

An Autonomous Chaotic System on Cylinder

Kazuyuki Kimura[†], Toshimichi Saito[†] and Tadashi Tsubone[‡]

[†]EE Dept., Hosei Univ., Tokyo 184-8584, Japan

[‡]ECS Dept. Nagaoka Univ. of Technology, Nigata 940-2188, Japan

Abstract—This paper presents a novel autonomous chaotic system: the manifold piecewise linear system on the cylinder. This system is defined by second order continuous flow on the cylinder with hysteresis switching and the trajectories do not diverge. The system on the cylinder is equivalent to the systems having infinite equilibria. This system can exhibit super expanding chaos characterized by very large positive Lyapunov exponent. Presenting a simple test circuit, the super expanding chaos is confirmed experimentally.

1. Introduction

The manifold piecewise linear system (MPL) is an autonomous chaotic system that consists of second-order continuous system and hysteresis switching of the equilibrium points [1]-[3]. As the most important characteristics of the MPL, it should be noted that the dynamics is integrated into a piecewise linear one-dimensional return map and chaotic generation is guaranteed theoretically. The MPL is realized by a simple electric circuit and chaotic behavior is confirmed in the laboratory. It should also be noted that the chaotic behavior is applicable to engineering systems such as chaos-based communication and radar systems [4][5]. Up to the present, a variety of autonomous chaotic systems have been presented and the dynamics have been analyzed [6]-[7]. These systems have been contributed to development of nonlinear dynamical system theory and its engineering applications.

This paper presents a novel autonomous chaotic system: the manifold piecewise linear system on the cylinder (CMPL). The CMPL is defined by second order continuous flow on the cylinder with hysteresis switching and the trajectories do not diverge. The system dynamics on the cylinder is equivalent to systems having infinite equilibria. Especially, the CMPL can exhibit super expanding chaos characterized by very large positive Lyapunov exponent. In the MPL, the divergent trajectory is inevitable and the super expanding chaos is impossible. Presenting a simple test circuit, the super expanding chaos is confirmed experimentally.

2. Manifold Piecewise Linear System

Here, we introduce the MPL as preparation to present the CMPL. The MPL is defined by the following second-order piecewise linear system with hysteresis switching.

$$\ddot{x} - 2\delta\dot{x} + x = \begin{cases} p & (A) \\ -p & (B) \end{cases}, \quad \mathbf{x} \equiv (x, \dot{x}) \quad (1)$$

This system has two equilibria $\pm p$. In order to define the switching rule, we define the following two segments

$$\begin{aligned} L_{2+} &\equiv \{\mathbf{x} \mid x \geq 0, \dot{x} = 0\} \\ L_{2-} &\equiv \{\mathbf{x} \mid x < 0, \dot{x} = 0\}. \end{aligned} \quad (2)$$

The right hand side is switched from (A) to (B) if the trajectory hits L_{2+} and is switched from (B) to (A) if the trajectory hits L_{2-} . The MPL is characterized by two parameters: damping δ , equilibrium point p . For simplicity, we assume:

$$0 < \delta < 1, (\omega \equiv \sqrt{1 - \delta^2}), p > 0 \quad (3)$$

The system has unstable complex characteristic roots $\delta \pm j\omega$. The trajectory rotates divergently around the equilibrium point p or $-p$. If the trajectory hits L_{2+} , the equilibrium point is switched from p to $-p$.

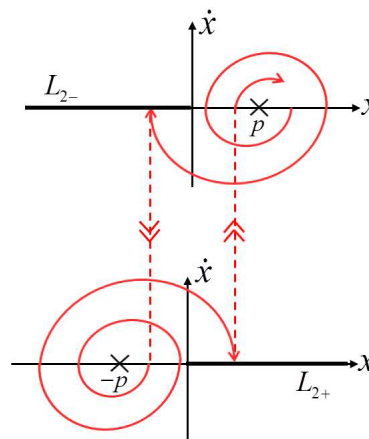


Figure 1: Switching of the MPL

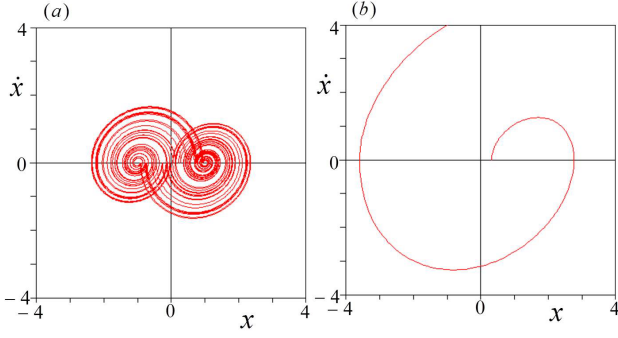


Figure 2: Trajectories. (a) Double-screw chaos for $\beta = \sqrt{2}$, (b) Divergence for $\beta = 2.6$

If the trajectory hits L_{2-} , the equilibrium point is switched from $-p$ to p . Note that the switching occur only on the x -axis. This system repeats in this manner. As shown in Fig. 2 (a), the MPL exhibits double-screw chaotic attractor. For convenience, we use the following parameter β hereafter.

$$\beta \equiv e^{\frac{\delta\pi}{\omega}} > 1 \quad (4)$$

If the trajectory does not diverge the MPL has a positive Lyapunov exponent $\ln\beta$. Chaos generation is guaranteed if

$$1 < \beta < 2 \quad (5)$$

Fig. 2 shows typical trajectories where β is used as a parameter instead of δ , for convenience.

3. MPL on Cylinder

We propose the CMPL based on the switching rule of the MPL. As a preparation, we define the infinite-screw MPL with infinite equilibria on x -axis. In order to define switching rule, we define the following two segment

$$\begin{aligned} L_{n+} &\equiv \{\mathbf{x} \mid 2nT < x \leq (2n+1)T, \dot{x} = 0\} \\ L_{n-} &\equiv \{\mathbf{x} \mid (2n-1)T < x \leq 2nT, \dot{x} = 0\} \\ L_n &\equiv L_{n+} \cup L_{n-}. \end{aligned} \quad (6)$$

where $n \in \mathbf{Z}$. The dynamics of the infinite-screw MPL on L_n is described by

$$\ddot{x} - 2\delta\dot{x} + x = \begin{cases} p + 2nT & (A_n) \\ -p + 2nT & (B_n) \end{cases}, \quad \mathbf{x} \in L_n \quad (7)$$

We consider the case: $0 < p < T$. The right hand side is switched to (A_n) if the trajectory hits L_{n+} and is switched to (B_n) if the trajectory hits L_{n-} . The trajectory rotates divergently around the equilibrium point $p + 2nT$ or $-p + 2nT$. If the trajectory hits L_{n+} , the equilibrium point is switched to $p + 2nT$. If the trajectory hits L_{n-} , the equilibrium point is switched

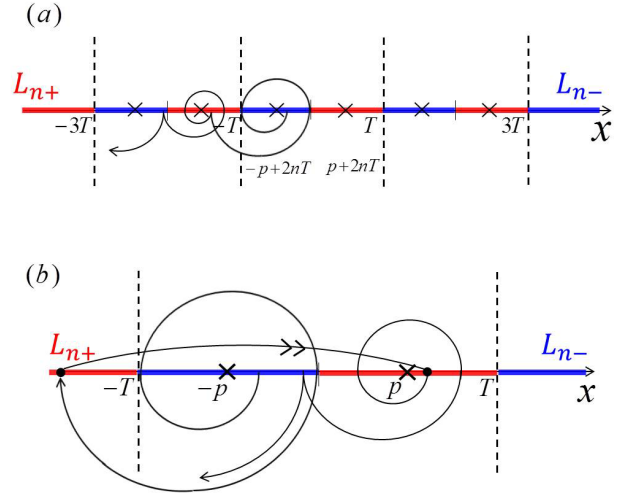


Figure 3: Switching rules. (a) Infinite-screw MPL (b) CMPL

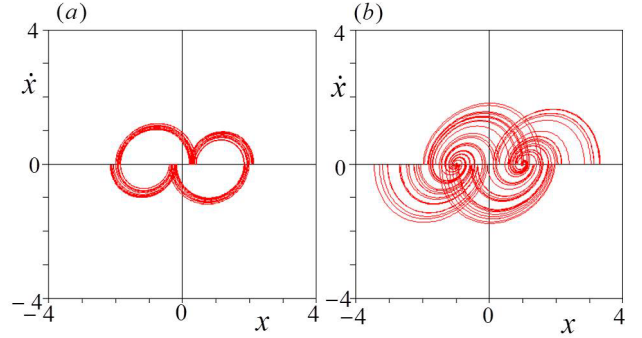


Figure 4: Trajectories. (a) Chaos for $\beta = \sqrt{2}$, (b) Super expanding chaos for $\beta = 2.6$

to $-p + 2nT$. The system repeats this switching. The trajectory does not diverge in this system if β is finite. The system can exhibit multi-screw chaotic attractor as shown in Fig. 3(a).

We propose the MPL on cylinder (CMPL). The dynamics of the CMPL is equivalent to the dynamics of the infinite-screw MPL.

If \mathbf{x} hits L_{n+} , the right hand side is switched to (A_n) and \mathbf{x} jumps to \mathbf{x}_0 on L_{0+} where $\mathbf{x}_0 \equiv (\mathbf{x} + T) \bmod (2T) - T$. If \mathbf{x} hits L_{n-} , the right hand side is switched to (B_n) and \mathbf{x} jumps to \mathbf{x}_0 on L_{0+} . The CMPL repeats this manner. Fig. 3 (b) illustrates switching of the CMPL. Note that the CMPL is characterized by three parameters: damping δ , the equilibrium point p and the circumference $2T$. For simplicity, we consider on the following parameter range:

$$1 < \beta < 3, \quad p = \frac{T}{2} = 1 \quad (8)$$

The CMPL exhibits various chaotic phenomena as shown in Fig. 4. The MPL for $\beta \geq 2$ as shown in Fig. 2(b). For $\beta \geq 2$, the MPL exhibits divergent

trajectory (Fig. 2 (b)), whereas the CMPL can exhibit chaos (Fig. 4 (b)), this is the super-expanding chaos characterized by very large Lyapunov exponent.

4. Laboratory Experiments

In order to observe the super expanding chaotic behavior, we have fabricated a simple test circuit as shown in Fig. 5. The test circuit of an equivalent to the CMPL. The test circuit is implemented by an OpAmp, a mono-stable multivibrator (M.M.), a flip-flop, and an analog switch. Here, v_1 and v_2 are capacitor voltages, V_{th} is a threshold voltage, and E is an equilibrium point voltage. If $v_1 < 0$ and $v_2 = 0$, an equilibrium point is switched from E to $-E$. If $v_1 > V_{th}$ and $v_2 = 0$, S_1 is closed. The capacitor voltage v_1 jumps to $v_1 - 2V_{th}$ and equilibrium point is switched from E to $-E$ simultaneously. Repeating in this manner, we can define the switching from $-E$ to E . This switching is equivalent to the CMPL. Figure 6 shows laboratory measurements. The data corresponds to Fig.4. We have verified the super expanding chaotic attractor for $\beta \geq 2$ in the laboratory

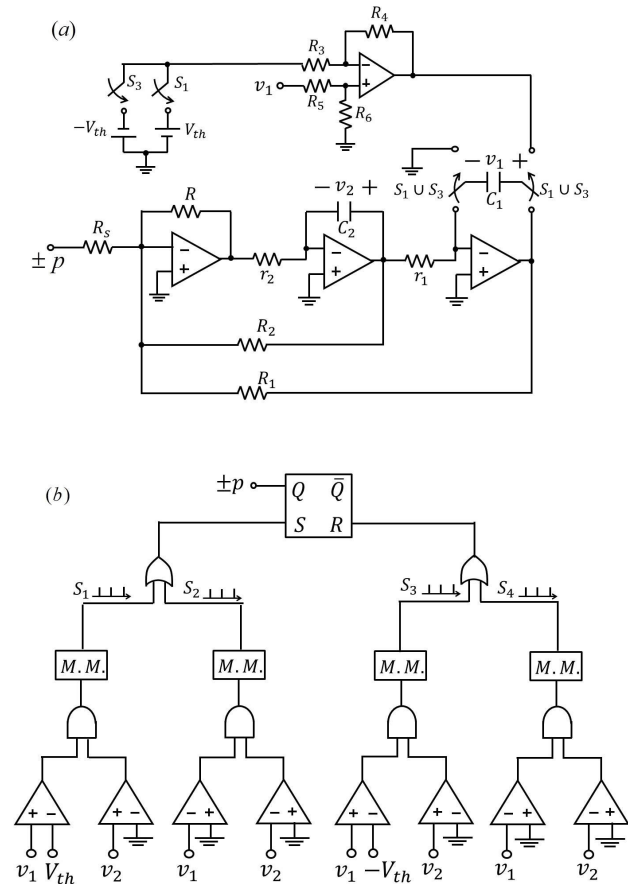


Figure 5: An implementation example. (a) Test circuit. (b) Switching circuit

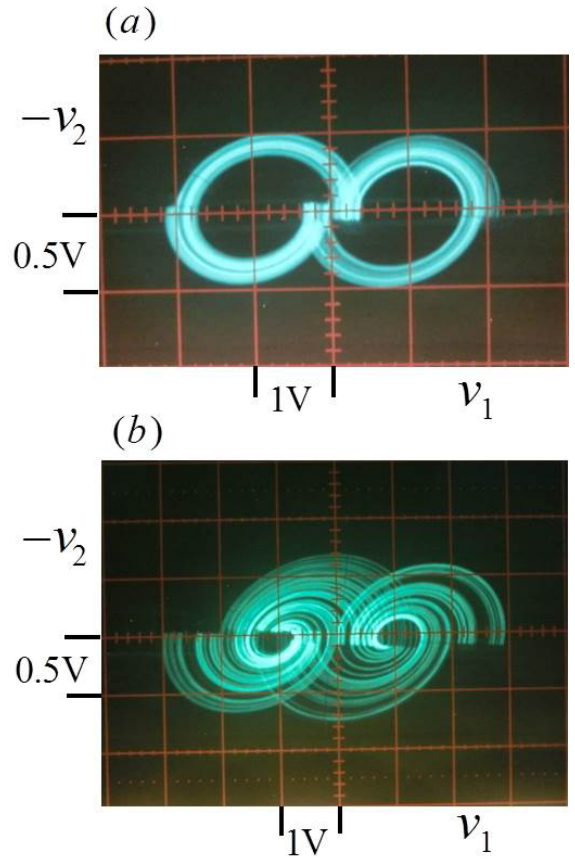


Figure 6: Laboratory measurements. $r_1 \simeq 1k\Omega$, $r_2 \simeq 1k\Omega$, $R_3 \simeq 1k\Omega$, $R_4 \simeq 5k\Omega$, $R_5 \simeq 5k\Omega$, $R_6 \simeq 1k\Omega$, $R_s \simeq 10k\Omega$, $C_1 \simeq 0.033\mu F$, $C_2 \simeq 0.033\mu F$, $V_{th} \simeq 2V$, $E \simeq 1V$ (a) Chaos for $\beta \simeq \sqrt{2}$, $R_1 \simeq 4.9k\Omega$, $R_2 \simeq 10k\Omega$ (b) Super expanding chaos for $\beta \simeq 0.26$, $R_1 \simeq 5.1k\Omega$, $R_2 \simeq 3.9k\Omega$

experiment as shown in Fig. 6(b).

5. Conclusions

In this paper, we have presented the CMPL. For $\beta \geq 2$, the trajectory diverges in the MPL. For $\beta \geq 2$, the CMPL exhibits super expanding chaos. The trajectory does not diverge on the cylinder if β is finite. The system can exhibit various chaotic attractor. Using a simple test circuit, super expanding chaos attractor can be verified in the laboratory experiment.

Future problems include analysis in wider parameter range, development into higher order systems and engineering applications.

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