# Analysis of Phase-Inversion Waves on In-And-Anti-Phase Synchronization in a Ladder Oscillator by Using Instantaneous Electric Power

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**Abstract**—We can observe a synchronization state that in-phase synchronizations and anti-phase synchronizations alternately exist on a coupled oscillators system as a ladder. We call the synchronization state an in-and-anti-phase synchronization. A wave-motion, which propagates and switches phase states between adjacent oscillators from the in-phase synchronization to the anti-phase synchronization or from the anti-phase synchronization to the in-phase synchronization, can be observed on the in-and-anti-phase synchronization. The wave-motion is called phase-inversion waves. In this study, we analyze the phase-inversion waves on the in-and-anti-phase synchronization by using instantaneous electric power, and make clear characteristics of the phase-inversion waves on the in-and-anti-phase synchronization.

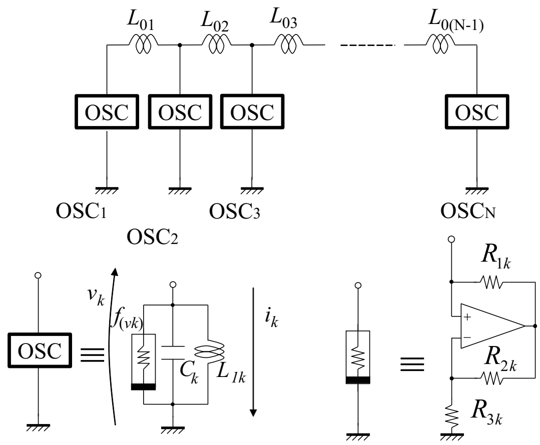


Figure 1: Circuit model.

## 1. Introduction

Nowadays, synchronization phenomena are attracting attentions in many fields [1], because a lot of synchronizations are observed in creations, the outer space, the atomic world, and so on. Furthermore, synchronization phenomena are used for industrial products which are the communication systems, the laser, and so on. In other words, we can not live without synchronization phenomena. The synchronization phenomena can be naturally observed in coupled oscillators systems. The in-phase synchronizations and the anti-phase synchronizations can be observed in the coupled oscillators systems that many van der Pol oscillators are coupled by inductors as a ladder, a 2D lattice or a 3D lattice. Furthermore, a synchronization phenomenon, that the in-phase synchronization phenomena and the anti-phase synchronization phenomena are alternately existing, can be observed in the ladder systems, the 2D lattice systems, and the 3D lattice systems. We call the synchronization phenomenon an in-and-anti-phase synchronization. A wave-motion which is propagating and switching phase states between adjacent oscillators from the in-phase synchronization to the anti-phase synchronization or from the anti-phase synchronization to the in-phase synchronization can be observed on the in-phase synchronization and the in-and-anti-phase synchronization in the ladder systems, the 2D lattice systems, and the 3D lattice systems [2]. We call the wave-motion phase-inversion waves. The phase-inversion waves can be observed in steady states. On the other hand, waves that phase differences between adjacent oscillators propagate in transient states can be observed. The waves are called phase-waves [3]-[4].

In our previous study, we analyzed the phase-waves and the phase-inversion waves on the in-phase synchronization in the ladder systems by using instantaneous electric powers. We investigated itinerancies of the instantaneous electric power of the phase-waves and of the phase-inversion waves, and clarified clear differences between the phase-waves and the phase-inversion waves.

In this study, itinerancies of the instantaneous electric powers of phase-waves and of phase-inversion waves are investigated on in-and-anti-phase synchronization. We make clear differences between phase-waves and phase-inversion waves. Furthermore, these phase-inversion waves on the in-and-anti-phase synchronization are compared from phase-inversion waves on the in-phase synchronization which are obtained by using a simulation and an actual experiment.

## 2. Circuit model

We show the circuit model of this study is shown in Fig. 1. The van der Pol oscillators are coupled by inductors as a ladder. The number of oscillators is assumed as "$N$." Each oscillator is named as OSC$_k$. A voltage of each oscillator is named $v_k$, and a current of an inductor of each oscillator is named $i_k$. An equation of the nonlinear negative resistor is shown as Eq. (1). The circuit equations are normalized by Eq. (2), and normalized equations are shown in Eqs. (3)–(4).

$$f(v_k) = -g_1 v_k + g_3 v_k^3 \qquad (g_1, g_3 > 0). \tag{1}$$

$$i_k = \sqrt{\frac{Cg_1}{3Lg_3}}x_k,\ v_k = \sqrt{\frac{g_1}{3g_3}}y_k,\ t = \tau\sqrt{L_1 C_\tau},$$

$$\alpha = \frac{L}{L_0},\ \varepsilon = g_1\sqrt{\frac{L}{C}}. \tag{2}$$

[Left Edge Oscillator] ($k = 1$).

$$\frac{dx_k}{d\tau} = y_k, \tag{3}$$

$$\frac{dy_k}{d\tau} = -x_k + \alpha\{x_{k+1} - x_k\} + \varepsilon(y_k - \frac{1}{3}y_k^3). \tag{4}$$

[Middle Oscillators] ($1 < k < N$).

$$\frac{dx_k}{d\tau} = y_k, \tag{5}$$

$$\frac{dy_k}{d\tau} = -x_k + \alpha\{x_{k+1} - 2x_k + x_{k-1}\} + \varepsilon(y_k - \frac{1}{3}y_k^3). \tag{6}$$

[Right Edge Oscillator] ($k = N$).

$$\frac{dx_k}{d\tau} = y_k, \tag{7}$$

$$\frac{dy_k}{d\tau} = -x_k + \alpha\{x_{k-1} - x_k\} + \varepsilon(y_k - \frac{1}{3}y_k^3). \tag{8}$$

Instantaneous electric power of each oscillator is named $P_k$. Instantaneous electric power of each coupling inductor $L_{0m}$ is named $P_{L0m}$. Instantaneous electric power of each inductor $L_{1k}$ and instantaneous electric power of each capacitor $C_k$ are named $P_{L1k}$ and $P_{Ck}$ respectively. Instantaneous electric power of each nonlinear negative resistor $f(v_k)$ is named $P_{ngk}$. A value of $P_k$ equal the sum of $P_{L1k}$, $P_{Ck}$, and $P_{ngk}$. These powers are calculated by using Eqs. (9)–(17).

[Coupling Inductors] ($1 \leq m \leq N-1$).

$$P_{L0m} = \delta\frac{1}{\varepsilon}\alpha(x_{m+1} - x_m)(y_{m+1} - y_m). \tag{9}$$

[Capacitor of Left Edge Oscillator].

$$P_{C1} = \delta\frac{1}{\varepsilon}y_1\{\alpha(x_2 - x_1) - x_1 - \varepsilon(\frac{1}{3}y_1^3 - y_1)\}. \tag{10}$$

[Capacitor of Middle Oscillators] ($2 \leq k \leq N-1$).

$$\begin{aligned} P_{Ck} = \ & \delta\tfrac{1}{\varepsilon}y_k\{\alpha(x_{k+1} - 2x_k + x_{k-1}) - x_k \\ & -\varepsilon(\tfrac{1}{3}y_k^3 - y_k)\}. \end{aligned} \tag{11}$$

[Capacitor of Right Edge Oscillator].

$$P_{CN} = -\delta\frac{1}{\varepsilon}y_N\{\alpha(x_N + x_{N-1}) - x_N - \varepsilon(\frac{1}{3}y_N^3 - y_N)\}. \tag{12}$$

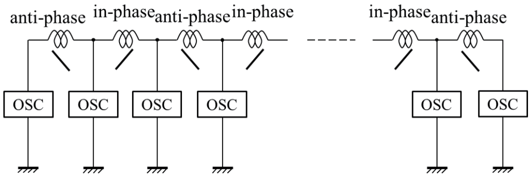


Figure 2: In-and-anti-phase synchronization.

[Inductors of Each Oscillator] ($1 \leq k \leq N$).

$$P_{L1k} = \delta\frac{1}{\varepsilon}x_k y_k. \tag{13}$$

[Nonlinear Negative Resistors of Each Oscillator] ($1 \leq k \leq N$).

$$P_{ngk} = \delta\alpha y_k(\frac{1}{3}y_k^3 - y_k). \tag{14}$$

[Left Edge Oscillator].

$$P_1 = \delta\frac{1}{\varepsilon}\alpha(x_2 - x_1). \tag{15}$$

[Middle Oscillators] ($2 \leq k \leq N-1$).

$$P_k = \delta\frac{1}{\varepsilon}\alpha(x_{k+1} - 2x_k + x_{k-1}). \tag{16}$$

[Right Edge Oscillator].

$$P_N = -\delta\frac{1}{\varepsilon}\alpha(x_N - x_{N-1}). \tag{17}$$

The $\alpha$ shows coupling parameter and the $\varepsilon$ expresses nonlinearity. The $\delta$ shows amplitude scale of instantaneous electric power.

## 3. Phase-inversion waves on the in-and-anti-phase synchronization.

We observe the phase-inversion waves propagates on an in-and-anti-phase synchronization in a ladder, and comparing the phase-inversion waves and the phase-waves.

<In-and-anti-phase synchronization.>
The in-and-anti-phase synchronization in a ladder is shown in Fig. 2. The in-phase synchronization and the anti-phase synchronization are alternately existing. When the phase states at the edge are the anti-phase synchronization, the in-and-anti-phase synchronization is stable.

<Observation conditions.>
We show the phase-waves and the phase-inversion waves in Figs. 3 and 4 respectively. A set of parameters of which phase-waves are observed is called Pattern-A, and a set of parameters of which phase-inversion waves are observed is called Pattern-B. In this study, we set the following five observation conditions.

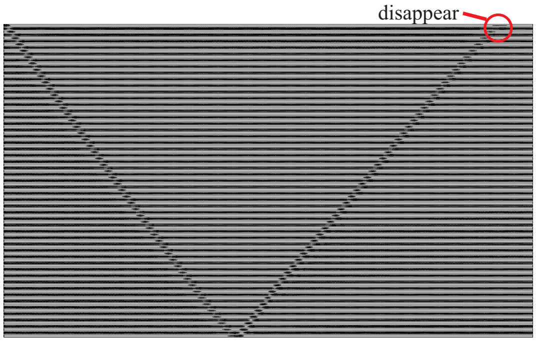


Figure 3: Phase-waves in 100 oscillators.

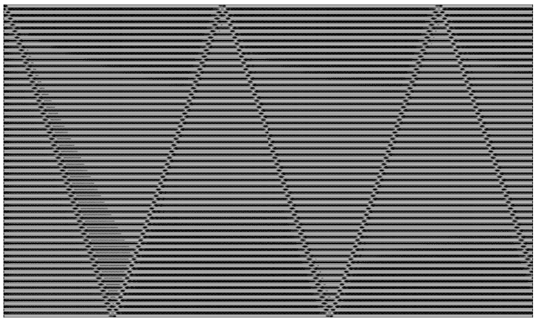

Figure 4: Phase-inversion waves in 100 oscillators.

1. $N$ = 100.
2. $\delta$ = 1.
3. Pattern-A : $\alpha$ is 0.06 and $\varepsilon$ is 0.30.
   Pattern-B : $\alpha$ is 0.10 and $\varepsilon$ is 0.10.
4. A basic phase state is fixed as the in-and-anti-phase synchronization.
5. The waves are generated by which a phase state between $OSC_1$ and $OSC_2$ is set the in-phase synchronization suddenly.

In the Figs. 3 and 4, 99 rectangular boxes are piled up vertically. Each box shows phase differences of between adjacent oscillators. Top of the box shows the phase state between $OSC_1$ and $OSC_2$, and bottom of the box shows the phase state between $OSC_{99}$ and $OSC_{100}$. The vertical axis is a sum of voltages of two adjacent oscillators and the horizontal axis is time in each box. In other words, the black areas are shown the in-phase synchronization and white areas are shown the anti-phase synchronization. In the Fig. 3, the phase-waves reflect at the bottom and disappear at the top. In the Fig. 4, we can observe the phase-inversion waves which are existing.

<Instantaneous electric powers.>
Instantaneous electric powers of $OSC_{19}$ of above result are shown in Figs. 5 and 6 respectively. In the Figs. 5 and 6, the vertical axes are instantaneous electric power and the horizontal axes are time. The dotted boxes of the Figs. 5 and 6 are closed up and shown in Figs. 7 and 8 respectively. Only one peak can be observed in propagating the phase-waves(see Fig. 7). However, two peaks can be observed when the phase-inversion waves propagating(see Fig. 8).

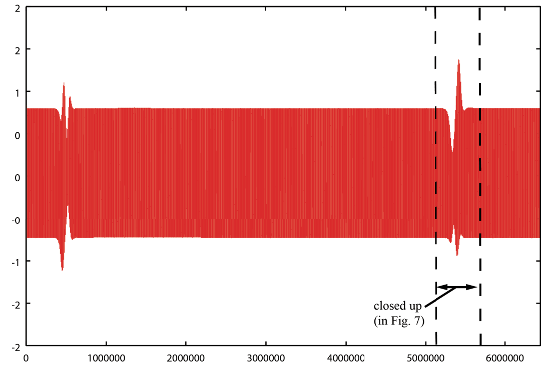


Figure 5: Instantaneous electric power of phase-waves at $OSC_{19}$.

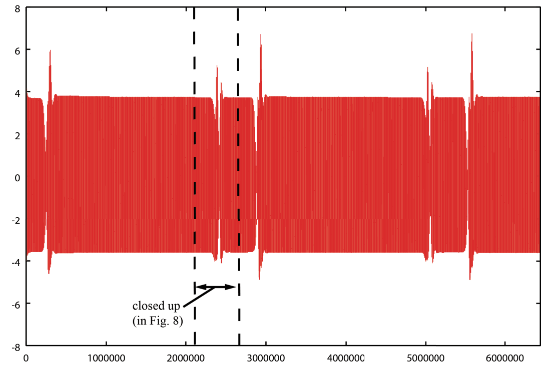


Figure 6: Instantaneous electric power of phase-inversion waves at $OSC_{19}$.

## 4. Comparison between phase-inversion waves on the in-and-anti-phase synchronizations and on the in-phase synchronizations

The phase-inversion waves can be observed on the in-phase synchronizations. We show a simulation result of the phase-inversion waves on the in-phase synchronizations in Fig. 9, and an actual circuit experimental result of the phase-inversion waves in the in-phase synchronizations in Fig. 10. The number of oscillators is 7 in these results. In this simulation, the coupling parameter $\alpha$ is 0.1, and the nonlinearity $\varepsilon$ is 0.2. In this actual circuit experiment, $L_{0k}$ is 494mH±1%, $L_{1k}$ is 52.2mH±1%, $C_k$ is 6.48nF±1%, $R_{1k}$ is 8.96kΩ±1%, $R_{2k}$ is 8.96kΩ±1%, $R_{3k}$ is 2.35kΩ±1% and OPAmps are TL082CP.

The instantaneous electric powers of $OSC_4$ of the simulation result and of the actual circuit experimental result are shown in Figs. 11 and 12. Two peaks are observed in both figures. We can observe the same results between the actual circuit experiment and the simulation. In these results, we can say that the itinerancy of the instantaneous electric power of the phase-inversion waves on the in-phase synchronization has same shape of the itinerancy of the instantaneous electric power of phase-inversion waves on the in-and-anti-phase synchronization.

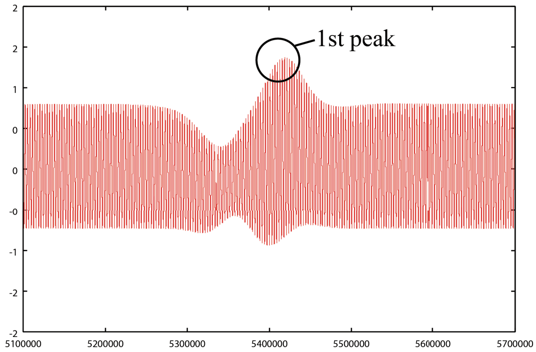


Figure 7: Close up of instantaneous electric power of phase-waves at $OSC_{19}$.

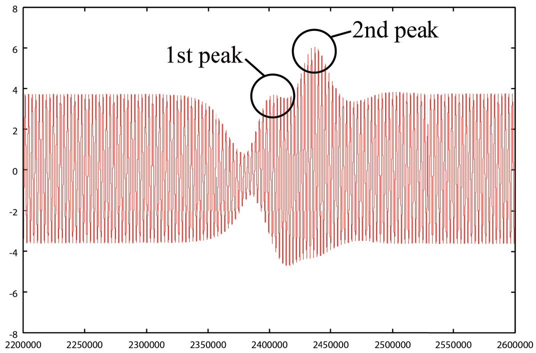


Figure 8: Close up of instantaneous electric power of phase-inversion waves at $OSC_{19}$.

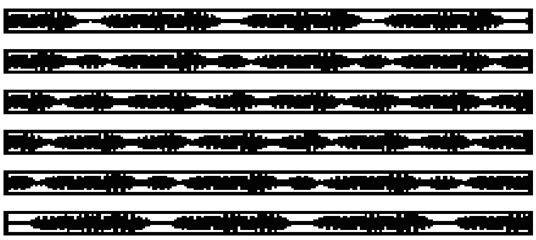

Figure 9: The simulation results of the phase-inversion waves in the in-phase synchronization.

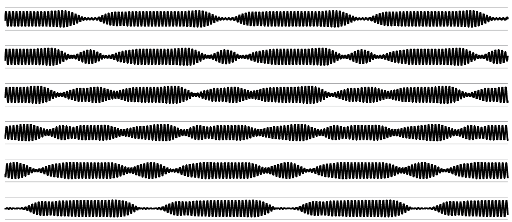

Figure 10: The actual experimental result of the phase-inversion waves in the in-phase synchronization.

## 5. Conclusion

We investigated the instantaneous electric powers of the phase-inversions wave and the phase-waves on the in-and-anti-phase synchronizations in the ladder of coupled van der Pol oscillators. The instantaneous electric power had

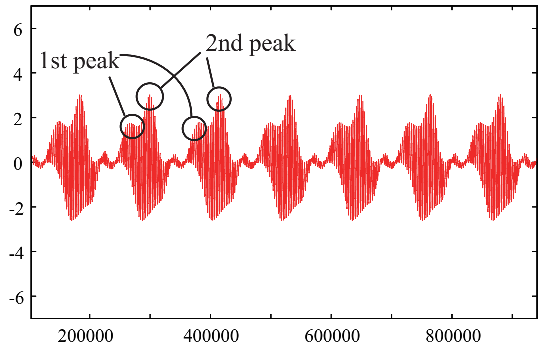


Figure 11: The simulation result of the power of $OSC_4$ in the in-phase synchronization.

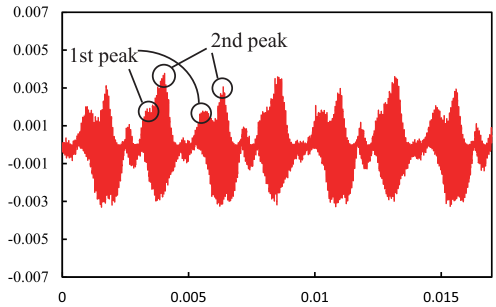


Figure 12: The actual experimental result of the power of $OSC_4$ in the in-phase synchronization.

two peaks when the phase-inversion waves were propagating, and the instantaneous electric power had only one peak when the phase-waves were propagating. It was clarified that the itinerancy of the instantaneous electric power of the phase-inversion waves on the in-and-anti-phase synchronization has same shape as the itinerancy of the instantaneous electric power of the phase-inversion waves on the in-phase synchronization.


## Acknowledgments

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