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Active porous media: waves and muscles

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Abstract—Many parts of biological organisms are comprised of deformable porous media. The biological media is both pliable enough to deform in response to an outside force and can deform by itself using the work of an embedded muscle. For example, the recent work [1] has demonstrated interesting 'sneezing' dynamics of a freshwater sponge, when the sponge contracts and expands to clear itself from surrounding polluted water. We derive the equations of motion for the dynamics of such an active porous media (*i.e.*, a deformable porous media that is capable of applying a force to itself with internal muscles), filled with an incompressible fluid. These equations of motion extend the earlier derived equation for a passive porous media filled with an incompressible fluid. We use a variational approach with a Lagrangian written as the sum of terms representing the kinetic and potential energy of the elastic matrix, and the kinetic energy of the fluid, coupled through the constraint of incompressibility. We then proceed to extend this theory by computing the case when both the active porous media and the fluid are incompressible, with the porous media still being deformable, which is often the case for biological applications. For the particular case of a uniform initial state, we rewrite the equations of motion in terms of two coupled telegraph-like equations for the material (Lagrangian) particles expressed in the Eulerian frame of reference, particularly suitable for numerical simulations, formulated for both the compressible media/incompressible fluid case and the doubly incompressible case. We derive interesting conservation laws for the motion, perform numerical simulations in both cases and show the possibility of self-propulsion of a biological organism due to particular running wave-like application of the muscle stress.

1. Short description of work and results

In this presentation, we described the initial variational approach to porous media developed in [2, 3]. We start by outlining the variational approach to the porous media by defining the media consisting of interacting variables of elastic matrix and fluid, with both parts may be either compressible or incompressible. We show that the additional complexity of matrix incompressibility for biologically relevant materials can be treated similarly, as the solid's incompressibility constraint introduces another Lagrange multiplier related to the pressure inside the solid. Mathematically, we base our methods on the classical Arnold's description of incompressible fluid [4] as geodesic motion on the group of volume-preserving diffeomorphisms of the fluid domain, in the absence of external forces. In Arnold's theory, the Lagrangian is simply the kinetic energy, as the potential energy of the fluid is absent, and the fluid pressure enters the equations from the incompressibility condition. We develop the linearized equations of motion to compute the wave propagation properties in the material, including wave speeds and attenuation. We also include the actions of the muscle by using a variational approach and show that the application of the muscle and its effect on the boundaries follows from a modified Lagrange-d'Alembert principle. We then show that there is an exact reduction of the fully nonlinear model for one-dimensional motion and derive integrals of motion, such as the net momenta and, in the case of double incompressibility, an affine relationship between the Lagrangian variables of the fluid and the solid. We perform numerical simulations of the resulting reduced one-dimensional equations for incompressible fluid, for both compressible and incompressible matrix. We illustrate the difference between these cases and show the possibility of self-propulsion of the porous matrix (solid) while preserving the net-zero momentum of the fluid and solid. We also discuss variational approach to the thermodynamics of porous media as recently described in [5]. That approach allows us to determine the thermodynamically consistent form of forces and stresses acting within the porous media.

References

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