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Designing IIR Filters with Variable Stopband Using Necessary and Sufficient Stability Criterion

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Abstract—In this paper, we propose a design method for variable IIR digital filters with changeable multifactors in the stopband using necessary and sufficient stability criterion. The variable IIR filter has a piecewise high attenuation in the stopband, and the filter varies stopband edge frequency, and both the frequency and its magnitude in the piecewise high attenuation. Stability criterion based on the system matrix as necessary and sufficient condition is used to guarantee the filter stability. Then, the proposed method has a potential to realize a higher stopband attenuation than one obtained by the conventional method using positive realness condition to guarantee the filter stability. Finally, we demonstrate effectiveness of the proposed method by design examples.

1. Introduction

Recently, digital signal processing is required various fields. In this paper, the filter can instantly change the frequency characteristics in digital signal processing is referred to as variable digital filters(VDFs). The VDFs are important in fields such as communications, measurement, sound, and image processing. Recently, several algorithms for the design of such filters have been proposed[1]–[3].

In the automatically dynamic measurement systems to measure object weight, we need filter with high stopband attenuation to perform an exact measurement. However, filters with high overall stopband attenuation need high filter order. Therefore, the delay for the calculation becomes very large. To reduce the delay, we designed the filter having a piecewise high attenuation in the stopband, unlike the overall stopband that achieves high attenuation. The range of the high attenuation in the stopband needs to vary according to differences of the measurement environments and the measurement objects to realize high speed and high accuracy measurement. Then, we have proposed variable IIR filter has piecewise high attenuation in the stopband, and the filter varies stopband edge frequency, and both the frequency and its magnitude in the piecewise high attenuation by using variable parameters[3]. The proposed algorithm needs quantity of large computer memory for a calculation to solve a design problem by semidefinite programming (SDP) and the algorithm is used only sufficient condition based on positive realness to guarantee the filter stability.

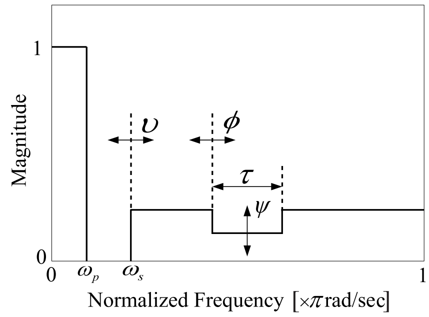


Figure 1: Desired characteristics

In this paper, we propose a design method for variable IIR digital filters with changeable multifactors in the stopband using necessary and sufficient stability criterion. Stability criterion based on the system matrix as necessary and sufficient condition is used to guarantee the filter stability. Then, the proposed method has a potential to realize a higher stopband attenuation than one obtained by the conventional method [3]. In the proposed method, we use the successive projection (SP) method to design the proposed filter. The SP method realizes less computer memory to design digital filters than SDP. Finally, we demonstrate effectiveness of the proposed method.

2. Design method of a IIR VDF using SP method

We assume that the desired variable amplitude response (see Fig. 1) is denoted by

$$F(\omega,\upsilon,\phi,\psi)=\begin{cases} A(\omega)e^{-j\theta(\omega)} & 0\le\omega\le\omega_p \\ 0 & \omega_s+\upsilon\le\omega\le\phi \\ 0 & \phi<\omega\le\phi+\tau \\ 0 & \phi+\tau<\omega\le\pi \end{cases}, \quad (1)$$

where ω_p and ω_s are the normalized angular frequencies in the passband and stop band edge frequencies, respectively. Moreover, $A(\cdot)$ and $\theta(\cdot)$ are the desired amplitude and the phase response, respectively. υ and ϕ are respectively variable parameters that change the normalized angular frequency at the stopband edge and the left side normalized frequency in a piecewise high stopband attenuation, and τ is the width of the piecewise high stopband attenuation.

Now, we define maximum allowable deviations in the passband and the stopband. Maximum allowable deviation is different from the stopband edge frequency and weight for the piecewise high attenuation in the stopband. Then, maximum allowable deviation at the each band is defined as

$$\lambda(\omega, \upsilon, \phi, \psi) = \begin{cases} \delta_p(\upsilon, \psi) & 0 \le \omega \le \omega_p \\ \delta_s(\upsilon, \psi) & \omega_s + \upsilon \le \omega \le \phi \\ \delta_s(\upsilon, \psi)/\psi & \phi < \omega \le \phi + \tau \\ \delta_s(\upsilon, \psi) & \phi + \tau < \omega \le \pi \end{cases}, \quad (2)$$

where ψ shows a variable parameter which generates the piecewise high attenuation in the stopband. The ranges of these variable parameters are

$$\upsilon \in [\upsilon_{\min}, \upsilon_{\max}], \quad (3)$$

$$\phi \in [\phi_{\min}, \phi_{\max}], \quad (4)$$

$$\psi \in [\psi_{\min}, \psi_{\max}]. \quad (5)$$

The values of each variable parameter are different variations. The variable parameters with small variations will be swamped by parameters with large variations. Consequently, the values of the variable parameters are not well reflected in the filter coefficients. These variable parameters are redefined as follows:

$$\tilde{\upsilon} = \frac{2(\upsilon - \upsilon_{\min})}{\upsilon_{\max} - \upsilon_{\min}} - 1, \quad (6)$$

$$\tilde{\phi} = \frac{2(\phi - \phi_{\min})}{\phi_{\max} - \phi_{\min}} - 1, \quad (7)$$

$$\tilde{\psi} = \frac{2(\log_{10}\psi - \log_{10}\psi_{\min})}{\log_{10}\psi_{\max} - \log_{10}\psi_{\min}} - 1, \quad (8)$$

where $-1 \le \tilde{\upsilon} \le 1$, $-1 \le \tilde{\phi} \le 1$ and $-1 \le \tilde{\psi} \le 1$.

The frequency response of the proposed IIR VDF is given by

$$H(\omega, \upsilon, \phi, \psi) = \frac{\sum_{i=0}^{M} a(i, \upsilon, \phi, \psi) e^{-ji\omega}}{1 + \sum_{i=1}^{N} b(i, \upsilon, \phi, \psi) e^{-ji\omega}} = \frac{N(\omega, \upsilon, \phi, \psi)}{D(\omega, \upsilon, \phi, \psi)}. \quad (9)$$

where M and N are filter orders of numerator and denominator, and $a(\cdot)$ and $b(\cdot)$ are filter coefficients of numerator and denominator, respectively. Then, filter coefficients $a(\cdot)$ and $b(\cdot)$ are approximated by a polynomial to vary filter characteristics according to variable parameters. Filter coefficients $a(\cdot)$ and $b(\cdot)$ are defined as

$$a(i, \upsilon, \phi, \psi) = \sum_{l_\upsilon=0}^{L_\upsilon} \sum_{l_\phi=0}^{L_\phi} \sum_{l_\psi=0}^{L_\psi} g_a\left(i, l_\upsilon, l_\phi, l_\psi\right) \upsilon^{l_\upsilon} \phi^{l_\phi} \psi^{l_\psi}, \quad (10)$$

$$b(i, \upsilon, \phi, \psi) = \sum_{l_\upsilon=0}^{L_\upsilon} \sum_{l_\phi=0}^{L_\phi} \sum_{l_\psi=0}^{L_\psi} g_b\left(i, l_\upsilon, l_\phi, l_\psi\right) \upsilon^{l_\upsilon} \phi^{l_\phi} \psi^{l_\psi}, \quad (11)$$

where $g_a(\cdot)$ and $g_b(\cdot)$ are polynomial coefficients. The design problem of the proposed IIR VDF to solve polynomial coefficients is

$$\begin{aligned} |\gamma(\omega, \upsilon, \phi, \psi)| &= \left|F(\omega, \upsilon, \phi, \psi) - H(\omega, \tilde{\upsilon}, \tilde{\phi}, \tilde{\psi})\right| \\ &= \left|F(\omega, \upsilon, \phi, \psi) - \frac{N(\omega, \tilde{\upsilon}, \tilde{\phi}, \tilde{\psi})}{D(\omega, \tilde{\upsilon}, \tilde{\phi}, \tilde{\psi})}\right| \\ &\le \lambda(\omega, \upsilon, \phi, \psi) \end{aligned}. \quad (12)$$

Then, the updating equation of numerator and denominator polynomial coefficients using the SP method in $n+1$-th iteration step are as follows (Details of derivation of equations refer to [4]):

$$g_a^{n+1}\left(i, l_\upsilon, l_\phi, l_\psi\right) = \begin{cases} g_a^n\left(i, l_\upsilon, l_\phi, l_\psi\right) + \delta_a\left(i, l_\upsilon, l_\phi, l_\psi\right) \\ \quad \text{if } e^n(\omega_M, \tilde{\upsilon}_M, \tilde{\phi}_M, \tilde{\psi}_M, t_{1M}, t_{2M}, \mathbf{g}_a^n, \mathbf{g}_b^n) \\ \qquad > \lambda(\omega_M, \upsilon_M, \phi_M, \psi_M) \\ g_a^n\left(i, l_\upsilon, l_\phi, l_\psi\right) \\ \quad \text{if } e^n(\omega_M, \tilde{\upsilon}_M, \tilde{\phi}_M, \tilde{\psi}_M, t_{1M}, t_{2M}, \mathbf{g}_a^n, \mathbf{g}_b^n) \\ \qquad \le \lambda(\omega_M, \upsilon_M, \phi_M, \psi_M) \end{cases}, \quad (13)$$

$$g_b^{n+1}\left(i, l_\upsilon, l_\phi, l_\psi\right) = \begin{cases} g_b^n\left(i, l_\upsilon, l_\phi, l_\psi\right) + \delta_b\left(i, l_\upsilon, l_\phi, l_\psi\right) \\ \quad \text{if } e^n(\omega_M, \tilde{\upsilon}_M, \tilde{\phi}_M, \tilde{\psi}_M, t_{1M}, t_{2M}, \mathbf{g}_a^n, \mathbf{g}_b^n) \\ \qquad > \lambda(\omega_M, \upsilon_M, \phi_M, \psi_M) \\ g_b^n\left(i, l_\upsilon, l_\phi, l_\psi\right) \\ \quad \text{if } e^n(\omega_M, \tilde{\upsilon}_M, \tilde{\phi}_M, \tilde{\psi}_M, t_{1M}, t_{2M}, \mathbf{g}_a^n, \mathbf{g}_b^n) \\ \qquad \le \lambda(\omega_M, \upsilon_M, \phi_M, \psi_M) \end{cases}, \quad (14)$$

where $\delta_a(\cdot)$ and $\delta_b(\cdot)$ are update value of polynomial coefficients, e^n shows error function, and t_1 and t_2 are rotation parameter to convert complex domain into the real domain. In addition, $\omega_M, \upsilon_M, \phi_M$, and ψ_M shows the maximum deviation point from the desired characteristics, that satisfies following equation:

$$\begin{aligned} &\left|e^n(\omega_M, \tilde{\upsilon}_M, \tilde{\phi}_M, \tilde{\psi}_M, t_{1M}, t_{2M}, \mathbf{g}_a^n, \mathbf{g}_b^n) - \lambda(\omega_M, \upsilon_M, \phi_M, \psi_M)\right| \\ &= \max_{\text{all } \omega, \upsilon, \phi, \psi} |e^n(\omega_s, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}, t_1, t_2, \mathbf{g}_a^n, \mathbf{g}_b^n) \\ &\qquad - \lambda\left(\omega_s, \upsilon_{s_\upsilon}, \phi_{s_\phi}, \psi_{s_\psi}\right)| \end{aligned}. \quad (15)$$

s, s_υ, s_ϕ, and s_ψ show evaluation point of approximate frequency band and variable variable parameters υ, ϕ, ψ, respectively, where $1 \le s \le S$, $1 \le s_\upsilon \le S_\upsilon$, $1 \le s_\phi \le S_\phi$, $1 \le s_\psi \le S_\psi$. Then, the numbers of the evaluation point of variable parameters limit as $S_\upsilon \ge L_\upsilon + 1$, $S_\phi \ge L_\phi + 1$, $S_\psi \ge L_\psi + 1$.

3. Guarantee of filter stability

In this paper, stability criteria based on system matrix in [5] is used to guarantee filter stability, and is necessary and sufficient condition. In the proposed method, filter coefficients of denominator are composed polynomial expression using variable parameters. Therefore, discrimination of filter stability needs to perform each evaluation points of variable parameters, because the filter coefficients are composed by variable parameters and updated polynomial coefficients. After updating of polynomial coefficients, denominator filter coefficients for each combination of variable parameters are

$$b^{n+1} = b^{n+1}\left(i, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}\right) = \sum_{l_\upsilon=0}^{L_\upsilon}\sum_{l_\phi=0}^{L_\phi}\sum_{l_\psi=0}^{L_\psi} g_b^{n+1}\left(i, l_\upsilon, l_\phi, l_\psi\right) \tilde{\upsilon}_{s_\upsilon}^{l_\upsilon} \tilde{\phi}_{s_\phi}^{l_\phi} \tilde{\psi}_{s_\psi}^{l_\psi} . \quad (16)$$

Here, the system matrix using Eq.(16) is

$$\mathbf{A}^{n+1} = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1 \\ -b_N^{n+1} & -b_{N-1}^{n+1} & -b_{N-2}^{n+1} & \cdots & -b_2^{n+1} & -b_1^{n+1} \end{bmatrix}. \quad (17)$$

As a result, if all absolute values of eigenvalues $\nu^{n+1}(k, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi})$ obtained from Eq.(17) satisfy

$$\left|\nu(k, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi})\right| \le \rho, \quad (18)$$

then the obtained filter in $n+1$-th iteration step is stable, where k is $1 \le k \le N$, and ρ is the prescribed maximum pole radius $0 < \rho < 1$. That is, the eigenvalues $\nu(\cdot)$ correspond to a pole of the transfer function. Next, we propose correction method of denominator polynomial coefficients when the obtained filter is unstable.

If the obtained filter is unstable, we find the largest eigenvalues unsatisfying Eq.(18) for each combination of variable parameters. Then, the unstable eigenvalues are corrected by following equation:

$$\hat{\nu}^{n+1}\left(k, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}\right) = \frac{\nu^{n+1}\left(k, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}\right)\rho}{\max\limits_{\text{all } k}\left|\nu^{n+1}\left(k, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}\right)\right|}, \quad (19)$$

where $\hat{\nu}^{n+1}(\cdot)$ show corrected eigenvalues. The stable denominator filter coefficients $\hat{b}^{n+1}(\cdot)$ are composed using the corrected eigenvalues as follows:

$$1 + \sum_{i=1}^{N} \hat{b}^{n+1}\left(i, \widetilde{\upsilon}_{s_\upsilon}, \widetilde{\phi}_{s\phi}, \widetilde{\psi}_{s\psi}\right) z^{-i} = \prod_{k=1}^{N}\left(1 + \hat{\nu}^{n+1}\left(k, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}\right) z^{-1}\right) . \quad (20)$$

Therefore, all of stable denominator filter coefficients for combination of variable parameters are

$$b_{stab}^{n+1}(i, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}) = \begin{cases} b^{n+1}(i, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}) & \text{if Stable} \\ \hat{b}^{n+1}\left(i, \widetilde{\upsilon}_{s_\upsilon}, \widetilde{\phi}_{s\phi}, \widetilde{\psi}_{s\psi}\right) & \text{if Unstable} \end{cases} . \quad (21)$$

Denominator filter coefficients are composed polynomial expression using variable parameters, then we have to correct polynomial coefficients $g_b(\cdot)$. In this paper, stable polynomial coefficients are computed by least squares as stable denominator filter coefficients $b_{stab}^{n+1}(\cdot)$ is set desired characteristics. Denominator polynomial coefficients $g_b^{n+1}(\cdot)$ to compose corrected i-th denominator filter coefficient are computed by squared evaluation function is Eq.(22). A solution for minimizing Eq.(22) is obtained by following equation:

$$\mathbf{g}_{b,i}^{n+1} = \mathbf{J}^{\dagger}\mathbf{b}_{stab,i}^{n+1} \quad (23)$$

where $\mathbf{J}^{\dagger}$ shows pseudo-inverse matrix of $\mathbf{J}$.

$$\mathbf{J} = \begin{bmatrix} \tilde{\upsilon}_1^0\tilde{\phi}_1^0\tilde{\psi}_1^0 & \cdots & \tilde{\upsilon}_1^{l_\upsilon}\tilde{\phi}_1^{l_\phi}\tilde{\psi}_1^{l_\psi} & \cdots & \tilde{\upsilon}_1^{L_\upsilon}\tilde{\phi}_1^{L_\phi}\tilde{\psi}_1^{L_\psi} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\upsilon}_{s_\upsilon}^0\tilde{\phi}_{s_\phi}^0\tilde{\psi}_{s_\psi}^0 & \cdots & \tilde{\upsilon}_{s_\upsilon}^{l_\upsilon}\tilde{\phi}_{s_\phi}^{l_\phi}\tilde{\psi}_{s_\psi}^{l_\psi} & \cdots & \tilde{\upsilon}_{s_\upsilon}^{L_\upsilon}\tilde{\phi}_{s_\phi}^{L_\phi}\tilde{\psi}_{s_\psi}^{L_\psi} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\upsilon}_{S_\upsilon}^0\tilde{\phi}_{S_\phi}^0\tilde{\psi}_{S_\psi}^0 & \cdots & \tilde{\upsilon}_{S_\upsilon}^{l_\upsilon}\tilde{\phi}_{S_\phi}^{l_\phi}\tilde{\psi}_{S_\psi}^{l_\psi} & \cdots & \tilde{\upsilon}_{S_\upsilon}^{L_\upsilon}\tilde{\phi}_{S_\phi}^{L_\phi}\tilde{\psi}_{S_\psi}^{L_\psi} \end{bmatrix} \quad (24)$$

$$\mathbf{b}_{stab,i}^{n+1} = \begin{bmatrix} b_{stab}^{n+1}\left(i, \tilde{\upsilon}_1, \tilde{\phi}_1, \tilde{\psi}_1\right) \\ \vdots \\ b_{stab}^{n+1}\left(i, \tilde{\upsilon}_{s_\upsilon}, \tilde{\phi}_{s_\phi}, \tilde{\psi}_{s_\psi}\right) \\ \vdots \\ b_{stab}^{n+1}\left(i, \tilde{\upsilon}_{S_\upsilon}, \tilde{\phi}_{S_\phi}, \tilde{\psi}_{S_\psi}\right) \end{bmatrix} \quad (25)$$

$$\mathbf{g}_{b,i}^{n+1} = \begin{bmatrix} g_b^{n+1}(i, 0, 0, 0) \\ \vdots \\ g_b^{n+1}\left(i, l_\upsilon, l_\phi, l_\psi\right) \\ \vdots \\ g_b^{n+1}\left(i, L_\upsilon, L_\phi, L_\psi\right) \end{bmatrix} \quad (26)$$

Then, we obtained denominator polynomial coefficients for stable filter by the closed equation form.

4. Design example

In this section, we demonstrate the effectiveness of the proposed method. The filter specifications to compare the conventional method using SDP [3] are as follows:

Filter order: $M = 29, N = 6$

Polynomial order: $L_\upsilon = L_\phi = L_\psi = 3$

Variation of frequency: $\upsilon = -0.02\pi \sim 0.02\pi$

Desired characterisitcs:

$$F(\omega, \upsilon, \phi, \psi) = \begin{cases} e^{-j16\omega} & 0 \le \omega \le 0.1\pi \\ 0 & 0.2\pi + \upsilon \le \omega \le \pi \end{cases}$$

High stopband attenuation

Variation of frequency: $\phi = 0.38\pi \sim 0.68\pi$

Variation of weight: $\psi = 10 \sim 100$

Bandwidth: $\tau = 0.2\pi$

In this example, 50 and 400 evaluation points were used in the passband and the stopband, respectively. The variable parameter υ for changing the stopband edge angular frequency was uniformly divided with a step size of 0.01π. The left side edge angular frequency of the piecewise high stopband attenuation ϕ was uniformly divided

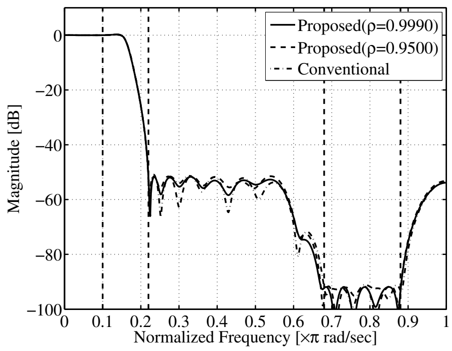
$$J(\mathbf{g}_{b,i}^{n+1}) = \frac{1}{2}\sum_{s_\upsilon=1}^{S_\upsilon}\sum_{s_\phi=1}^{S_\phi}\sum_{s_\psi=1}^{S_\psi}\left(b_{stab}\left(i,\widetilde{\upsilon}_{s_\upsilon},\widetilde{\phi}_{s\phi},\widetilde{\psi}_{s\psi}\right) - \sum_{l_\upsilon=0}^{L_\upsilon}\sum_{l_\phi=0}^{L_\phi}\sum_{l_\psi=0}^{L_\psi} g_b\left(i,l_\upsilon,l_\phi,l_\psi\right)\widetilde{\upsilon}_{s_\upsilon}^{l_\upsilon}\widetilde{\phi}_{s\phi}^{l_\phi}\widetilde{\psi}_{s\psi}^{l_\psi}\right)^2 \quad (22)$$

Table 1: Comparing stopband attenuations.

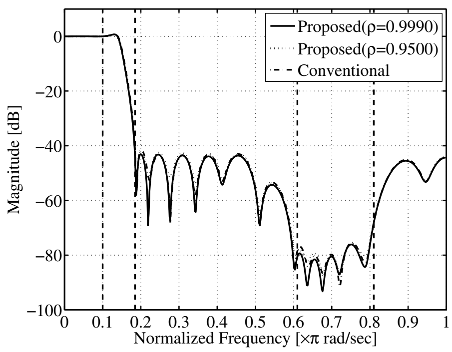
$\upsilon = 0.020\pi$ $\phi = 0.680\pi$ $\psi = 100$	Piecewise high stopband attenuation	Stopband attenuation
Proposed (ρ = 0.9990)	91.8957[dB]	51.7120[dB]
Proposed (ρ = 0.9500)	91.6271[dB]	51.4376[dB]
Conventional [3]	91.0870[dB]	50.9140[dB]

with a step size of 0.075π, and the weighting parameter of the piecewise high stopband attenuation ψ was divided at [10 18 32 56 100]. In addition, maximum allowable deviation in the passband was used as same as the conventional method [3].

Table 1 shows maximum piecewise high stopband attenuation of the proposed method and the conventional method, respectively. Then, the variable parameters are shown in the table. From table 1, the proposed method realizes higher attenuation than the conventional method. In Fig.2, we show the amplitude response obtained from the proposed method and the conventional method. It is clear form Fig.2, the amplitude response of the obtained filter can be changed according to variable parameters, and that has equiripple characteristic. We verified that the group responses of the obtained filter approximated desired characteristics in the all examples. Next, we verified the filter stability when changing the variable parameters. The variable parameters were changed that υ, ϕ and ψ are evenly 30, 50 and 12 point, respectively. As a result, the maximum pole radius in the case of $\rho = 0.9500$ becomes 0.9503. Then, the example does not determinate maximum allowable pole radius. In the proposed method, correction method of polynomial coefficients uses least squares. We conclude above result that least squares includes approximation error.



(a) $\upsilon = 0.020\pi, \phi = 0.680\pi, \psi = 100$



(b) $\upsilon = -0.015\pi, \phi = 0.610\pi, \psi = 60$

Figure 2: Amplitude response of obtained VDFs.

5. conclusion

In this paper, we proposed a design method for variable IIR digital filters with changeable multifactors in the stopband using necessary and sufficient stability criterion. Stability criterion based on the system matrix as necessary and sufficient condition is used to guarantee the filter stability. Then, the proposed method realized a higher stopband attenuation than one obtained by the conventional method using SDP [3] . Finally, we demonstrated effectiveness of the proposed method.

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