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A Fast and Accurate Algorithm for IIR Adaptive Notch Filter Using Monotonically Increasing Function

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Abstract—In this paper, we propose a fast and accurate algorithm for the IIR adaptive notch filter. The adaptive notch filter is used to remove a narrow-band signal embedded in a wide-band signal. For updating the coefficient of the adaptive notch filter, the NLMS algorithm is often used. The NLMS algorithm, however, provides the low convergence performance due to the high correlation of the input signal. Other algorithms which orthogonalizes the input vector, such as the APA, is also inefficient, since the number of the updating coefficient is one. To solve this problem, we introduce a monotonically increasing function to the gradient vector in the LMS algorithm. Several computer simulations of the narrow-band signal reduction shows the efficiency of the proposed method.

1. Introduction

To remove a narrow-band signal embedded in a wide-band signal is one of the essential techniques in many signal processing fields, such as speech and audio signal processing, biomedical signal processing, and image processing [1, 2, 3]. In many practical situations, the frequency of the narrow-band signal is unknown and often changes with time. Hence, it is an important issue to remove the narrow-band signal fast and accurately.

For reduction of the unknown narrow-band signal, an adaptive notch filter is used [4, 5, 6]. The adaptive notch filter is composed by a second-oder IIR filter, and its amplitude response has a steep rejection characteristic at the frequency of the zero (notch frequency). By adjusting the filter coefficient which determines the notch frequency so that the notch frequency and the frequency of the narrow-band signal are corresponding, the adaptive notch filter achieves the reduction of the narrow-band signal

For updating the coefficient of the adaptive notch filter, the NLMS algorithm is often used [7]. The NLMS algorithm is one of the gradient based algorithm, and the algorithm can be performed with low computational complexity and implemented easily. The update term of the NLMS algorithm is derived based on the gradient of the error function, which is described by the average power of the filter output. In the NLMS algorithm, there are significant problems which are the low speed convergence and the low accuracy (, which means large estimation error variance), since the error function does not become a convex function, but becomes a quasi-convex function. The other orthogonalized algorithms, such as the APA or the Newton-LMS algorithm [8], are also inefficient, since the adaptive notch filter has only one updating coefficient.

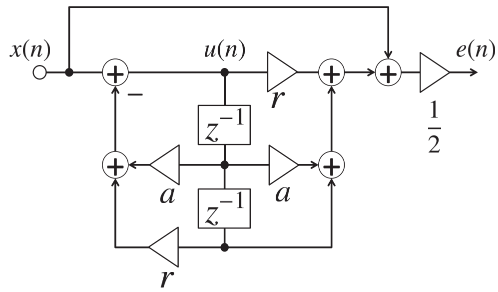

Figure 1: Structure of notch filter.

In this paper, to improve the convergence speed and accuracy in the adaptive notch filter, we propose a new gradient based algorithm. In the proposed method, we employ the monotonically increasing gradient, which is derived from the filter output power and the direction information of the NLMS algorithm's gradient. The monotonically increasing gradient achieves the convex error function, so that it improve the convergence speed and accuracy. Several computational simulation show the efficiency of the proposed method.

2. Adaptive Notch Filter

In this section, we show the fundamental structure of an adaptive notch filter. Also we show the NLMS algorithm as the conventional updating method of the filter coefficient.

2.1. Structure of Adaptive Notch Filter

Figure 2.1 shows the structure of a second-order adaptive IIR notch filter [5]. Here, the signal x_n is the input signal, e_n is the output signal, and u_n is the signal described by $u_n = x_n - au_{n-1} - ru_{n-2}$. The input signal x_n is described by using a narrow-band signal s_n and an additive wide-band desired signal w_n, i.e.,

$$x_n = w_n + s_n. \tag{1}$$

In this paper, we assume that w_n is given by a white Gaussian signal with the mean 0 and the variance σ^2, and s_n is given by a sinusoidal signal with the frequency ω_s, the amplitude p, and the phase ϕ.

The transfer function of the adaptive notch filter is given by

$$H(z) = \frac{1}{2}\left(1 + \frac{r + az^{-1} + z^{-2}}{1 + az^{-1} + rz^{-2}}\right), \tag{2}$$

where r $(-1 < r < 1)$ is a parameter which controls the rejection bandwidth. Specifically, the bandwidth becomes narrow along with the increasing value of r toward 1. The parameter a determines the notch frequency ω_N. The relation between a and ω_N is given by

$$a = -(1 + r)\cos(\omega_N). \tag{3}$$

The adaptive notch filter can remove the narrow-band signal by adjusting a so that $\omega_N = \omega_s$.

2.2. NLMS algorithm

In this chapter, we explain the NLMS algorithm which is one of the gradient based algorithm. By using the gradient of the error function $E[e_n^2]/E[u_{n-1}^2]$, we can obtain the updating term of NLMS algorithm as

$$a_{n+1} \quad = \quad a_n - \mu\frac{e_n u_{n-1}}{E[u_{n-1}^2]}, \tag{4}$$

where μ is a step size parameter which satisfies $0 < \mu < 2$. The parameter μ can adjust the convergence speed and accuracy under the trade-off, e.g., the large value of μ provides the fast convergence and the low estimation accuracy. Since $E[e_n u_{n-1}]$ becomes 0 at $\omega_N = \omega_s$, the notch frequency convergences toward the frequency of the narrow-band signal.

Figure 2.2 shows the gradient of the NLMS algorithm with $\omega_s = \pi/2$ and $r = 0.6, 0.8, 0.95$, where the horizontal axis is the notch frequency ω_N. As seen from this figure, the gradient of NLMS algorithm has the large amplitude around ω_s, and it has the small amplitude with distance from ω_s. Due to this property, the NLMS algorithm has the problems of low accuracy around ω_s and low convergence speed with distance from ω_s.

3. Monotonically Increasing Gradient Algorithm

To solve the convergence problem of NLMS algorithm, it is required to improve the gradient characteristic to be a monotonically increasing function which becomes 0 at ω_s. In this paper, we achieve the monotonically increasing gradient by multiplying the quasi-convex function $E[e_n^2]$ which

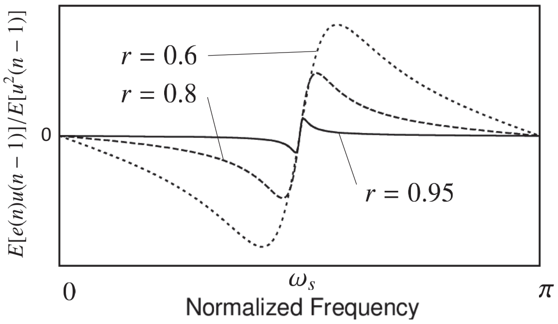

Figure 2: Gradient of NLMS algorithm.

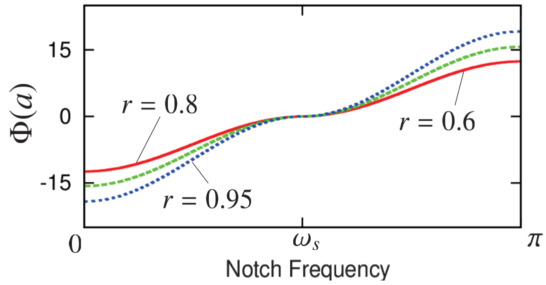

Figure 3: Gradient of proposed method.

has the minimum at ω_s and the direction information of the gradient of the NLMS algorithm. The proposed updating term using the monotonically increasing gradient is given by

$$a_{n+1} = a_n - \mu \mathrm{sgn}\Big(E[e_n u_{n-1}]\Big)\frac{E[e_n^2]}{E[u_{n-1}^2]}. \tag{5}$$

Here, we define the sign function $\mathrm{sgn}(E[e_n u_{n-1}])$ as

$$\mathrm{sgn}(E[e_n u_{n-1}]) = \begin{cases} -1, & E[e_n u_{n-1}] < -\epsilon \\ E[e_n u_{n-1}], & -\epsilon \le E[e_n u_{n-1}] \le \epsilon \\ 1, & \epsilon < E[e_n u_{n-1}] \end{cases}, \tag{6}$$

where the parameter ϵ $(0 \le \epsilon \le 1)$ smooths the shape of the sign function in the vicinity of $E[e_n u_{n-1}] = 0$. The large value of ϵ can reduce the degradation of the estimation accuracy which is caused by the discontinuity of sign function. Figure 3 shows the proposed gradient with $r = 0.6, 0.8, 0.95$. As seen from this figure, the proposed gradients becomes monotonically increasing functions.

Next, we evaluate the convergence speed of the proposed method. For the evaluation criterion, we use the ratio of the convergence speeds, which is defined as the ratio of the proposed gradient divided by the NLMS algorithm's gradient, i.e., the ratio of the convergence speed is given by

$$R \quad = \quad \frac{\mathrm{sgn}\Big(E[e_n u_{n-1}]\Big)E[e_n^2]/E[u_{n-1}^2]}{E[e_n u_{n-1}]/E[u_{n-1}^2]}$$

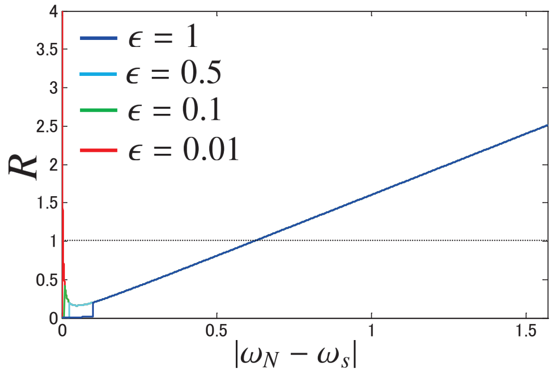

Figure 4: Ratio of the convergence speed.

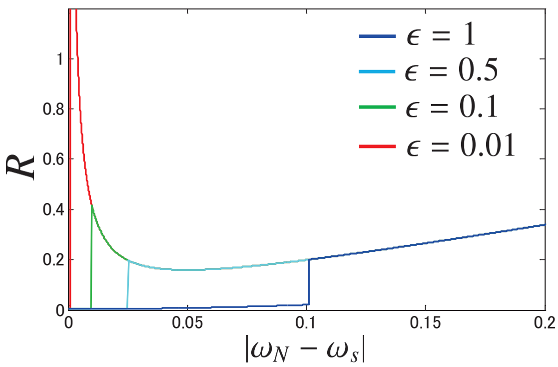

Figure 5: Expansion of figure 4 in $0 \leq |\omega_N - \omega_s| \leq 0.2$.

$$= \operatorname{sgn}\Big(E[e_n u_{n-1}]\Big)\frac{E[e_n^2]}{E[e_n u_{n-1}]}. \tag{7}$$

If the value of R is large, the proposed method provides the fast convergence speed compared with the NLMS algorithm. Figure 4 shows the several R curves with $\epsilon = 1, 0.01, 0$, where we put $r = 0.9$, $\omega_s = \pi/2$, $p = \sqrt{2}$, and $\sigma^2 = 1$. In this figure, the horizontal axis denotes the distance between the notch frequency and the frequency of the narrow-band signal, i.e., $|\omega_N - \omega_s|$. As seen from Fig. 4, the convergence speed of the proposed method is faster than one of the NLMS algorithm with the distance from ω_s because of $R > 1$. Additionally, the proposed method can provide the high accurate estimation around ω_s, since the proposed method provides slow convergence speed in $R < 1$. Figure 5 shows the expansion of Fig. 4 in $0 \leq |\omega_N - \omega_s| \leq 0.2$. In the nearest neighbor of ω_s, it can be seen that $R > 1$ when ϵ is set to a small value. It caused by the discontinuity of sign function, and it decreases the estimation accuracy. The deterioration of the estimation accuracy can be reduced by setting ϵ to a large value. In this case, the proposed method can provide the fast and accurate convergence.

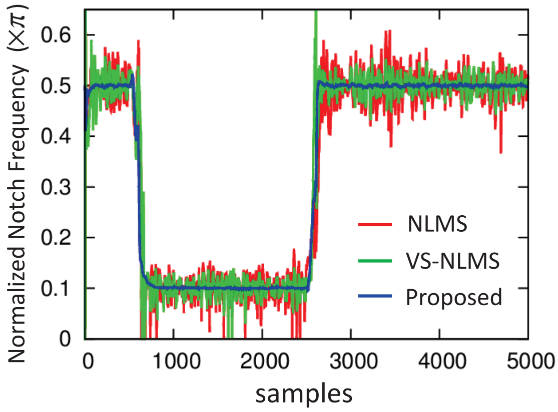

Figure 6: Convergence behaviors for NLMS algorithm, VS=NLMS algorithm, and Proposed method with $\epsilon = 0.1$.

4. Simulation

To confirm the effectiveness of the proposed algorithm in (5), we carried out several simulations for the sinusoidal noise reduction. In this simulation, we compare the convergence accuracy of the proposed method, the NLMS algorithm, and the variable step-size NLMS (VS-NLMS) algorithm [9], where the VS-NLMS algorithm is one of the typical method which improves the convergence property of the NLMS algorithm. As the input signal, we use x_n in (1) composed of the sinusoidal signal s_n with $p = \sqrt{2}$ and white Gaussian signal w_n with $\sigma^2 = 1$, i.e., SNR= 0. The length of x_n is 5000 samples. The frequency of s_n is changed with time as follows:

$$\omega_s = \begin{cases} \pi/2, & 1 \leq n < 500 \\ \pi/10, & 500 \leq n < 2500 \\ \pi/2, & 2500 < n \leq 5000 \end{cases}. \tag{8}$$

The parameter settings are following. In the proposed method, we set $\mu = 0.3$. In the NLMS algorithm, we set $\mu = 0.4$. In the VS-NLMS algorithm, we set $\mu = 0.3$, $\alpha = 0.98$, and $C = 0.0001$. We set above parameters so that all convergence speeds becomes same level.

Figure 6 shows convergence behaviors of the notch frequency ω_N, where we put $\epsilon = 0.1$. It can be seen in this figure that the proposed methods effectively reduce the variance around ω_s compared with other methods. Figure 7 shows MSE behaviors between ω_N and ω_s with 100 trials average, where we put $\epsilon = 0.1, 0.00$. Note that the proposed method with $\epsilon = 0.00$ is equal to the method in [] As seen from this figure, the MSE of the proposed method is lower than the conventional methods. In the proposed method, the MSE with $\epsilon = 0.05$ is lower than one with $\epsilon = 0.00$, especially it can be seen from 3000 to 5000 samples. These result shows the proposed method improves the trade-off of the convergence speed and the estimation accuracy. Hence, by adjusting the parameters of the pro-

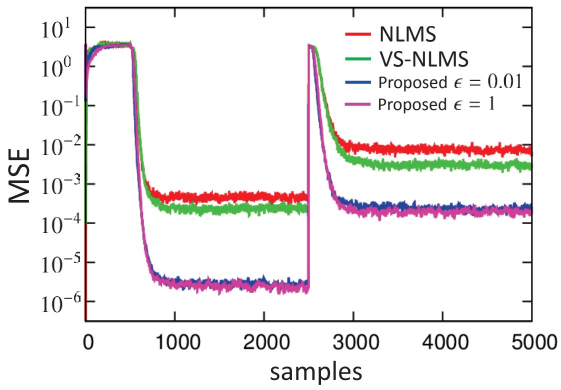

Figure 7: MSE behaviors for NLMS algorithm, VS=NLMS algorithm, and Proposed method with $\epsilon = 0.05, 0.00$.

posed method, the proposed method can provide the fast and accurate convergence, simultaneously.

5. Conclusion

In this paper, we proposed a fast and accurate algorithm for the IIR adaptive notch filter. The proposed method employs the monotonically increasing gradient. Several computational simulations shows that the proposed method can provide high estimation accuracy compared with the NLMS algorithm.

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