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Pseudo periodic orbits and their stabilization

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Abstract— In a chaotic attractor, some orbits looking like unstable periodic orbit are recognized. We call them pseudo periodic orbits and they are not proper periodic orbits but, we consider that they preserve similar properties to unstable periodic orbits. In this paper, we propose the method stabilizing the pseudo periodic orbit by using the external force control. Although, the pseudo periodic orbit is an approximative unstable periodic orbit but it is found easily without any analytic approaches. We demonstrate the chaos controlling by using the pseudo periodic orbit for the Duffing equation with hysteresis characteristics.

1. Introduction

The aim of the controlling chaos is to stabilize an unstable periodic orbit(UPO). Various applications have been studied as a category related controlling chaos[1–5]. As typical methods are OGY method[1], Delayed Feedback Control(DFC)[6][7] and External Force Control(EFC)[8]. These methods are equivalently represented by Eq.(1), where $\boldsymbol{u}(t)$ is the controlling signal.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), t) + \boldsymbol{u}(t). \tag{1}$$

The DFC feeds back the past state with a certain latency. The controlling signal $\boldsymbol{u}(t)$ is expressed by Eq.(2).

$$\boldsymbol{u}(t) = \boldsymbol{K}(\boldsymbol{x}(t-\tau) - \boldsymbol{x}(t)). \tag{2}$$

By applying of controlling signal, the controller can stabilize UPOs with the period τ for a chaotic attractor. When the controlling signal converges to zero, the behavior of system is periodic with cycle τ. The advantage of DFC is that necessary information is the delay time τ only. Additionally, this method is easy to provide by using a hardware memory. However, a disadvantage of DFC is that one must configure the control gain K by trial and error. If multiple UPOs with period τ exist in the given chaotic attractor, we cannot know which UPO will be stabilized.

The EFC is a modified version of the DFC. This controller feeds back time series of a UPO to the system. The controller is expressed by Eq.(3), and the block diagram is shown Fig. 1.

$$\boldsymbol{u}(t) = \boldsymbol{K}(\boldsymbol{G}(t) - \boldsymbol{x}(t)), \tag{3}$$

where, $\boldsymbol{G}(t)$ is the target orbit. The UPO sets as the target orbit. By doing this, EFC can stabilize the UPO. The advantages of EFC is a fast response and high robustness, but this controller requires the shape of the target orbit preliminary. The orbit time series should be prepared by using appropriate methods.

In this study, we propose the method stabilizing a pseudo periodic orbit by EFC. A pseudo periodic orbit is an approximated UPO and it can be found easily without the analytic technique. Firstly, the controller observes the chaotic system without the controlling, checks the distance between the current state and the last state on the Poincaré section. If the distance is short enough, this trajectory is defined as a pseudo periodic orbit. Note that, the orbit is similar to a closed curve. If the controller find the pseudo periodic orbit, the controlling is started, and the controller feeds back the pseudo periodic orbit as $\boldsymbol{G}(t)$. Therefore, it can be considered that our method is modification of the EFC. With this way, the controller can design without

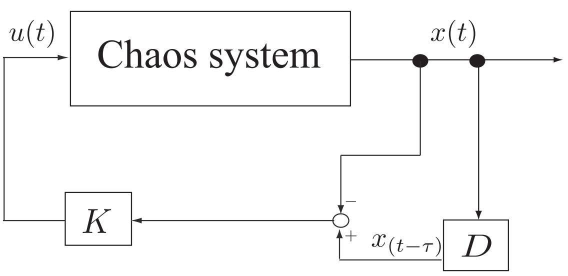

Fig. 1: Block diagram of the DFC

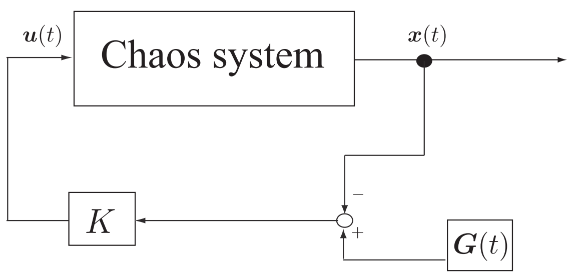

Fig. 2: Block diagram of the EFC

detail information of UPOs. Our method resolves the disadvantage of the EFC method. We demonstrate the chaos controlling with the pseudo periodic orbit for the Duffing equation with hysteresis characteristics.

2. Pseudo periodic orbits

We propose the method stabilizing a pseudo periodic orbit. As is well known that chaotic orbits are never return to the same point. In addition, chaotic orbits visit around the neighborhood orbit again of UPOs.

First of all, the controller observes the chaotic system without controlling. And, detect whether the current orbit visits around the neighborhood of the initial point on the Poincaré map. Note that, the orbit looks similar to a closed curve, but is not closed. We define the orbit as a pseudo periodic orbit. The Poincaré map is defined as Eq.(4).

$$\Pi = \{\boldsymbol{x} \in \boldsymbol{R}^n;\, q(\boldsymbol{x}) = 0\}. \tag{4}$$

When discretize chaotic orbits using Eq. (4), chaotic orbits are represented as a discrete map. If trajectory satisfies the condition $||\boldsymbol{x}_0 - \boldsymbol{x}_\tau|| \leq \epsilon, \boldsymbol{x}_0, \boldsymbol{x}_\tau \in \pi$, this is defined as the pseudo periodic orbit. This condition means the recursiveness of the chaotic attractor and express the phenomenon that pseudo periodic orbits go around again neighborhood of the points passed in the past. In this study, we employ $\epsilon = 10^{-2}$. Pseudo periodic orbits are provided as the reference orbit $\boldsymbol{G}(t)$ in EFC. Assume that any pseudo periodic orbit looks the certain UPO since it is in the given chaotic orbit and is almost a closed orbit. In the case of the 1-periodic orbit, the controller save the trajectory during that it reaches to the Poincaré section. If the mapped point is similar to the last one enough, the controller set the saved orbit as the target orbit $\boldsymbol{G}(t)$, and starts the controlling.

3. Result

We consider the Duffing equation with hysteresis characteristics. This equation is as follows:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -ky - f(x) + B_0 + B\cos t, \end{aligned} \tag{5}$$

where $\boldsymbol{x} = (x,\, y)^\top$ is state, k, B_0 and B are parameters. $f(x)$ is set the hysteresis characteristics. It is increased monotonically by odd function that satisfies the condition $f(x) \to \pm\infty (x \to \pm\infty)$.

Hysteresis characteristics are that high-power magnetic material has one of the feature. The high-power magnetic material is tend to magnet by the influence of the magnetic field. It is used the permanent magnet, electric magnet and core of the coil. The loss energy increase according to the gap of hysteresis. This is called hysteresis loss, and it is important evaluating, when one investigates the system include the hysteresis. Therefore, various model of the hysteresis have been studied, e. g. Stoner-Wohlfarth model[9] and Preisach model[10] as typical model. In this study, hysteresis characteristics are expressed by Eq (6).

$$f(x) = \begin{cases} x^3 - 1 \ (x \geq -1), \\ x^3 + 1 \ (x \leq 1). \end{cases} \tag{6}$$

Chaotic orbits are observed when parameters are set $k = 0.2$, $B_0 = 0.5$ and $B = 6.9$. Here, we define gain as $\boldsymbol{K} = I$. Figure 3 show controlling results of the 1-periodic orbit. It is confirmed that the chaotic attractor is controlled to the pseudo periodic orbit. Figure 4 show the time wave of the system and the amount of controlling signal. In Fig. 4, the gray region shows the search phase of the pseudo periodic orbit. In this phase, the controller observes the state of system only. At time, the controller finds the pseudo periodic orbit, and starts the controlling phase. The white region shows the controlling phase. The controller converge to the pseudo periodic orbit without transient variation because the system already behaves similar to $\boldsymbol{G}(t)$ when the controlling started. From the time wave of the amount of controlling, you notice that controlling signals are kept the small value. However, $\boldsymbol{u}(t)$ is not vanished finally, because the pseudo periodic orbit is not a genuine UPO. Our method can be controlled the another long periodic orbit by changing the judgment of the pseudo periodic orbit, e. g. the controller can be applied to 3-periodic orbit by checking the error on the Poincaré map with respect three times. In this way, our method can apply to any periodic orbits. Figures 5-10 show controlling result for 3 and 5-periodic orbits. They also can be performed similarly. Additionally, if the controlling gain $\boldsymbol{K}$ is changed, our method can stabilizing the system. By these results, it can be said that our method has a high performance and robustness.

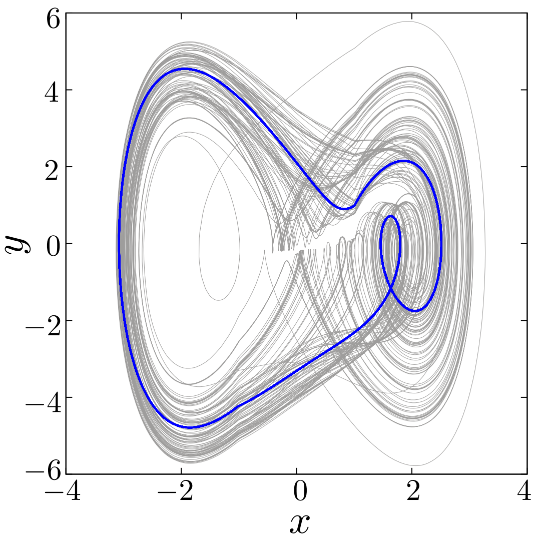

Fig. 3: Controlling result of pseudo periodic orbits

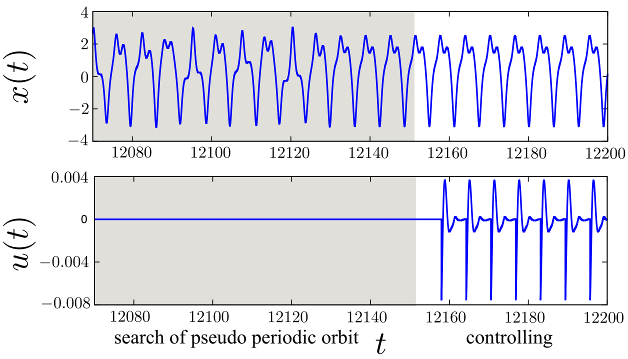

Fig. 4: Time wave of the system and time response of the controlling signal Fig. 3

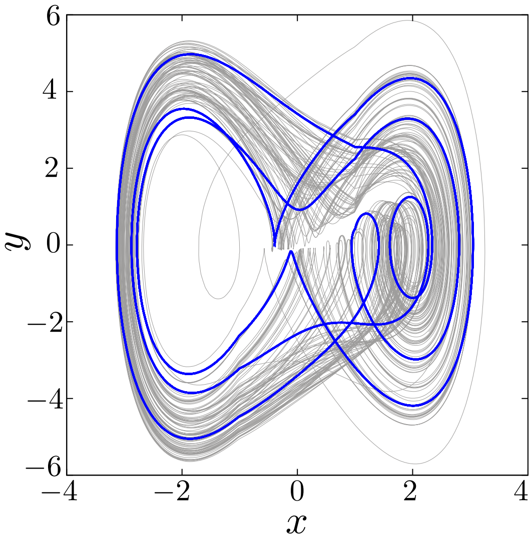

Fig. 5: Controlling result of pseudo periodic orbits in the three cycle

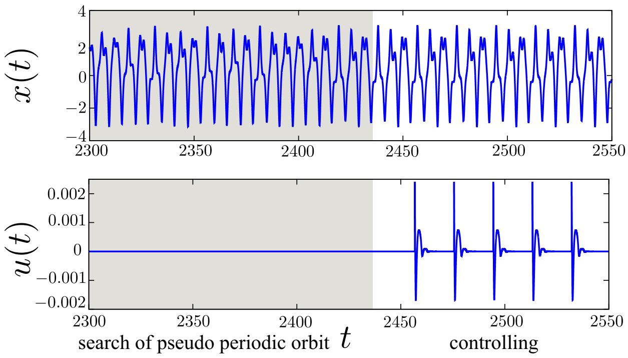

Fig. 6: Time wave of the system and time response of the controlling signal Fig. 5

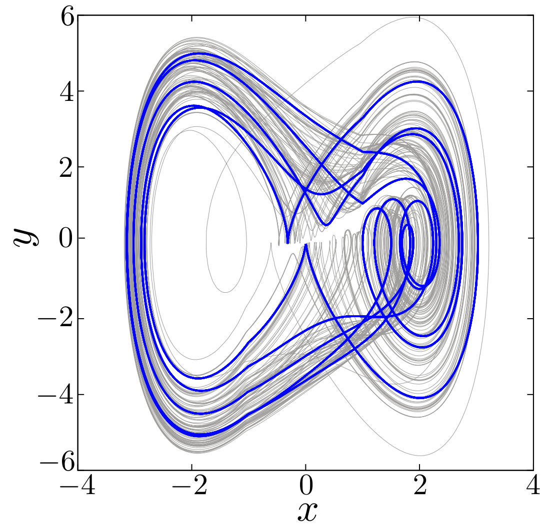

Fig. 7: Controlling result of pseudo periodic orbits in the five cycle

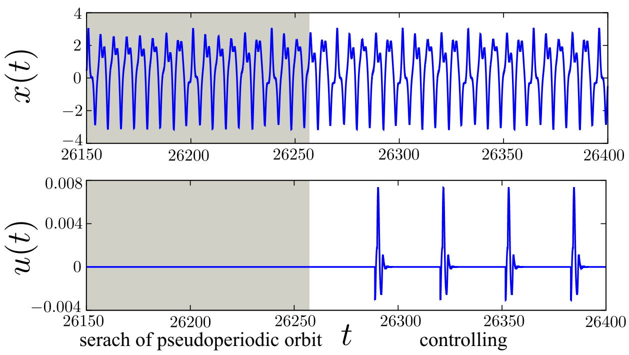

Fig. 8: Time wave of the system and time response of the controlling signal Fig. 7

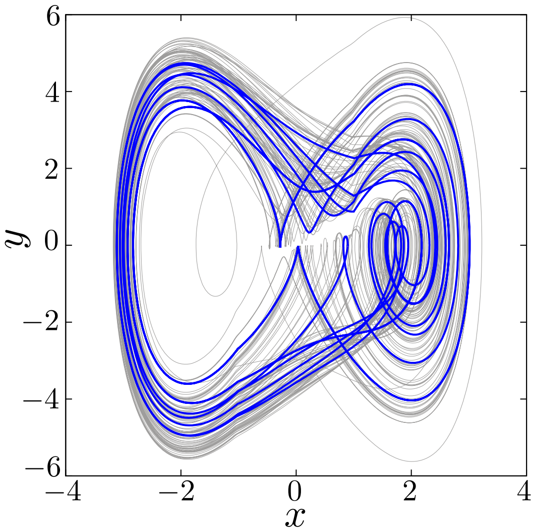

Fig. 9: Controlling result of pseudo periodic orbits in the seven cycle

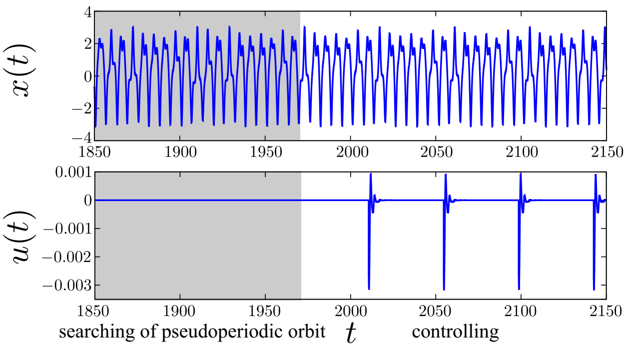

Fig. 10: Time wave of the system and time response of the controlling signal Fig. 9

4. Conclusion

In this study, we propose the method stabilizing the pseudo periodic orbit by using EFC method. When the system shows the recurrence property, the pseudo periodic orbit that is not closed loop is appeared. Pseudo periodic orbits are provided as the reference orbit $\boldsymbol{G}(t)$ in EFC. The EFC method has the disadvantage that the UPO is not obtained easily. However, the pseudo periodic orbit is the approximated UPO and it can be found easily without the analytical approach. We have used the Duffing equation with hysteresis characteristics as an example, and show controlling results by numerical simulation. It is confirmed that our controller can control the chaotic system by controlling with pseudo periodic orbit. In addition, our method can also control for long period UPOs such as 3, 5 and 7-periodic orbit.

Acknowledgement

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