the left *randomly* (i.e., following the uniform probabilistic distribution).

The first three actions, namely 0:unload, 1:load and 2:move, may be *infeasible* depending on the circumstance of the robot in concern; e.g., a robot cannot move into a non-empty cell. In such a case, 3:Turn will be chosen instead.

Then, let us turn to define an agent's perception region. The perception region is the set of 5 cells in front of an

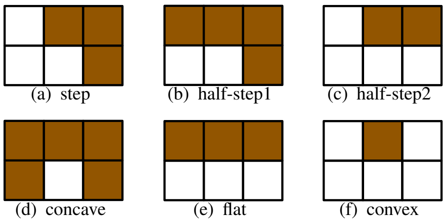


Figure 3: Possible patterns of neighboring blocks

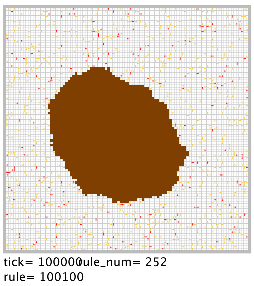
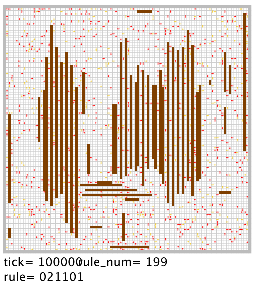
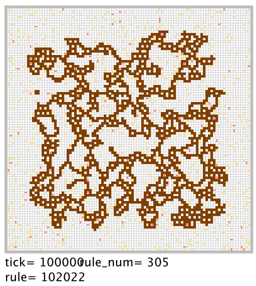
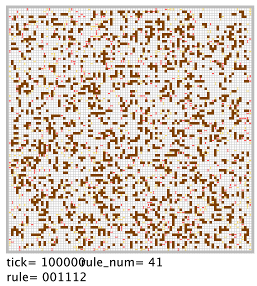
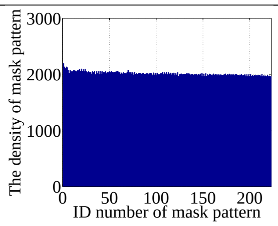
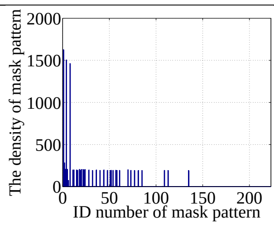
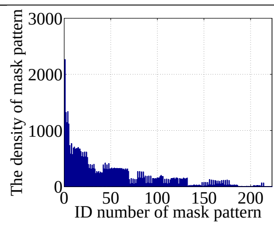
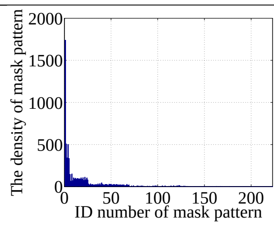
agent. By eliminating the configurations which are equivalent by the symmetry of the robot, the number of patterns are reduced to 6 patterns (Fig. 3).

If the front cell is empty, a robot tries to choose *randomly* move or turn. Otherwise, a robot tries to select from 3 action rules: unload, load, or turn. By selecting from 3 action rules for 6 local configurations respectively (Fig. 3(1)-(6)), all combinations are calculated as $3^6 = 729$. In this paper, a combination of action rules for local configurations is identified with "Code". Code is supposed to count based on ternary to decimal conversion. For example, if a robot is supposed to select a combination as Table 1, this action rule is treated as 100100 in base-3 numeral system. By converting to base-10 numeral system, Table 1 is expressed as Code 252.

3. Analysis of 2-D pattern formation

Suppose a field is occupied by $100 \times 100 = 10000$ cells. 2000 objects are randomly distributed within a square region surrounded by $(10, 10), (90, 10), (90, 90), (10, 90)$. 1000 robots are also placed at 1000 empty cells, and multiple robots are supposed to never occupy in a single cell. A robot tries to select *turn* at boundary condition. Numerical simulations are carried out for 729 possible combinations.

Table 2: Relations between formed pattern and feature values (D_K and HLAC)

Code	252 [100100]	199 [021101]	305 [102022]	41 [001112]
Formed pattern				
D_k	1.578	1.642	1.687	1.782
HLAC				

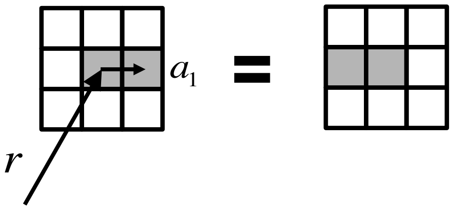


Figure 4: Invariance of parallel shift by integration

4 typical formed patterns are expressed in Table 2. It seems that different types of clusters are formed by different action selects for local curvatures.

3.1. Evaluation: Kolmogorov dimension

In order to measure 'complexity' of resulting clusters, we propose to classify the results using *Kolmogorov dimension* [6]. Suppose a number $N(\epsilon)$ is treated as the smallest number in order to cover a set $X \in R_d$ by a set of convex (sphere, cube) whose diameter is ϵ. Here, *Kolmogorov dimension* is defined as

$$D_k = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log\left(\frac{1}{\epsilon}\right)} \tag{1}$$

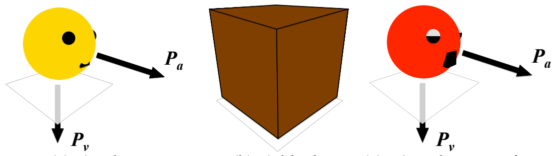
where $\log N(\epsilon)$ is treated as an entropy on a matric space. Thus, Kolmogorov dimension tries to express the pattern complication as a degree of information entropy. Table 2 indicates the results based on Kolmogorov dimensions. It can be verified that complication of formed patterns are expressed quantitatively.

3.2. Evaluation: HLAC

Let us turn to evaluate the resulting patterns quantitatively, with an index called HLAC (higher order local autocorrelation) [7][8]. Suppose an image plane is denoted by P. Images on P are expressed by functions $f(r) \geq 0$ defined within P, where $r \in P$. Suppose x denote a feature of the image $f(r)$ extracted over P. The m th-order autocorrelation functions with m displacements $a_1, ..., a_m$ are defined by

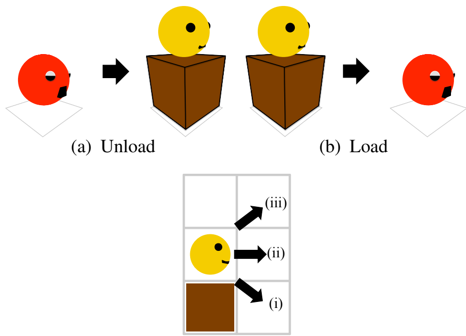
$$x = \int_P f(r)f(r + a_1)\cdots f(r + a_m)dr \tag{2}$$

Since the number of these autocorrelation functions obtained by the combination of the displacements over the image f is enormous, we try to reduce them for practical application. Then, we consider the order m up to the eighth ($m = 0, 1, ..., 8$). We also limit the range of displacements to within a local 3×3 cell, and the center of 9 cells is treated as the reference point. By eliminating the equivalent displacements, the number of the patterns of the displacements is reduced to 223.



(a) A robot (b) A block (c) A robot carrying a block

Figure 5: Objects in the three dimensional space



(a) Unload (b) Load

(c) Move

Figure 6: The robot is capable of doing these actions in the three dimensional space.

Table 2 indicates the results classified based on HLAC. It seems that the difference of cluster patterns are effectively expressed by ID number of mask pattern.

4. Extention to 3-D pattern formation

Let us turn to extend object pattern formation in 3-dimentional space. As we have mentioned in 2-dimentional space, each cell is supposed to be empty or occupied by a robot, a block, or a robot carrying a block (Fig. 5). If an object (a robot or a block) is on an empty cell, every object drops one cell in every step. A robot is supposed to have its orientation as shown in Fig. 5(a) and move based on 5 action rules as follows:
Primitive action of a robot:

0: Unload Put down the carrying block and climb on top of it.

1: Load Pick up the block underneath the robot.

2: Move Step forward.

3: Turn Rotate 90 degree horizontally *randomly*.

4: Cling Rotate 90 degree in a *random* direction staying on a block or a robot

Fig. 7 indicates the procedure of action rules. First, a robot tries to select from 3 action rules: *unload*, *load*, or *turn*. Second, a robot changes its action rule to *turn*. Third, a robot selects *cling*. Finally, a robot drops one cell.

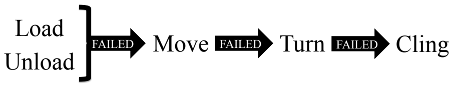


Figure 7: The procedure of selecting action rules

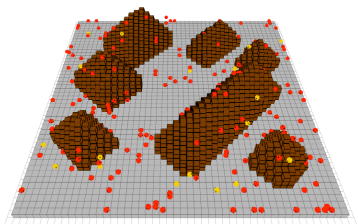


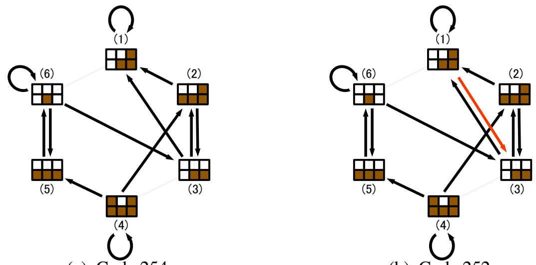
Figure 8: Structure formation based on code 254

Suppose the field is occupied by $50 \times 50 \times 50 = 12500$ cells. 3000 objects are randomly distributed within a square region surrounded by $(2, 48), (48, 2), (48, 48), (2, 2)$. 200 robots are also placed on objects, and multiple robots can never occupy in a single cell. Fig. 8 shows the simulation result based on Code 254. It seems that robots with Code 254 tries to form some quadrangular pyramids. On the other hand, it can be verified that the structures are on the way to form complete quadrangular pyramids.

Let Code 254 be treated as a visually based (directed graph) representation of parallel programs as shown in Fig. 9(a), (1) step structure is verified to be an equilibrium state. Thus, stable step structures leads to form multiple quadrangular pyramids. If we try to form one large quadrangular pyramid, we need to feed a disturbance to (1) step structure as shown in Fig. 9(b). Fig. 10 shows the simulation result with disturbance. It seems that one large quadrangular pyramid is formed.

5. Conclusions with future works

In this paper, we examined pattern formations of objects by autonomous transporting agents by cellular automata approach. After demonstrating some pattern formations by simulations, we evaluated the complexity of the several patterns based on Kolmogorov dimension and HLAC. In addition, we expanded pattern formations in 3-dimensional space. There remains several issues to be discussed in the future works: (a) mathematical analysis of *stability of patterns*, and (b) its application to design engineering.



(a) Code 254 (b) Code 252

Figure 9: State transition graph in 3 example Codes

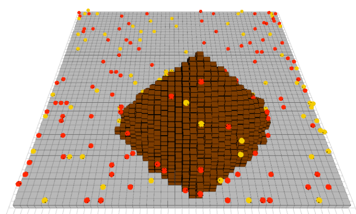


Figure 10: Four pyramid structure formation in the three dimensional space

Acknowledgments

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References

[1] D. Gordon, *Ants At Work: How An Insect Society Is Organized*. The Free Press, 1999.

[2] M. Hansell, *Built by Animals: The Natural History of Animal Architecture*. Oxford University Press, USA, 2009.

[3] T. Tahara, Y. Sueoka, M. Ishikawa, and K. Osuka, "Autonomous formation of object clusters on vertical plane," in *Proc. of International Symposium on Nonlinear Theory and its Applications*, 2013, pp. 57–60.

[4] S. Wolfram, *Cellular Automata and Complexity : collected papers*. Westview, 1994.

[5] J. Schiff, *Cellular Automata : A Discrete View of the World*. John Wiley & Sons, 2008.

[6] G. J. Chaitin, "Algorithmic information theory," *IBM Journal of Research and Development*, vol. 21, no. 4, pp. 350–359, July 1977.

[7] T. Kurita, N. Otsu, and T. Sato, "A face recognition method using higher order local autocorrelation and multivariate analysis," in *Proc. of 11th IAPR International Conference*, 1992, pp. 213–216.

[8] N. Otsu and T. Kurita, "A new scheme for practical flexible and intelligent vision systems." in *MVA*, 1988, pp. 431–435.