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Influence of Local Bridge on a Complex Network of Coupled Chaotic Circuits

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Abstract—In this study, we investigate the influence of local bridge on a complex network of 25 coupled chaotic circuits. From synchronization phenomena of coupled chaotic circuits, we show that synchronization of local bridge is easy to break down. By means of computer simulations, the network switches to global synchronization and partial synchronization. In order to analyze synchronization, we define asynchronous probability during a certain time interval. Moreover, we statistically analyze the sojourn time of synchronization of each edge including local bridge.

1. Introduction

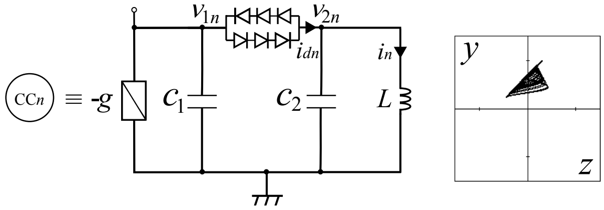
Complex networks have attracted a great deal of attention from various fields since the discovery of "small-world" network [1] and "scale-free" network [2]. In particular, how network topological structure influences its dynamical behaviors, is currently becoming a topic of great important. On the other hand, synchronization phenomena on the networks of coupled chaotic systems are very interested. However, there are not many studies of large-scale network of continuous-time real physical systems such as electrical circuits. Additionally we focus on synchronization phenomena of coupled chaotic circuit network with community structure based on social theory.

In sociology, there is a famous theory called "The strength of weak ties" by Granovetter [3]. This is the theory that weak networks (weak ties) are important more than strong networks (strong ties). Because strong networks are easy to isolate by centripetal force for homogeneity and affinity. Therefore weak ties are essential for information propagation and so on. Weak ties connect strong networks with each other as the bridge. In large-scale network, the bridging function may be provided locally. This kind of the bridge is called "local bridge".

In this study, synchronization phenomena on 25 coupled chaotic circuit network with local bridge are investigated. We show that synchronization of local bridge is easy to break down. By means of computer simulations, the network switches to global synchronization and partial synchronization. In order to analyze synchronization, we define asynchronous probability during a certain time interval. Moreover, we statistically analyze the sojourn time of synchronization of each edge including local bridge.

2. Network Model

Figure 1(a) shows the chaotic circuit which is three–dimensional autonomous circuit proposed by Shinriki *et al*. [4][5]. This circuit is composed by an inductor, a negative resistance, two condensers and dual–directional diodes. This circuit generates asymmetric attractor as shown in Fig. 1(b). A proposed network model of 25 coupled chaotic circuits with local bridge is shown in Fig. 2. In this study, chaotic circuits (CCn) are applied to each node of the network and each edge corresponds to resistors R. In this model, local bridges are 1-25, 8-9, 14-15, 15-16, and 22-23.



(a) Chaotic circuit. (b) Attractor ($\alpha = 0.4, \beta = 20, \gamma = 0.5$).

Figure 1: Chaotic circuit and attractor.

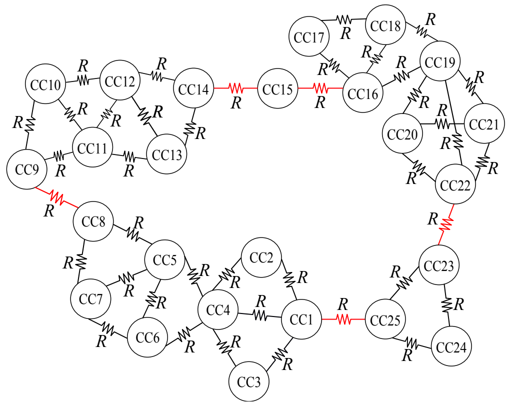


Figure 2: Network model.

First, we approximate the $i-v$ characteristics of the nonlinear resistors consisting of the diodes by the following three-segment piecewise-linear function as follows:

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \leq V) \\ G_d(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \quad (1)$$

By using the parameters and the variables as follows:

$$\begin{cases} i_n = \sqrt{\dfrac{C_2}{L}} V x_n, \; v_{1n} = V y_n, \; v_{2n} = V z_n \\ t = \sqrt{LC_2}\tau, \; \text{“}\cdot\text{”} = \dfrac{d}{d\tau}, \; \alpha = \dfrac{C_2}{C_1} \\ \beta = \sqrt{\dfrac{L}{C_2}} G_d, \; \gamma = \sqrt{\dfrac{L}{C_2}} g, \; \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C_2}}, \end{cases} \tag{2}$$

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x}_n = z_n \\ \dot{y}_n = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta \sum_{k \in S_n} (y_n - y_k) \\ \dot{z}_n = f(y_n - z_n) - x_n, \end{cases} \tag{3}$$

where $n = 1, 2, 3, ..., 25$ and S_n is set of nodes which are connected to CCn. The nonlinear function $f()$ corresponds to the $i - v$ characteristics of the nonlinear resistors consisting of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \leq 1) \\ \beta(y_n - z_n + 1) & (y_n - z_n < -1). \end{cases} \tag{4}$$

3. Synchronization States

In this study, we fix the same parameters as $\alpha = 0.4$, $\beta = 20$, $\gamma = 0.5$ and δ on all circuits. Each circuit is given different initial values each other. We show the dynamics of synchronization of the coupled chaotic circuits. In Fig. 3, the vertical axes are the differences between the voltage (corresponding to v_1) of the two chaotic circuits. Namely, if the two chaotic circuits synchronize, the value of the graph should be almost zero like 19-20. We can confirm that synchronizations of local bridges (8-9, 14-15, 15-16, 22-23 and 1-25) are easy to break down compared with others. In this study, we define two synchronization states of “global synchronization” and “partial synchronization”. Figure 3 shows that the network switches to two synchronization states. Additionally, partial synchronization is almost occurred from local bridge.

4. Statistical Analysis

In this section, we fix a certain time interval as ($\tau = 10,000$ and step $= 0.01\tau$) and we statistically analyze the synchronization phenomena observed from 25 coupled chaotic circuits with local bridge. First, we show the coupling strength dependency of global synchronization. In order to analyze synchronization state, we define the synchronization as following equation,

$$|y_n - y_k| < 0.01 \quad (k \in S_n). \tag{5}$$

Figure 4 shows the coupling strength dependency of global synchronization. We checked whether if the all edges are

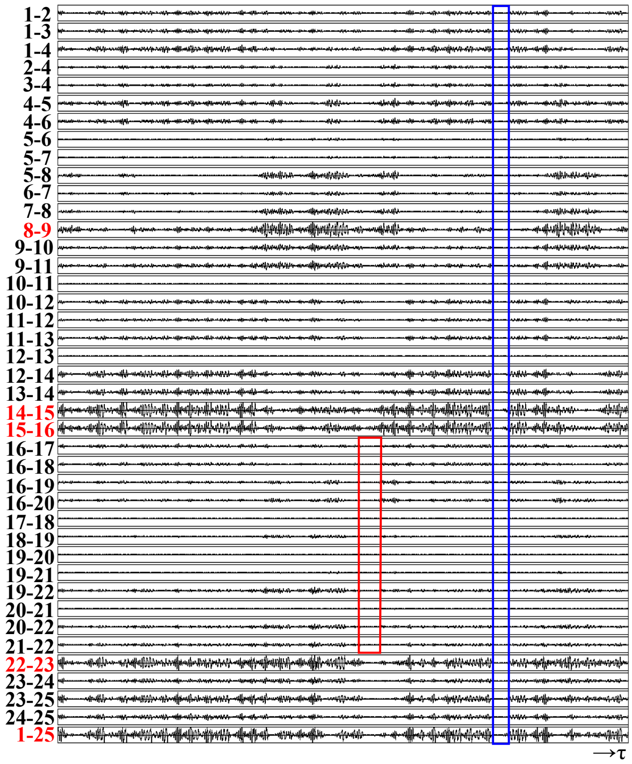


Figure 3: Phase difference waveform ($\delta = 1.0$).

synchronized during a certain time interval. We confirm that the network easy to become global synchronization state by increasing the coupling strength δ. In particular, around $\delta = 1.6 \sim 1.7$ are rapidly become to high global synchronization probability.

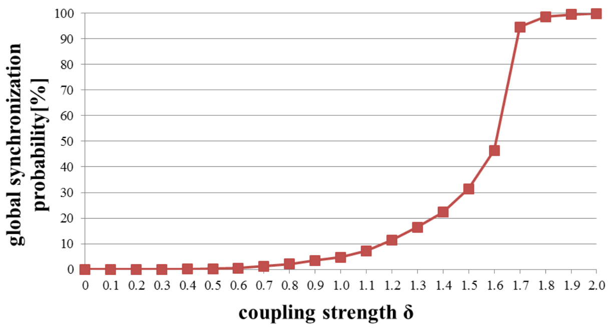


Figure 4: The coupling strength dependency of global synchronization.

From this point forward, we choose the coupling strength as $\delta = 1.0$ and we analyze synchronization focusing edge more statistically. Figure 5 shows that sorted all edges in order from highest to lowest of asynchronous probability and the number of switching between synchronization and asynchronous each edge. Local bridges are the top five of asynchronous probability among all edges. Additionally, local bridges are large number of switching between synchronization and asynchronous compared with other edges. Figure 6 shows the distribution of the number of synchronized edges during a certain time interval where the 41 of synchronized edge shows global synchronization and the 0 of edge shows fully asynchronous. In this parameters, we consider that various partial synchronizations exist on the network. In Fig. 6, it is interested that the distribution of the 41 edges is the second highest.

Next, node 15 is located in between two local bridges (14-15 and 15-16). Therefore the investigating of the states of special node 15 in this network, is important. Figure 7 shows the distribution of the states of node 15 during a certain time interval. In Fig. 7, for example, state D shows that nodes 14 and 15 are synchronized however nodes 15 and 16 are not synchronized. In four states, state A is the highest distribution. Therefore node 15 is easy to isolate however state B is the second highest distribution that is interested.

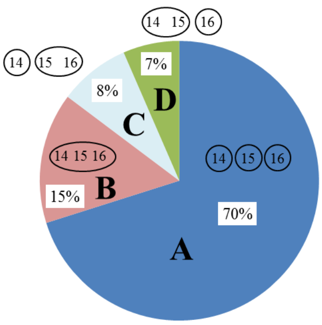


Figure 7: Distribution of the states of node 15.

Moreover, we focus on the sojourn time of synchronization of each edge including local bridge. Figure 8 shows the distribution of the sojourn time of synchronization. The slots in the horizontal axes of the figure denote the ranges of the sojourn time in Tab. 1. From Fig. 8(a), the graphs of local bridges is very similar and the ratio of slot 1 is predominantly high. Namely, synchronizations of local bridges disappear immediately. On the other hand, from Fig. 8(b), graphs of various edges show that the sojourn time of the synchronization is longer than local bridges. In particular, the sojourn time of synchronization of edge 23-25 that is located in between two local bridges (22-23 and 1-25), is similar to the local bridges. From this result, edge 23-25 is considered quasi-local bridge.

Table 1: Ranges of slots in Fig. 8.

Slot	Sojourn time (τ)	Slot	Sojourn time (τ)
1	τ<1	4	3 $\leq\tau$<4
2	1 $\leq\tau$< 2	5	4 $\leq\tau$<5
3	2 $\leq\tau$< 3	6	τ $\geq$5

5. Conclusion

In this study, we have proposed the network of 25 coupled chaotic circuits with local bridge. By means of computer simulations, synchronization of local bridge is easy to break down. Additionally, we confirmed that two synchronization states of global synchronization and partial synchronization. Moreover, we statistically analyzed that the sojourn time of synchronizations of local bridges are shorter than other edges. Namely, local bridge almost behaves asynchronously. These phenomena show that local bridge is the weak ties for promoting information propagation. In order to understand the phenomena correctly, more detailed investigation considering cluster should be carried out in our future works.

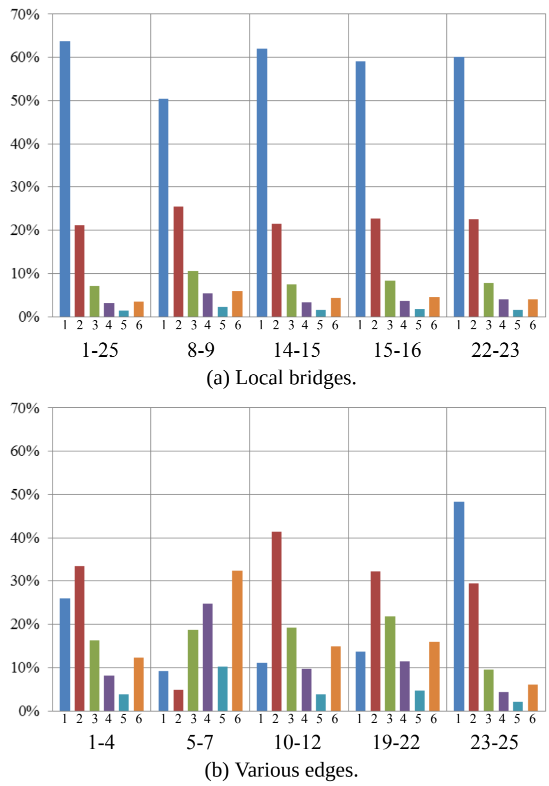


Figure 8: Distribution of the sojourn time τ of synchronization.

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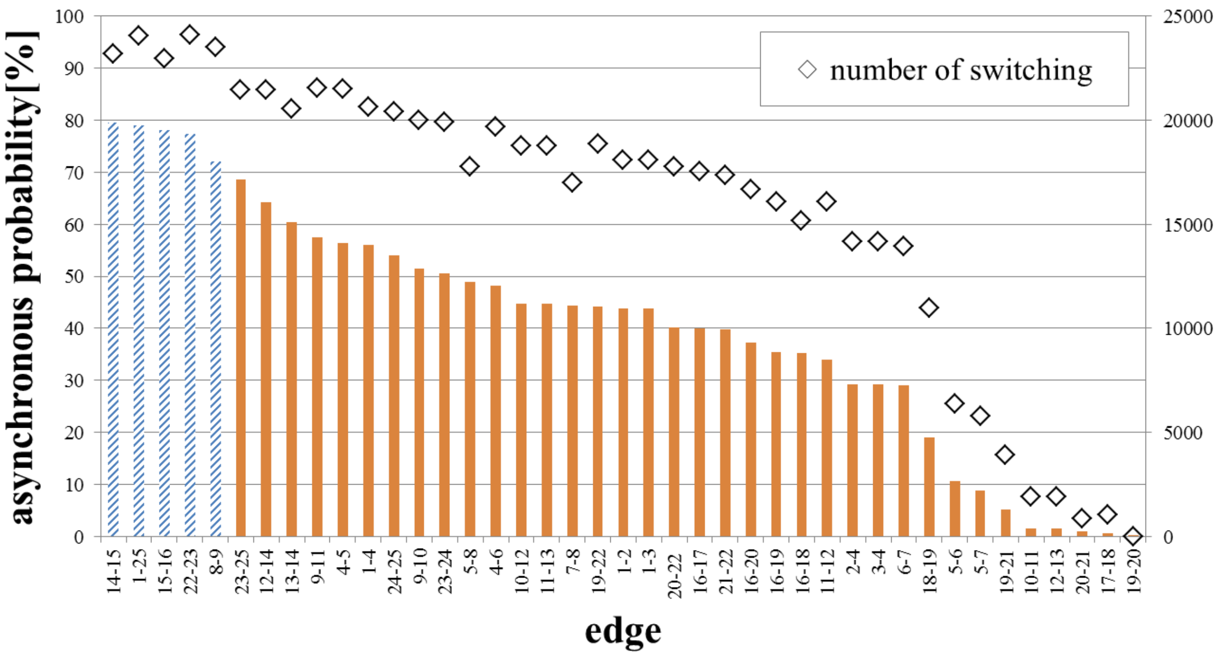

Figure 5: Asynchronous probability and the number of switching between synchronization and asynchronous (δ = 1.0).

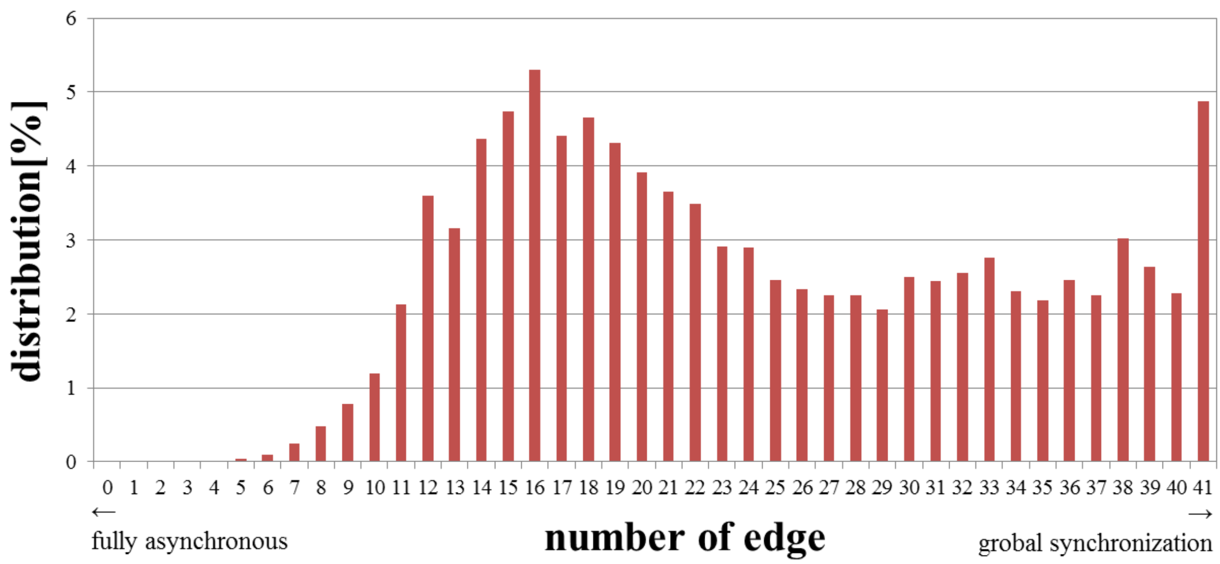

Figure 6: Distribution of the number of synchronized edges during a certain time interval (δ = 1.0).