A Feedback Suppression of Spatiotemporal Nonlinear Phenomena in a Two-dimensional Excitable Medium with Parameter Uncertainty

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Abstract—The present paper considers a feedback control method for suppressing spirals and spatiotemporal chaos in a two-dimensional excitable medium. This method uniformly applies small impulsive external forces to the medium with feedback based on real-time rough information on the medium. It is numerically shown that the control performance of this method does not decline even if the medium has a parameter uncertainty.

1. Introduction

Nonlinear phenomena in excitable media, such as propagating waves, spirals, and spatiotemporal chaos, have been analytically and experimentally investigated for many years. In recent years, there has been some interest in *control* of such phenomena as follows: unstable propagating waves in a photosensitive Belousov-Zhabotinsky reaction are experimentally stabilized by a feedback light-intensity control [1]; the spirals and the spatiotemporal chaos in excitable media, known as a cause for irregular heartbeat, are suppressed by various control methods [2].

These methods for suppression can be classified into two categories: *nonfeedback* control and *feedback* control. Most studies on suppression employ the *nonfeedback* control (e.g., see Refs. [3, 4, 5, 6]) due to simplicity of control structure. By contrast, even though the *feedback* control has been widely used for industrial applications in the field of control engineering, only a few feedback control methods have been used for the suppression [7, 8, 9, 10, 11, 12]. It is well accepted in the field of control theory that the control performance with feedback does not decline even if the controlled objects have some uncertainty; that is, the feedback control methods are generally robust over uncertainty of controlled objects, while the nonfeedback is not.

Yoneshima *et al.* proposed a feedback control method for suppressing spirals in a cellular automata [13]. This feedback control system is described by a cellular automaton (i.e., discrete time, value, and space systems); thus, its behavior can be easily demonstrated on computer simulations because of unnecessity of numerical integration. On the other hand, it is difficult to deal analytically with such behavior, since popular mathematical tools for analyzing nonlinear dynamics are generally meant for partial differential equations (i.e., continuous time, value, space systems). Furthermore, the main advantage of feedback, that is, the robustness over parameter uncertainty, has never been examined.

The aim of the present work is to propose a feedback control method for suppressing spirals and spatiotemporal chaos in an excitable medium described by a partial differential equation: this method uniformly applies small impulsive external force, on the basis of real-time rough information on the medium, to whole area on the medium. Its robustness over parameter uncertainty is investigated using numerical simulations.

2. Feedback control

The Bär model with no-flux boundary [14], one of the most popular models used in examining the control methods for elimination of the spirals and the spatiotemporal chaos,

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{1}{\varepsilon} u(1-u)\left(u - \dfrac{v+b}{a}\right) + \nabla^2 u \\ \dfrac{\partial v}{\partial t} = g(u) - \alpha v + e \end{cases}, \qquad (1)$$

is here considered, where $u \in \boldsymbol{R}$ and $v \in \boldsymbol{R}$ are fast and slow variables. $\nabla^2 := \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$ denotes the Laplace operator. $x_{1,2} \in [0, L]$ represents the position on the medium with width L. According to Ref. [14], the parameters, $\varepsilon = 0.08$, $a = 0.84$, and $b = 0.07$, are fixed and the nonlinear function $g(u)$ is given by

$$g(u) = \begin{cases} 0 & u < 1/3 \\ 1 - 6.75u(u-1)^2 & 1/3 \le u \le 1 \\ 1 & u > 1 \end{cases}. \qquad (2)$$

The present work introduces an uncertain parameter $\alpha > 0$ in order to investigate the robustness of control

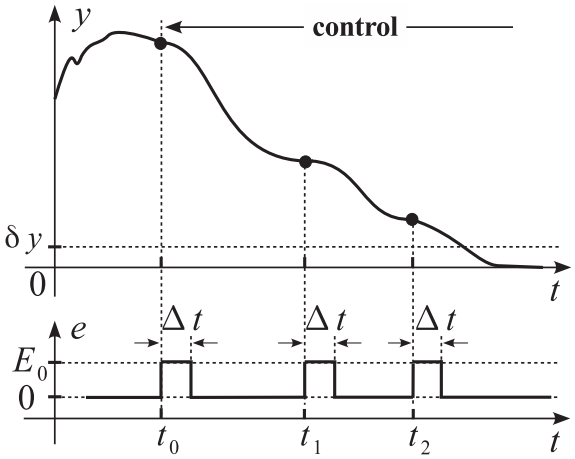


Figure 1: Schematic illustration of measured information y and feedback force e

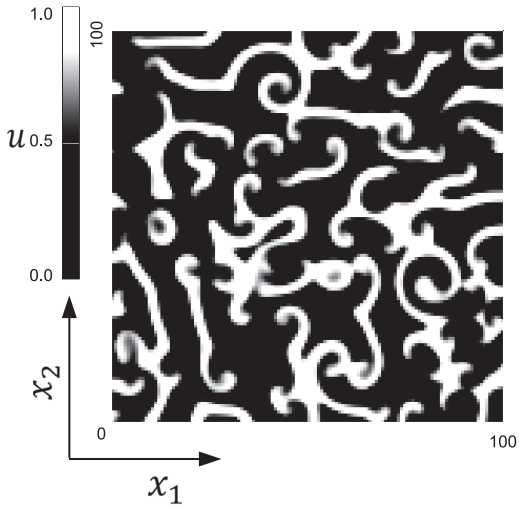


Figure 2: Snapshot of spatial behavior with $\alpha = 1.0$

systems over parameter uncertainty[1]. Medium (1) for $\alpha = 1$ is equivalent to the original Bär model.

The external force with the amplitude E_0 and for the small interval $\Delta t \ll 1$,

$$e = \begin{cases} E_0 & t \in [t_i, t_i + \Delta t] \\ 0 & \text{otherwise} \end{cases}, \ (i = 0, \ldots, M-1), \quad (3)$$

is applied with spatially uniformity to the medium M times. Remark that the force is added to the dynamics of slow variable[2]. A single impulsive force ($M = 1$) [3] can induce an elimination of propagating wave as follows: the force increases v in the entire medium; the wave front (back) velocity decreases (increases) due to the increase of v at the wave front (back); this fact leads to narrow the wave width; if the amplitude E_0 is large, the wave back hits the wave front and then the propagating wave vanishes. In contrast, if E_0 is not large, the wave back cannot hit the wave front and then the propagating wave survives. Our previous study [10] showed that the periodic impulsive force (3) with $t_i = t_0 + iT$, where T denotes the period of impulsive force, can eliminate the propagating wave even with small amplitude E_0 by a repeat of the above actions. However, the periodic force has a problem how to design T when the medium has some uncertain parameters. This is because T depends on the parameters. It should be emphasized that these forces mentioned above are classified into the nonfeedback control.

[1]The parameter α directly influences the dynamics of slow variable v: an increase of α expands the width of the propagating waves on the medium.

[2]Most of the previous studies on control of excitable media employ the fast variable for the force; however, the present paper employs the slow variable. The reason is described in Refs. [10, 15].

The present paper considers feedback control, which measures a rough real-time information on the medium, that is y, and then applies the impulsive force with the amplitude E_0 at time t_i (see Fig. 1). The information y, the ratio of a space mean excited area to the whole medium area L^2, is given by

$$y = \frac{1}{L^2} \int_0^L \int_0^L H(u - u_0) \mathrm{d}x_1 \mathrm{d}x_2, \quad (4)$$

where H is a step function and $u_0 := b/a$ is the threshold for excitation. t_i is the time when y is at a local minimum, that is,

$$\left. \frac{\mathrm{d}y}{\mathrm{d}t} \right|_{t=t_i} = 0, \quad \left. \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} \right|_{t=t_i} > 0. \quad (5)$$

The first impulsive force is applied at $t = t_0$ independently of y and then the above control law is used from $t = t_1$. In order to avoid the feedback force with short intervals[3], we define t_i such that condition (5) and $t_i - t_{i-1} \geq T_{\min}$ hold. Further, the feedback force is applied until y becomes almost zero: $y < \delta y \ll 1$ holds. The feedback force can be considered as an extension of the previous report [13] to continuous systems.

3. Numerical simulations

This section will numerically investigate the control performance. The parameters are set as follows: $L = 100$, $\Delta t = 0.1$, time step for numerical integration $h = 0.02$, number of space grid $N = 128$, threshold of short intervals $T_{\min} = 0.12$, and threshold of suppression $\delta y = 10^{-5}$. The numerical simulations are

[3]The impulsive forces with short intervals would not be easily realized in practical situations such as a defibrillator.

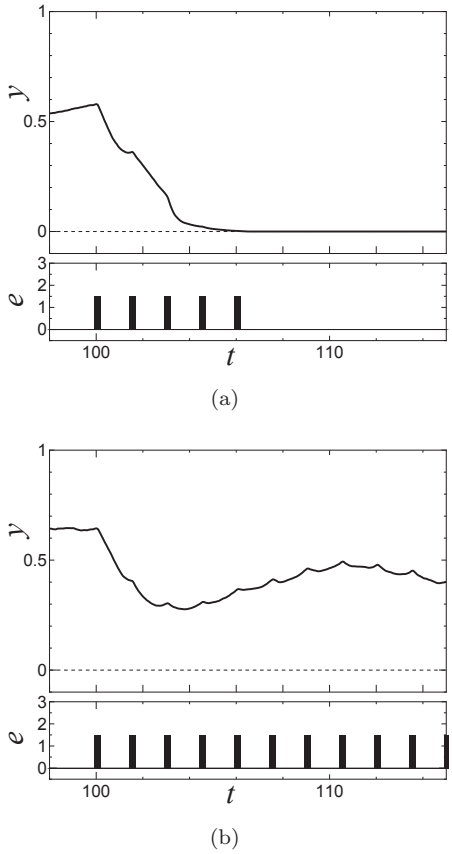


Figure 3: Time-series data of y and e with periodic force ($E_0 = 1.5, T = 1.5$): (a) $\alpha = 0.8$, (b) $\alpha = 1.0$.

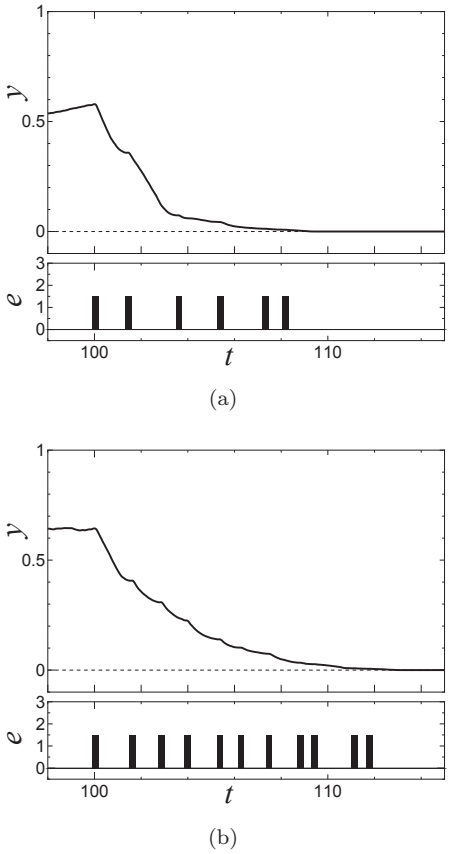


Figure 4: Time-series data of y and e with feedback force ($E_0 = 1.5$): (a) $\alpha = 0.8$, (b) $\alpha = 1.0$.

achieved by the popular forward-time centered-space method[4]. Figure 2 shows a snapshot of spatial behavior on medium (1) without control (i.e., $e \equiv 0$).

In order to compare the robustness of the feedback force with that of typical nonfeedback control, that is, the periodic force (3) [10]. Figure 3a illustrates the time-series data of spatiotemporal behavior on medium (1) for $\alpha = 0.8$ with the periodic force. The excited ratio y drops at every impulsive forces; as a consequence, the five small periodic impulsive forces eliminate the excited region. On the other hand, for $\alpha = 1.0$, as demonstrated in Fig. 3b, the periodic force fails to eliminate it even with the same period. These results suggest that the suitable period T depends on the parameter α; in other word, this method is not robust over the uncertain parameter. Therefore, for the situation where the parameter α is unknown, one cannot design the period T in advance.

In contrast, as shown in Figs. 4a and 4b, the feedback force succeeds in eliminating the excited region both for $\alpha = 0.8$ and $\alpha = 1.0$: it can be seen that the impulsive forces are added with proper timing. From these results, we can see that the feedback force is robust over the uncertain parameter compared with the periodic force.

[4]The time series y on numerical simulations is not smooth due to space discretization; thus, $t_{\mathrm{ave}} = 0.06$ backward moving average is used for smoothing y.

4. Discussions

Let us investigate the control performances of the periodic force and the feedback force from practical viewpoints. We define two performance indices: the total energy required to eliminate y,

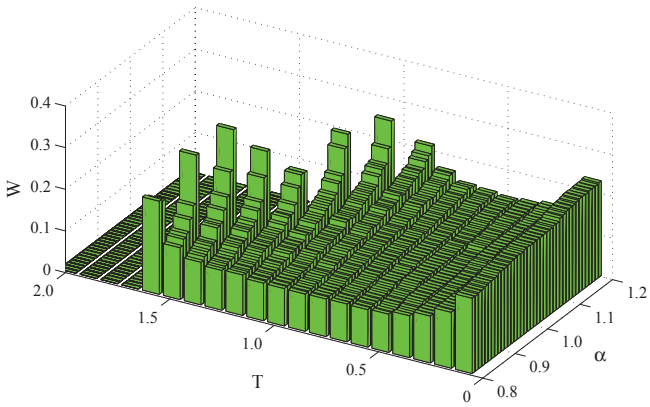
$$W = \int_0^\infty e^2 \mathrm{d}t, \tag{6}$$

and the average period of forces,

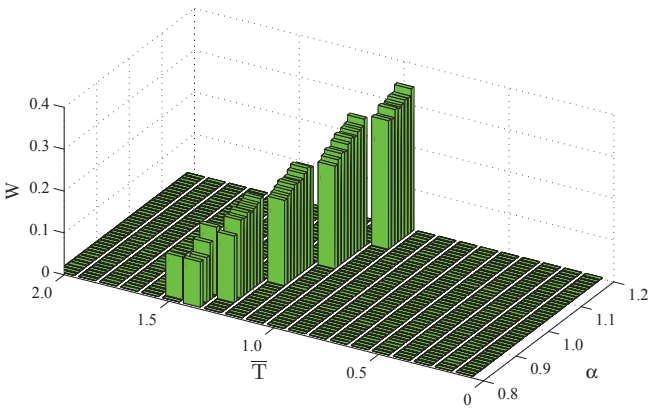
$$\overline{T} = \frac{1}{M-1} \sum_{i=1}^{M-1} (t_i - t_{i-1}) . \tag{7}$$

It is easy to understand that the small energy and the long period, which reduces damage to a medium, are desired for practical situations.

Figure 5a shows the total energy W plotted as a function of the force period T and the uncertain parameter α for the periodic force. It should be noted that the periodic force with the parameter set (α, T)



(a)



(b)

Figure 5: Plots of total energy W against force period T (or $\overline{T}$) and uncertain parameter $\alpha \in [0.8, 1.2]$ with $E_0 = 1.5$: (a) periodic force, (b) feedback force.

at $W = 0$ fails to eliminate it as demonstrated in Fig. 3b. From Fig. 5a, we see that $T = 1.5$, which was used in Fig. 3, may be one of good candidates for suppression, since we have a long period and a not-so-high energy. We notice that this is true for small α (e.g., $\alpha = 0.8$), but not for large α (e.g., $\alpha = 1.0$), as shown in Fig. 3. This result implies that it is not easy to choose a long T if α is unknown.

Now let us consider the performance of the feedback force. For a given α, the total energy W and the average period $\overline{T}$ estimated by Eq. (7) are plotted in Fig. 5b. It can be seen that the feedback force, which can automatically choose the long average period (i.e., $\overline{T} \in [1.0, 1.5]$), succeeds in eliminating it for any $\alpha \in [0.8, 1.2]$. Therefore, it may be concluded that the feedback force is robust over the uncertain parameter.

5. Conclusion

This study investigated the robustness of the feedback force over an uncertain parameter, compared with the periodic force: the feedback force can automatically choose the long average period for any uncertain parameter within a range. These results would be useful information for development of a less-invasive defibrillator for irregular heartbeat. This is because behavior of human hearts depends on individuals; the defibrillator should be robust over the parameter uncertainty, individual variation.

Acknowledgments

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