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Liquid State Machine with Heterogeneous Connections for Information Networks

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Abstract—Liquid State Machine (LSM) is a recently proposed model of cortical computation. The model consists of a random network of large number of neurons interconnected with almost uniform connection strengths. Despite task-independent topology of the network , the LSM successfully performs them only if connection strengths to a readout unit are tuned properly. It implies that an information network based on LSM does not need costly topology maintenance. Recent experiments, however, revealed that strengths of synaptic connections in cortical network are far from uniform but widely distributed on highly heterogeneous distribution with a heavy tail, that can help reliable spike information transmission. In this paper, we first introduce highly heterogeneous connection strengths into the LSM and show that heterogeneity actually improves ability to store input sequences. We then limit the number of output neurons directly projecting to a readout unit. While it degrades the communication ability across the LSM, we can partly compensate for the degradation by utilizing finite history of states of projection neurons.

1. Introduction

Both our brain and information communication systems are a complicated network consisting of a huge number of devices. While both networks need to reliably transmit information across distances, their solution seems very different. In an information network, a router forwards packets to a carefully chosen downstream router to organize an optimal path to a destination. A neuron in the cortical network, however, sends spike trains to all of the connected postsynaptic neurons along almost random network topology [1]. Thus input information injected into the cortical network seems rapidly diffuse over the entire network without optimally delivered to a destination.

For information networks with extremely large number of devices, routing with optimal path to a destination can face difficulty because it requires a large amount of computational resources, communication overhead and energy consumption. This difficulty is especially crucial for wireless sensor network [2] with extremely small devices such as the smart dust [3] or nano-sensors [4] where computational and communication capability and energy consumption is tightly restricted due to limited power and space of devices. We thus need to develop novel information communication technologies without optimal routing for these applications. If we can apply information communication mechanisms of the brain that realized on random network topology without optimal routing to large-scale information network, it must offer fascinating opportunity for wireless sensor network.

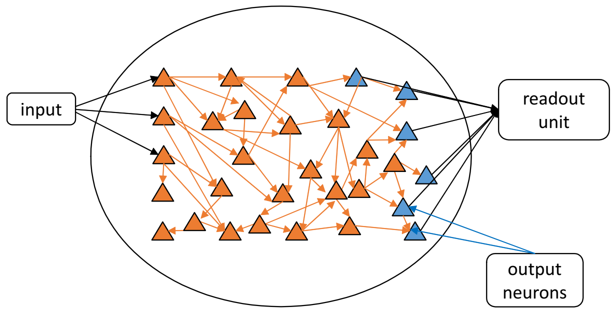


Figure 1: Liquid State Machine

In order to study the possibility of information communication on random networks without optimal routing, we focus on the recently proposed model of cerebellum cortex known as the Liquid State Machine (LSM) [5] that is also called as the Echo State Network [6] and the computation realized on the network is often denoted as the Reservoir Computation [7]. The liquid state machine consists of a randomly connected network of neurons (Fig. 1) called reservoir that models local circuit of cortex. Restricted numbers of neurons in the reservoir work as input neurons and directly receive input to the reservoir. Similarly, certain numbers of neurons work as output neurons and directly project output connection onto a readout unit. Note that, in the originally proposed LSM, all neurons in the reservoir work as output neuron. Interestingly, while input signals does not systematically transmitted to readout unit but randomly diffuse into the reservoir network, proper tuning of only connections onto the readout unit allows activity of readout unit reproduces input sequences even if connections inside the reservoir remain intact.

In the original LSM, strengths of synaptic connections among neurons in the reservoir are almost uniform. Recent biological experiments, however, reported that synaptic strengths within the cortical network is far from uniform but distributed on highly heterogeneous heavy-tailed distribution [8]. This means that a few connections are extremely strong while most of connections in the network are very weak. Recent theoretical studies reveals that the heterogeneity largely contributes to reliable spike information transmission among neurons [9].

In the paper, considering application of the LSM to information communication on networks with random topology, we first introduce heterogeneous connection strengths into the reservoir and show that the heterogeneity improves ability of the LSM to store input sequences, which is known as the most fundamental ability of the LSM [10]. In a case of a wireless sensor network, connection strengths determine virtual weights of data in processing at a receiver. LSM-based communication in a wireless sensor network can be regarded as in-network information processing. Then we reduce the number of output neurons and study how it influences on communication precision between input and the readout unit. If the reduction is successful, we can avoid deploying many output neurons, which correspond to sensors directly connected to a sink, in a wireless sensor network. While communication ability decreases with decreasing of the number of readout neurons, we show that the degradation can be partly recovered by the novel method of input estimation utilizing finite history of state of limited output neurons.

2. Model

In this section, we explain Liquid State Machine which is a subject of the research.

2.1. Liquid State Machine

Reservoir of the Liquid State Machine consists of $N = 500$ neurons whose dynamics are given as

$$x_i(t+1) = f\Big(\sum_{i=1}^{N} w_{ij}x_j(t) + w_i^{\mathrm{in}}s(t) - \theta\Big), \quad (1)$$

where $x_i(t)$ is the state of the ith neuron at time t, w_{ij} is the connection strength from the jth neuron to the ith neuron, $\theta = 100$ is the firing threshold of neurons. $f(x)$ is the step function defined as

$$f(x) = \begin{cases} 0 & (x < 0) \\ 1 & (x \geq 0) \end{cases} . \quad (2)$$

The value of the input signal at time t, $s(t)$, is randomly chosen from the uniform distribution ranging from 0 to 1. Connection strength from the signal to the ith neuron is w_i^{in}. The value of the w_i^{in} is zero excepting $N_{in} = 20$ input neurons. We chose nonzero values of w_i^{in} as

$$P(x) = \begin{cases} 0 & (w_i^{\mathrm{in}} < \theta) \\ \theta(w_i^{\mathrm{in}})^{-2} & (w_i^{\mathrm{in}} \geq \theta) \end{cases} , \quad (3)$$

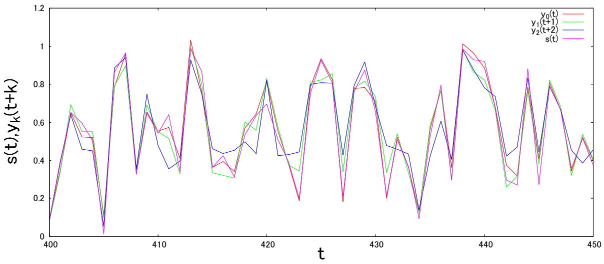


Figure 2: Input signal and estimated outputs

such that the average number of active neurons, or neurons whose state is one, is proportional to $s(t)$. This means input neurons represent input signals as average number of active neurons or population firing rate of them. While we choose this simplest input representation, other choices of input weight are also possible. Study with the other input representation will be an important future subject. The readout unit receives inputs from N_{out} output neurons to estimate sequences of input signals $s(t-k)$ as a weighted linear summation of them,

$$y_i(t) = \sum_{i \in \mathrm{output}} x_i(t) w_{ki}^{\mathrm{out}}. \quad (4)$$

Values of output weights, w_{ki}^{out}, are derived by the linear regression from only T_L training data

$$w_k^{\mathrm{out}} = (X^{\mathrm{T}}X + \lambda I)^{-1} X^{\mathrm{T}} s_k. \quad (5)$$

Eq(5) satisfies

$$\min[(y_k(t) - s(t-k))^2 + \lambda |w_k^{\mathrm{out}}|^2], \quad (6)$$

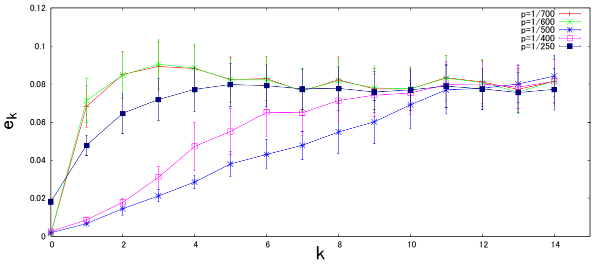
where s_k is a vector whose component $s_{ki} = s(i-k)$, X is the $T_{\mathrm{L}} \times (N_{\mathrm{out}} + 1)$ matrix whose component X_{ij} is $x_j(i)$ for $1 \leq j \leq N_{\mathrm{out}}$ and $X_{ij} = 1$ for $j = N_{\mathrm{out}} + 1$. $\lambda = 0.1$ is the regularization coefficient. Fig. 2 shows examples of input signals $s(t)$ and estimated output sequences $y_k(t)$. We can evaluate storage ability of the LSM using error of the signal estimation defined as mean square error e_k between $s(t-k)$ and $y_k(t)$:

$$e_k = \frac{1}{T} \sum_{t}^{T} (y_k(t) - s(t-k))^2. \quad (7)$$

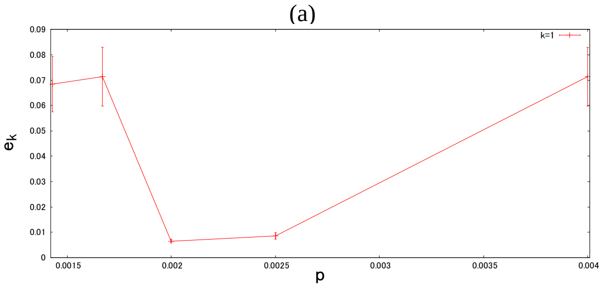
Here, average should be taken many sample data that include no training data. Because input information diffuse and decay in the reservoir, we can expect that e_k will increase as k increase. Similarly, in order to evaluate ability of LSM as an information communication channel we use minimum value of e_k over $k > 0$:

$$e = \min_k e_k. \quad (8)$$

Smaller value of e means that the readout unit can reconstruct input sequences more precisely, or much amount of information of input signals is transmitted from the input neurons to readout neurons through the randomly connected reservoir network.



(a)



(b)

Figure 3: (a)Estimation error as a function of k for various values of p. (b) Error e as a funcion of p

3. Results

In this section, we describe the contents of simulation performed and the results.

3.1. Heterogeneous distribution of connection strengths in the reservoir network

In order to study whether heterogeneous connection strengths improve ability of the LSM, we define random connection matrixes w_{ij} for the reservoir as

$$P(w_{ij}) = p\delta(w_0) + (1-p)\delta(0), \tag{9}$$

and change values of p and w_0 with keeping the product $pw_0 = c$ constant. As the constant value c, we use θ in order to ensure that neurons can turn to active state if all of presynaptic neurons are simultaneously active. If p is close to one, most of the reservoir neuron connected each other with the uniform θ/p strength. Contrary, if value of p closes to zero, the connection strengths are highly heterogeneous; most of connections in the reservoir vanish whereas only a few connections are extremely strong.

Fig. 3a shows e_k as functions of k for various values of p. We can see that e_k are increase function of k as expected. Also we find that values of p both close to one and very close to zero do not give good storage ability. To see the result clearly, we plot e, or minimum values of e_k over $k > 0$, as a function of p (Fig. 3b). The result clearly shows that p close to $1/N$ gives best storage ability. In the network with $p = 1/N$, each neuron averagely receives one input with strong connection and also send the received information to again averagely one neuron via very strong connections. Thus, signal information represented by population firing rate of the input neurons are most precisely transmitted into the reservoir without rapidly decaying or diverging.

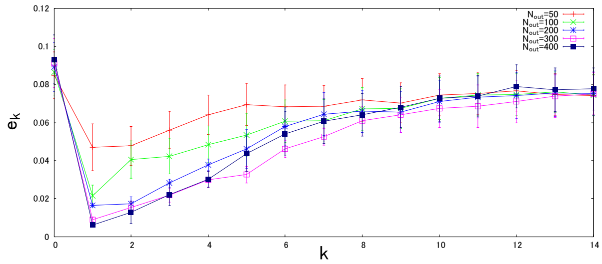


Figure 4: Estimation error for various values of N_{out}

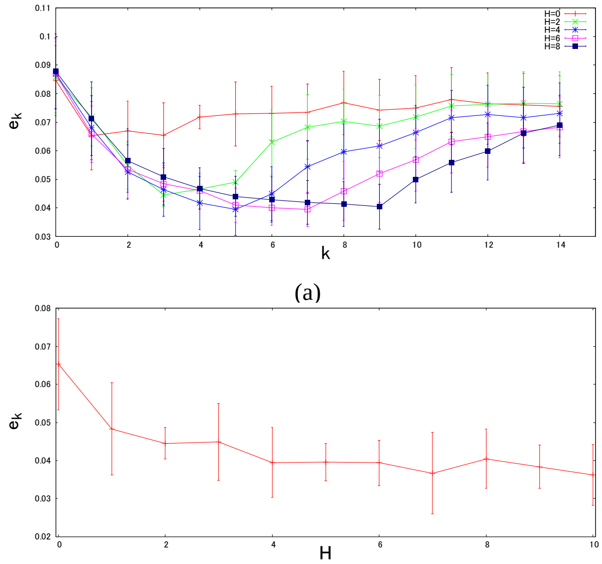
3.2. Liquid State Machine as an information communication network

Most of previous studies of the LSM, including the originally proposed model, assumed that population of the output neurons are the same with the entire population of the reservoir neurons. Thus, readout unit can access all of the neuron in the random reservoir network. This assumption is, however, clearly unsuitable when we discuss ability of the LSM as a communication network because a source of information is in generally in nonzero distance from destinations. Moreover, if we consider applications of the LSM to a sensor network, it is also unrealistic that a readout unit can directly access to entire sensors. Rather, it is reasonable assumption that a user or observer of the sensor network, corresponding to the readout unit, can directly access to states of only limited number of devices may be chosen randomly.

In order to see communication ability on the LSM and study effects of reduction of the number of output neurons, we numerically calculate e_k with changing the number of output neurons (Fig .4). Here, for simulation with $N_{out} < N$, we randomly chosen N_{out} output neurons from all reservoir neurons but input neurons exclusively. As decrease of the number of readout neurons, e_k almost monotonically increase for all k. Note that due to the exclusive choice of output neurons from input neurons, input signal at $k = 0$ have not reached to the output neurons yet. This is the reason why e_k for $k = 0$ is very high for $N_{out} < N$.

3.3. Input-signal estimation utilizing finite history of limited number of output neurons

In order to recover the reduction of the information communication capacity of the LSM due to limited number of output neurons, here we propose the novel estimation method of input signals as an alternative of eq.(5). Instead of estimating previous values of input signals $s(t-k)$ only from present values of output neurons $x_i(t)$, the proposed method use finite history of states of output neurons



(a)

(b)

Figure 5: (a)Estimation error for various values of H. (b) Error e as a funcion of H

$x_i(t-h)$, $0 \leq h \leq H$, to estimate $s(t-k)$ as

$$w_k^{\mathrm{out}} = (X^{\mathrm{T}}X + \lambda I)^{-1} X^{\mathrm{T}} s_k, \tag{10}$$

where X is a $T_{\mathrm{L}} \times (N_{\mathrm{out}}H + 1)$ matrix given as

$$\begin{cases} X = (X_0, X_1, \cdots, X_H, 1) \\ (X_h)_{ij} = x_j(i-h) \\ 1 = (1, 1, \cdots, 1)^{\mathrm{T}} \end{cases} . \tag{11}$$

The novel method can be easily implemented in information network including wireless sensor networks as far as required memory size that proportional with H is not very large.

Fig. 5a shows e_k as a function of k for various values of H. Fig. 5b showing e as function of H summarizes the result. We can see that by introducing finite length of memory, we can decrease estimation error again even for network with limited number of output neurons.

4. Conclusion

Here we discuss storage and communication ability of the Liquid State Machine. We find that heterogeneous connection strengths in the reservoir network improve storage ability of the reservoir probably because input signals neither decay nor diverge but stay longer in the network. We also propose the novel method of input-signal estimation for LSM with finite history of states of limited number of output neurons. By using certain length of history, we can partly compensate degraded communication ability of the LSM.

Acknowledgements

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