Desynchronization Transitions Caused by Random Attacks in Neural Networks

Fabiano A.S. Ferrari ‡†; Ricardo L. Viana‡ and Ruedi Stoop†

‡Federal University of Parana, Physics Department
post office box 19044, 81531-980, Curitiba, Brazil
†Institute of Neuroinformatics, University of Zurich and ETH Zurich
Winterthurerstrasse 190, CH-8057 Zurich, Switzerland
Email: fabianosferrari@gmail.com

Abstract—Lesions in the brain can break the neuronal synchronization and significantly affect brain function. In this work we propose a simplified model to study the dynamical effect of random lesions. We show the existence of a phase transition from the synchronized state to the desynchronized state caused by the desynchronization of the non-attacked neurons.

1. Introduction

The neurons in the brain are connected according to a complex pattern of connectivity [1]. The topological organization of the brain has been associated with integration and segregation properties [2]. The presence of lesions in the brain causes abnormalities in the topological organization and has been associated with Alzheimer's Disease, Autism, Schizophrenia and other mental illnesses [6]. As the synchronization plays an important role in information processing and cognition [3, 4, 5], neural desynchronization can be responsible for symptoms caused by neurodegenerative diseases.

Several studies have been made to understand the impact of lesions in the brain [7, 8]. However the basic properties behind the lesions remain unclear. Based on this we analyze in this work the coupling effect caused by random lesions in a synchronized neural network of globally coupled Rulkov neurons. Despite the simplicity of the model we have observed a non-trivial behavior in the synchronization-desynchronization transition.

2. Neural Network

To study the transition from the synchronized to the desynchronized state we will consider a simplified model of globally coupled Rulkov neurons (all to all connections)

$$x_{n+1}^{(j)} = \frac{\alpha^{(j)}}{(1+(x_n^{(j)})^2)} + y_n^{(j)} + \frac{\varepsilon}{N}\sum_{i=1}^{N} x_n^{(i)}, \quad (1)$$

$$y_{n+1}^{(j)} = y_n^{(j)} - \sigma x_n^{(j)} - \beta, \quad (2)$$

where x is called the fast variable and y is the slow variable, α is associated with the burst frequency and is chosen by a uniform random probability such that $\alpha \in [4.1, 4.3]$, N is the network size, ε is the coupling strength and the other parameters are set as $\sigma = \beta = 0.001$.

Several works have shown that this kind of network exhibits phase synchronization [9, 10]. This property can be evaluated by defining a phase according to burst frequency such that for each neuron we have

$$\varphi_n^{(j)} = 2\pi k + 2\pi \frac{n - n_k^{(j)}}{n_{k+1}^{(j)} - n_k^{(j)}}, \quad (3)$$

in which n_k and n_{k+1} are the times of two succesive bursts. The phase synchronization can be determined by the Kuramoto order parameter,

$$r_n = \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\varphi_n^{(j)}}\right|, \quad (4)$$

$$< R > = \frac{1}{n'}\sum_{n=1}^{n'} r_n, \quad (5)$$

in which n' is sufficiently large. The Kuramoto order parameter, $< R >$, is 1 when the network is completely synchronized and 0 when desynchronized.

3. Phase Syncrhonization for Globally Coupled Rulkov Neurons

For global coupling there is a critical value, ε_c, such the network goes from the desynchronized state to the partial synchronized state. For Rulkov neurons the critical value is $\varepsilon_c \approx 0.02$ as shown in the Figure 1, the phase transition is independent of the network size. Figure 1 shows that when the coupling strength is $\varepsilon = 0.04$ the network reaches the maximum possible value for the order parameter and we consider the network completely synchronized in phase.

4. Desynchronization Induced by Random Attacks

Decoupling neurons randomly, it is possible to desynchronize the network. To analyze this effect we first choose an ε in which the network is completely synchronized in

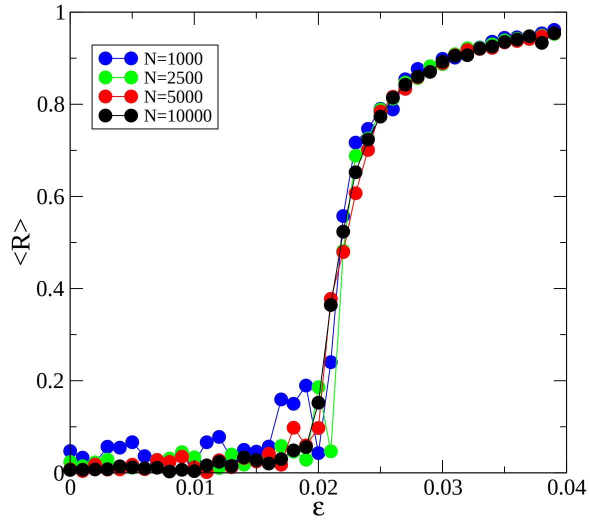

Figure 1: Kuramoto order parameter for Rulkov neurons globally coupled. The different colors represent different network sizes.

phase and also other two cases, $\varepsilon = 0.3$ and $\varepsilon = 0.2$, for comparison. For each specific ε we disconnect the neurons one by one evaluating how the order parameter decreases. When a neuron is disconnected from the network the coupling strength is reduced by the factor

$$\frac{(N-l)}{N}, \qquad (6)$$

in which l is the number of affected neurons. Figures 2 (a) and 3 (a) show that the network desynchronizes with a lower coupling, the lower the coupling strength is in the begining of the lesion. In Figure 2 (b) the order parameter for the whole network decays linearly with the number of disconnected neurons before decaying towards the critical value in which the whole network desynchronize. The Figure 3 (b) shows that the non-attacked neurons desynchronize almost all together.

5. Conclusions

For a network of globally coupled Rulkov neurons the network is completely synchronized for $\varepsilon = 0.04$ and increasing the coupling strength above this value then increases the robustness of the network making it necessary to attack more neurons to desynchronize the network. Analysing the cluster of non-attacked neurons we can see that the critical point for the phase transition from the synchronized to the desynchronized state is caused by the desynchronization of this cluster.

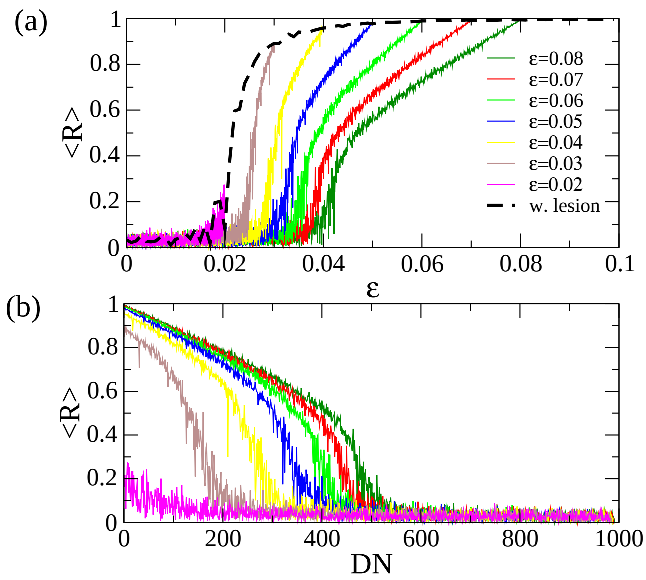

Figure 2: Desynchronization effect caused by random attacks in the whole network. In both figures the colors represent different initial coupling strength for the moment in which we start the attacks in the network. In the figure (a) we present the Kuramoto order parameter as a function of the coupling strength and in the figure (b) we present the Kuramoto order parameter as a function of the disconnected neurons (DN). The black dashed line in the figure (a) is the order parameter for the case without attacks (w. lesion).

Acknowledgments

The authors thanks to Thomas Lorimer for his fruitful suggestions and comments. This work was made possible through the partial financial support from the Brazilian research agencies: CAPES and CNPq.

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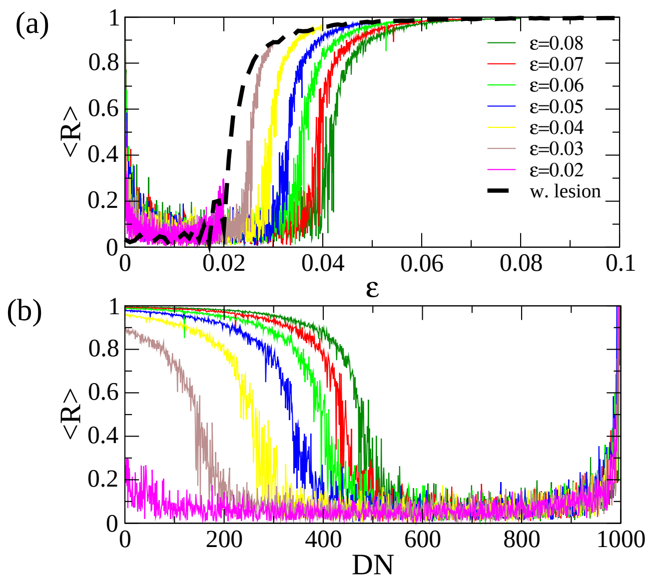

Figure 3: Desynchronization effect in the non-attacked cluster in the neural network. In both figures the colors represent different initial coupling strength for the moment in which we start the attacks in the network. In the figure (a) we present the Kuramoto order parameter as a function of the coupling strength and in the figure (b) we present the Kuramoto order parameter as a function of the disconnected neurons (DN). The black dashed line in the figure (a) is the order parameter for the case without attacks (w. lesion).

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