Heterogenous cluster formation in the activity-dependent time-varying network

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Abstract—In this work, we investigated the organization of activity-dependent time-varying network of coupled phase oscillators, where the coupling weights dynamically change depending on the relative timings between the oscillators. We determined the phase coupling function $\Gamma(\phi)$ of the oscillator model using conductance-based neuron models and examined the effects of the Fourier zero mode of $\Gamma(\phi)$. We demonstrated that heterogeneous layered clusters with different frequencies emerge from homogeneous populations as the Fourier zero mode increased. Our findings would provide new insights into the self-assembly mechanisms of neural networks related to synaptic plasticity.

1. Introduction

Synaptic plasticity plays a vital role in learning in the brain, and it has been intensively investigated to understand the mechanism underlying learning. It induces changes in the structures of synaptic connections associated with neuronal activity, facilitating the organization of memory-related functional neural assemblies [1]. Recent neurophysiological experiments revealed that changes in synaptic connections depend on the relative spike timing between neurons during spike-timing-dependent plasticity (STDP) [2–4]. This observation implies that the temporal spike patterns of neurons determine synaptic patterns, raising the question of how STDP organizes neural networks into functional neuronal assemblies. This query remains an open question in the field of theoretical neuroscience, particularly when a network has rich recurrent connections.

Several numerical studies reported that STDP-organized recurrent networks exhibit interesting behaviors, including the emergence of clusters with neurons that fire synchronously [5–10] and feed-forward networks [11–15].

The interplay between neurons and their synapses makes it difficult to analyze the dynamics of STDP-organized recurrent networks. In the presence of plasticity, the spike pattern alters the structure of the synaptic connections, resulting in the formation of new spike patterns. In other words, the synaptic connections and neuronal activities evolve simultaneously. To elucidate the essential nature of this co-evolving neural network dynamics, we developed a simple, co-evolving dynamical model of neuronal oscillators [16, 17]. In this talk, we consider the Fourier zero mode of the phase coupling function of the oscillator. Previous studies on phase oscillator have ignored this mode. However, we find that this constant term is critically important for STDP-organized recurrent networks, as evidenced by the result that heterogeneous layered clusters with different frequencies emerge from homogeneous populations with identical natural frequencies.

2. Model

We consider the following equation of a coupled dynamical system:

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) + \sum_j \mathbf{f}_{ij}(\mathbf{x}_i, \mathbf{x}_j), \tag{1}$$

where, $\mathbf{x}_i$ denotes the state of the i-th neuron in a network of N neurons. The first term describes the intrinsic dynamics of the neurons (e.g. several types of ion channels) and the second term describes coupling with other neurons via synapses.

The activity of a neuron is assumed to be oscillatory, rather than random. Thus, we consider that the neuron model undergoes a limit-cycle oscillation, which is perturbed by synaptic inputs and noises. This assumption enables us to reduce the description of the neuron to a simple form with the variable, ϕ. Using a standard reduction technique [18], the coupled limit-cycle system can be described as follows:

$$\frac{d\phi_i}{dt} = \omega_i + \frac{1}{N} \sum_j^N k_{ij} \Gamma(\phi_i - \phi_j), \tag{2}$$

where, ϕ_i denotes the phase of the limit-cycle oscillation of the i-th neuron in the network ($i = 1, \ldots N$), ω_i is its natural frequency, and k_{ij} is the coupling weight of the connection from the j-th to the i-th neuron. The coupling function $\Gamma(\phi)$ is a 2π-periodic function.

Next, we introduce the dynamics of the synaptic weights due to the plasticity. The evolution of the weights depends on the relative timing between the neurons, similar to the case with STDP.

$$\frac{dk_{ij}}{dt} = \epsilon \Lambda(\phi_i - \phi_j), \quad |k_{ij}| \le 1. \tag{3}$$

The function $\Lambda(\phi)$, which we refer to as a learning function, determines the evolution of the weights. The learning parameter ϵ has a very small value because the dynamics of the synaptic weights are much slower than those of the neurons. The condition $|k_{ij}| \leq 1$ means that the synaptic weight is bounded. If the weight has a value outside [1,1], then the weight is immediately set to the appropriate bounded value (1 or 1). This rule is reasonable because the weight cannot increase indefinitely.

The learning function $\Lambda(\phi)$ is periodic. Therefore, for the sake of simplicity, we assume that $\Lambda(\phi)$ takes the form:

$$\Lambda(\phi) = -\sin(\phi_i - \phi_j + \beta) \tag{4}$$

where β is the shift parameter that characterizes the learning function (top panels in Fig. 1). For example, when $\beta \sim -\pi/2$, the weights for a pair of in-phase (or anti-phase) neurons will increase (or decrease). This relationship can be considered as a Hebbian-like rule. When $\beta \sim 0$, the dependency on the relative timing becomes similar to temporally asymmetric Hebbian rule. When $\beta \sim \pi/2$, the learning function has the opposite form to the Hebbian-like rule.

In this talk, we consider the effect of the constant term of the coupling function $\Gamma(\phi)$ [19]. This constant term has been ignored in previous studies of Kuramoto phase oscillators because it can be absorbed into the natural frequency term:

$$\frac{d\phi_i}{dt} = \omega_i + \frac{1}{N}\sum k_{ij}\Gamma(\phi_i - \phi_j) \tag{5}$$

$$= \omega_i + \frac{1}{N}\sum k_{ij}\left[\Gamma_0 - \sin(\phi_i - \phi_j + \alpha) + \cdots\right] \tag{6}$$

$$= \tilde{\omega}_i - \frac{1}{N}\sum k_{ij}\left[\sin(\phi_i - \phi_j + \alpha) + \cdots\right], \tag{7}$$

where

$$\tilde{\omega}_i = \omega_i + \frac{\Gamma_0}{N}\sum k_{ij}. \tag{8}$$

In an adaptive network with synaptic plasticity, however, the frequency $\tilde{\omega}_i$ depends on the coupling strength k_{ij}, which changes dynamically over time. In other words, Γ_0 can provide a mechanism for the adaptive control of the frequency during synaptic plasticity. Therefore, we examined the effects of the constant term Γ_0 on the network organization due to the synaptic plasticity using the following equations:

$$\frac{d\phi_i}{dt} = \omega + \frac{1}{N}\sum k_{ij}\left[\Gamma_0 - \sin(\phi_i - \phi_j + \alpha)\right] \tag{9}$$

$$\frac{dk_{ij}}{dt} = -\epsilon\sin(\phi_i - \phi_j + \beta), \quad |k_{ij}| \leq 1. \tag{10}$$

The oscillators were assumed to have identical natural frequencies.

3. Results: Emergence of heterogeneous layered clusters from homogeneous populations

When $\Gamma_0 \sim 0$, the model given by the above equations exhibits the same behavior as that described in [16, 17]. As Γ_0 increases, a novel type of network organization emerges at $\beta \in (-0.3\pi, 0.1\pi)$, with a coherent state being observed at $\Gamma_0 = 0$.

When $\Gamma_0 = 0.3$, layered clusters of the same frequency are organized (Fig. **??**). The order parameters converge to nonzero values, and the normalized rate of change of the coupling weights reaches zero, indicating a fixed structure of the phase pattern and the coupling weights. the order parameters is given by: $R_m = |\frac{1}{N}\sum_j e^{im\phi_j}|$, and the normalized rate of change in the weights, averaged over all connections, given by $\Delta K(t) = \frac{1}{N(N-1)}\sum_{i\neq j}\frac{|k_{ij}(t)-k_{ij}(t-\Delta)|}{\Delta}$. The sampling interval is $\Delta \sim \frac{2\pi}{\omega} \ll \frac{1}{\epsilon}$. The raster plot and phase distribution demonstrate the organization of several clusters of synchronized oscillators. The phase differences between the clusters are locked, and their frequencies are the same. A type of layered connection is organized throughout this phase pattern, in which neurons within a cluster are symmetrically connected via positive couplings. Connections between the clusters become asymmetric according to their phase differences.

As Γ_0 increases ($\Gamma_0 = 0.5$), the state of layered clusters with the same frequency transitions to another type of layered clusters (Fig. 1). The order parameters and the normalized rate of change of the coupling weights do not converge to fixed values, and they oscillate quasi-periodically (Fig. 1 (a)). The global synchronization is broken, and multiple clusters with different frequencies emerge. In fact, the raster plot clearly shows that the oscillators are organized into several synchronized clusters, and the clusters are not phase-locked (Fig. 1 (c)). These clusters have different frequencies as indicated by the several peaks in the frequency distribution (Fig. 1 (d)). Moreover, the phase distribution has several peaks indicating the emergence of multiple synchronized clusters (Fig. 1 (e)). The coupling weights indicate that the layered connection type is the same as when $\Gamma_0 = 0.3$ (Fig. 1 (f)). Note that the oscillators have the same natural frequencies. The heterogeneity of their frequencies is caused by the organization of the neural network as a result of the synaptic plasticity.

4. Discussions

In this study, we investigated a network of coupled neural oscillators in which the connections between the oscillators change dynamically due to synaptic plasticity. We demonstrated that the co-evolving dynamics of the connections and neural oscillators lead to the emergence of several specific types of structured connections and neural activities. We examined the effects of the Fourier zero mode of the phase coupling function $\Gamma(\phi)$. This constant Γ_0 has been ignored in previous studies of the Kuramoto model, but it has significant effects on the organization of adaptive networks due to synaptic plasticity. For example, we demonstrated that heterogeneous layered clusters with different frequencies emerge from a homogeneous population of neurons as Γ_0 increases.
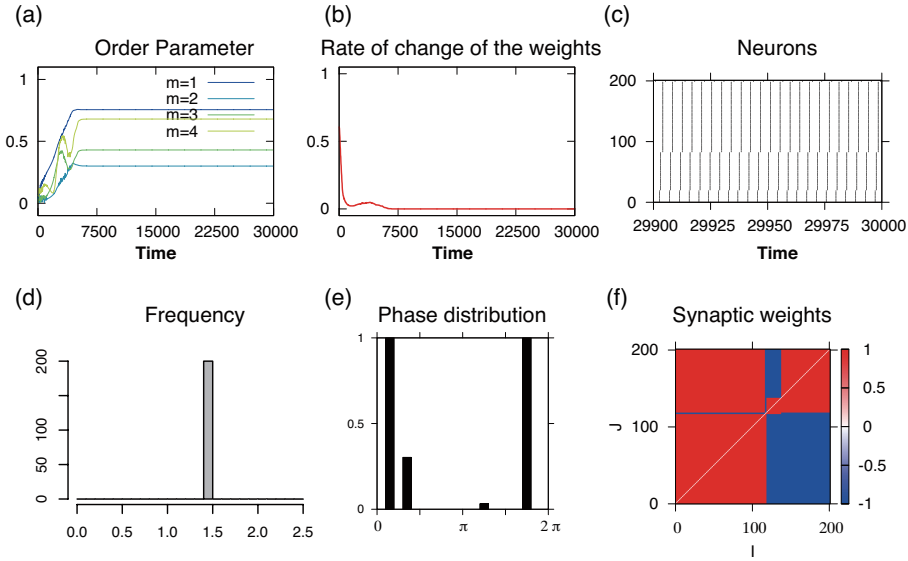
Figure 1: Emergence of layered clusters with the same frequency, where $\alpha = 0.1\pi$, $\beta = -0.1\pi$ and $\Gamma_0 = 0.3$. Time development of (a) the order parameters and (b) the normalized rate of change of the coupling weights ΔK. (c) Raster plot of the stationary state. Index i of the neurons is arranged in order of increasing phase ϕ_i. (d) Frequency and (e) phase distribution at $t = 30,000$. (f) Matrix of synaptic weights k_{ij} at $t = 30,000$.

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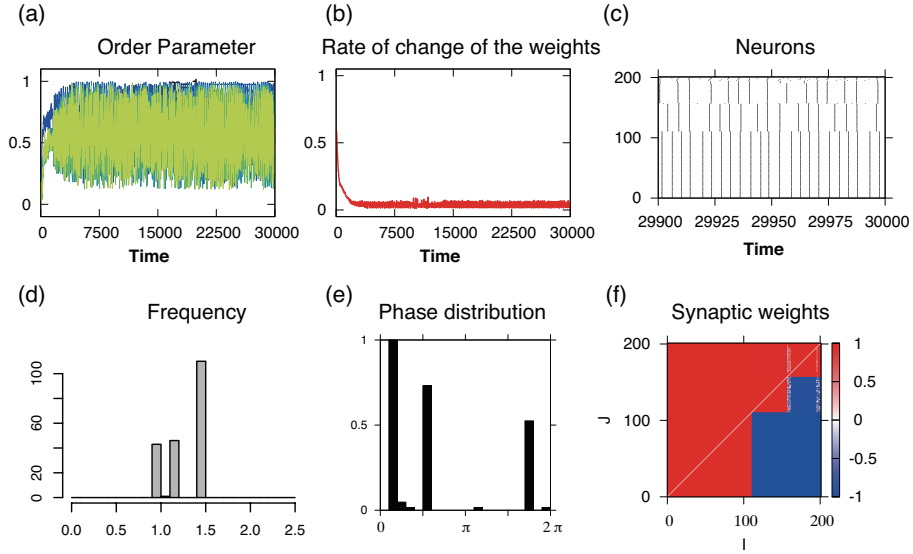
Figure 2: Emergence of layered clusters with different frequencies, where $\alpha = 0.1\pi$, $\beta = -0.1\pi$ and $\Gamma_0 = 0.5$. Time development of (a) the order parameters and (b) the normalized rate of change of the coupling weights ΔK. (c) Raster plot of the stationary state. Index i of the neurons is arranged in order of increasing phase ϕ_i. (d) Frequency and (e) phase distribution at $t = 30,000$. (f) Matrix of the synaptic weights k_{ij} at $t = 30,000$.

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