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Intrinsic Localized Modes in a Magnetically Coupled Two-degree-of-freedom Resonator Array

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Summary

Nonlinearly coupled resonator array is known to have spatially localized and temporally periodic solutions which are called intrinsic localized modes (ILMs) or discrete breathers (DBs)[1]. It is well known that the ILM/DB exist not only in one-dimensional array but also in two- or three-dimensional lattice[2]. Many studies of ILM/DB for one-dimensional arrays assume the array is rigid *i.e.* each oscillator moves only along the axis of the array.

A flexible nonlinear chain in which each oscillator can move two-dimensionally is known to have two types of intrinsic localized mode(ILM). One is the same as the traditional ILM that oscillates along the chain axis. We call it longitudinal ILM (L-ILM). The other mainly oscillates perpendicularly, which is called transverse ILM (T-ILM). In a flexible Fermi–Pasta–Ulam chain which has cubic nonlinearity in the coupling force, L-ILM loses its stability because of the buckling effect of the chain[3]. For T-ILM, almost all the solutions are unstable when the chain is not stretched[4]. If the chain has nonlinearity in the on-site restoring force, stable L- and T-ILM can be found for certain regions in the ω-β plane where ω is the angular frequency of ILM and β is the ratio of on-site and inter-site nonlinearity[5].

A magnetically coupled two-degree-of-freedom resonator array is designed for experimental studies for L- and T-ILM in a flexible chain. An elastic rod with a cylindrical permanent magnet at the tip is a two-degree-of-freedom oscillator. The permanent magnet induces nonlinearity in the coupling force of adjacent oscillators. In this study, the magnets are attached to generate repulsive force and it becomes stronger if the distance between adjacent oscillators becomes closer. Then the nonlinearity of the coupling force is classified as hard spring nonlinearity. ILM will appear above the upper bound of the phonon band. If the edge of the array is sinusoidally excited by an external exciter at a frequency higher than the upper bound, several moving L-ILMs appear and travel along the chain[6, 7]. The moving L-ILMs can keep their shape of localized energy distribution unless the excitation amplitude is rather small. However, if the amplitude becomes large, transverse oscillations are induced and the moving L-ILMs become

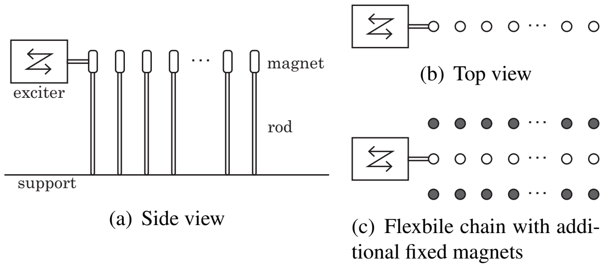

Figure 1: Magnetically coupled two-degree-of-freedom resonator array

unstable. The instability will come from the flexibility of the chain. There is no force to prevent the deformation of the flexible chain except the restoring force of the rod. Additional permanent magnet arrays are placed parallel to the flexible chain. Nonlinearity is added to the restoring force against the transverse displacement. In this report, the stability of static L- and T-ILMs is investigated for the flexible chains. We will discuss the effect of the surrounding resonator arrays on the stability of ILMs.

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