# A Predistorter Based on the Least-Mean Squares Newton Algorithm

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Abstract— This work presents an efficient digital predistorter based on a modified Least Mean Squares Newton (LMSN) adaptive filtering algorithm. The proposed predistorter is shown to consistently outperform the Recursive Least Squares (RLS) algorithm by 1dB in terms of error performance while requiring on average 10% less computational time. The performance of the proposed predistorter is validated through linearizing a Doherty power amplifier driven by two 20MHz wideband code division multiple access (WCDMA) signals having different carrier configurations. The results presented in this work demonstrate the desirable performance of the algorithm compared to the well-known RLS algorithm.

#### 1. Introduction

Power amplifiers (PAs) are vital components in radio frequency (RF) systems and base station transmitters, whose nonlinear behaviour limits the bandwidths and levels of multiplexing used. As such, compensating for these nonlinearities is of a great importance to researchers and industry alike [1]. One popular technique for linearizing PAs is digital predistortion; in which the input signal is pre-conditioned using a digital signal processor (DSP) and then fed to the nonlinear PA to obtain an output signal which is as linear as possible with respect to the predistorter's input signal [1]-[4]. In this study, the well-known memory polynomial (MP) model is used to describe both the PA and its digital predistorter (DPD) [5].

In the literature, a variety of adaptive algorithms have been used to implement DPDs with good results [1]-[3]. In this work, a modified version of the least mean-squares-Newton (LMSN) algorithm is used to construct the DPD. The LMSN algorithm is a slightly simplified version of the recursive least squares (RLS) algorithm [6] that requires a lower number of computations in addition to being more numerically robust. Through the simulations performed in this work, this algorithm was found to outperform the RLS algorithm in terms of both estimation accuracy and speed.

The remainder of this paper is organized as follows: Section II discusses the predistortion problem, presents the model used and illustrates the issue of high correlation in the model data, which motivates the development of the proposed predistorter. Section III presents the LMSN-based predistortion algorithm, and Section IV reports

simulation results validating the performance of the proposed method. Finally, Section V concludes this article by summarizing the work done and discussing the benefits of this work.

## 2. Behavioral Modeling and Predistortion Background

In order to motivate the work done in this article, the basics of DPD and the mathematical model used in this study are briefly discussed in this section.

### 2.1. DPD setup and architecture used

In this work, the well-known indirect learning method for implementing predistorters is used [7]. In this architecture, an input signal is first passed through the PA to be linearized and the resulting output signal is recorded. The input and output signals of the device under test are then aligned in time domain. Next, the acquired output signal is normalized by the amplifier's small-signal gain , and an appropriate model for training the predistorter is fitted using the pair of input-output signals. This process is illustrated through Figure 1. After the predistorter is trained as described above, the input signal is used as input to the DPD, producing an intermediate output signal which is then passed to the PA, producing an output signal that is linear. During the training phase, the signals and are equal.

In the literature, a wide variety of models are available to choose from, differing in complexity, structure and modeling accuracy [1]. In this work, the well-known memory polynomial model is utilized due to its good performance and low complexity. This model is briefly discussed next.

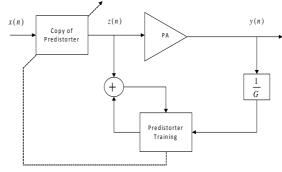


Figure 1. Block diagram illustrating the implementation of DPD.

### 2.2. The Memory Polynomial Model

This model was chosen for this study due to its high accuracy and low computational complexity. The equation used for training a predistorter using this model is given by:

$$x(n) = \sum_{k=0}^{K} \sum_{l=0}^{L-1} h_{kl} \frac{y(n-l)}{G} \left| \frac{y(n-l)}{G} \right|^{k}$$
 (1)

where is the delayed version of the complex output sample of the PA normalized by its small-signal gain , is the input signal shown in Figure 1, and are the set of complex model coefficients. The constants L and K are known as the memory depth and the nonlinearity order, respectively.

In practical terms, the memory depth is of great importance when modelling and predistorting amplifiers driven by wideband signals that emulate memory effects. The more nonlinear a device is, the higher the value of the parameter K required [5]. To compute one sample of the memory polynomial output, a vector of entries is constructed by taking L samples of the output signal, through and combining them in the manner specified by (1).

Due to the particular structure of the model, the correlation observed in the data used grows higher as the memory depth and nonlinearity order increased [8] This high level of correlation present in the data matrices of the memory polynomial model means that the estimation of the model's coefficients cannot be carried out using simpler adaptive algorithms such as the least mean squares (LMS) algorithm and its family, thereby forcing the use of more complex algorithms such as RLS or LMSN [5][8]. In the literature, the RLS algorithm has been used to implement digital predistorters, such as in [9].

# 3. The Least Mean-Squares Newton (LMSN) Algorithm

The algorithm proposed in [10], is known to outperform the standard least mean squares (LMS) algorithm when the data used has a large eigenvalue spread or when the regressor matrices involved are ill-conditioned, such as is the case with the MP model, as explained previously. This algorithm exhibits many desirable characteristics such as stability, robustness and accuracy [9]. An additional advantage of the LMSN algorithm is that it uses less parameters than the RLS algorithm, which utilizes a forgetting factor. Due to this, the LMSN algorithm is more robust to parameter choices than RLS. The equations defining the LMSN algorithm are found in [9], and are given in Table 1 for reference, noting that in this version of the algorithm, the dependence of the LMSN algorithm on a user-selected constant is removed by dropping the parameter  $\alpha$  used in usual implementations

for 
$$n = 1:N$$

$$\mathbf{R}^{-1}(0) = \delta \mathbf{I}$$

$$\mathbf{w}(0) = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^T$$
for  $n = 1 : \mathbf{N}$ 

$$e(n) = d(n) - \hat{\mathbf{y}}(n)$$

$$\mathbf{R}^{-1}(n) = \begin{bmatrix} \mathbf{R}^{-1}(n-1) - \frac{\mathbf{R}^{-1}(n-1)\mathbf{u}^H(\mathbf{n})\mathbf{u}(n)\mathbf{R}^{-1}(n-1)}{1 + \mathbf{u}(n)\mathbf{R}^{-1}(n-1)\mathbf{u}^H(\mathbf{n})} \end{bmatrix}$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mu \mathbf{R}^{-1}(n)\mathbf{u}^H(\mathbf{n})e(n)$$

where

 $\delta$  is a constant usually having large values ( $\geq 10^3$  in this study).  $\mu$  is the step size, where  $0 < \mu < 1$ 

of the algorithm. This has the advantage of increasing the robustness of the algorithm.

### 4. Experimental Validation of the Proposed Algorithm

To validate the performance of the proposed LMSN variant, it was used to build a DPD for a highly nonlinear Doherty PA operating at 2.140GHz using four-carrier wideband code division multiple access (WCDMA 1001 and WCDMA 1111) signals having a bandwidth of 20MHz and sampled at 92.6MHz. In the WCDMA 1001 signal, only the outer carriers are used to transmit, while in the WCDMA 1111 all four carrier are used for transmission. The proposed NLMS algorithm was benchmarked against RLS for estimating the coefficients of a memory polynomial DPD with memory depths varying between 3 and 5, and a nonlinearity order K between 5 and 6. The resulting values of normalized mean square error (NMSE) were collected and are presented in Table 2 along with the total amount of CPU time required for the identification process.

From the results in Table 2, it can be seen that the proposed algorithm consistently outperforms the RLS algorithm by a margin of 1~1.5dB in terms of error while requiring about 10% less computational time. This supports the value of the proposed algorithm and demonstrates its potential in various applications.

To further illustrate the value of the LMSN algorithm, Figures 2 and 3 present the frequency-domain performance of the LMSN- and RLS-based DPDs for the WCDMA 1001 and 1111 signals, respectively. Looking at the spectra in Figures 2 and 3 it can be seen that the LMSN-based predistorter outperforms the RLS-based one. These results demonstrate the value of the LMSN algorithm when used for digital predistortion, as its performance manages to exceed the benchmark set by RLS in both the time- and frequency domains.

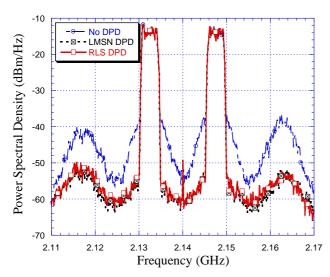


Figure 2. Frequency-domain performance of the RLS- and LMSN-based DPDs when the WCDMA 1001 signals are used (L=5, K=6).

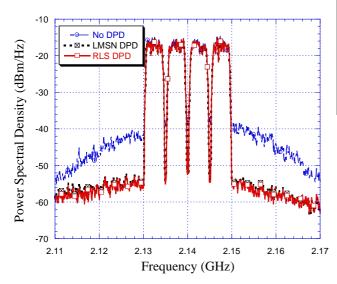


Figure 3. Frequency-domain performance of the RLS- and LMSN-based DPDs when the WCDMA 1111 signals are used (L=5,K=5).

### 5. Conclusions

In this work, the use of the LMSN algorithm to construct efficient and accurate DPDs was proposed. Through simulation results, based on measured power amplifier characteristics, the advantage of this algorithm was demonstrated by comparing its performance to that of RLS in the context of digital predistortion in both the time- and frequency- domains. The results show that the LMSN based DPD leads to better results that the RLS counterpart while requiring less computation time

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TABLE. II. NMSE AND CPU TIME ACHIEVED BY EACH ALGORITHM FOR VARIOUS SIZES OF THE MP MODEL.

	Model		Algorithm	NMSE	Total CPU
	Dimensions		C	(dB)	Time (sec)
WCDMA 1001	L=3	K=5	LMSN	-30.97	0.8
			RLS	-30.96	0.9
		K=6	LMSN	-35.50	1.21
			RLS	-34.90	1.23
	L=4	K=5	LMSN	-32.75	0.90
			RLS	-31.04	0.99
		K=6	LMSN	-35.40	1.00
			RLS	-34.18	1.13
	L=5	K=5	LMSN	-33.43	1.11
			RLS	-31.02	1.16
		K=6	LMSN	-35.68	1.22
			RLS	-34.12	1.26
WCDMA 1111	L=3	K=5	LMSN	-33.10	1.00
			RLS	-32.86	1.21
		K=6	LMSN	-36.62	1.07
			RLS	-36.55	1.23
	L=4	K=5	LMSN	-33.13	1.13
			RLS	-32.89	1.28
		K=6	LMSN	-36.92	1.25
			RLS	-36.84	1.35
	L=5	K=5	LMSN	-33.18	1.46
			RLS	-32.93	1.52
		K=6	LMSN	-36.94	1.64
			RLS	-36.86	1.62

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