Breakdown of Inter-Cluster Synchronization of Coupled Chaotic Circuits Arranged in One-Dimensional Coordinate

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Abstract—In this study, we investigate synchronization phenomena of coupled chaotic circuits. The chaotic circuits are combined by resisters on one-dimensional coordinate system. We change the distance between the circuits to adapt the coupling strength. We investigate synchronization phenomena when the distance between the circuits in the group is changed. Also, we measure the phase difference using computer simulations. From the computer simulations, we could make sure of the breakdown of inter-cluster synchronization when the system is changed from the symmetric system to the asymmetric system.

1. Introduction

Synchronization phenomenon is one of the typical phenomena observed in nature. Recently, many studies have been investigated synchronization of chaotic circuits [1]~[5]. It is focused how the differences of the network structure impact on the whole circuits. Additionally, it is applicable to the fields of medical science and biology and so on.

In our research group, we have investigated the clustering phenomena resulting from the synchronization phenomena observed in coupled chaotic circuits when the chaotic circuits are arranged in two-dimensional coordinate [6], [7]. We observed that the chaotic circuits arranged in the near distance are synchronized at in-phase state, and the coupled circuits with the far distance could not be synchronized. From the results we confirmed the relationship between clustering and synchronization phenomena.

The more detailed researches are needed for applying to the investigation of more large scale network and general network. So in this study, we investigate the synchronization phenomena observed from symmetric coupled chaotic circuits and asymmetric coupled chaotic circuits arranged in one-dimensional coordinate. We combine chaotic circuits by resister, and the circuits are arranged in one-dimensional coordinate system. The number of the circuits is always ten and we investigate symmetric systems and asymmetric systems. The distance between the central circuits is fixed. We investigate synchronization phenomena by changing the distance between the circuits.

In this study, we use ladder system. In the ladder system, chaotic circuits are connected to only adjacent circuits. Figure 1 shows the system model of the ladder system. We measure the phase difference using computer simulations.

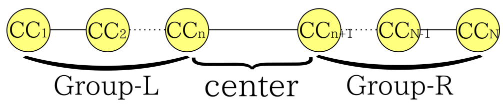


Figure 1: Ladder system.

2. Circuit Model

Figure 2 shows the circuit model. This is a chaotic circuit called Nishi-Inaba circuit [8]~[10].

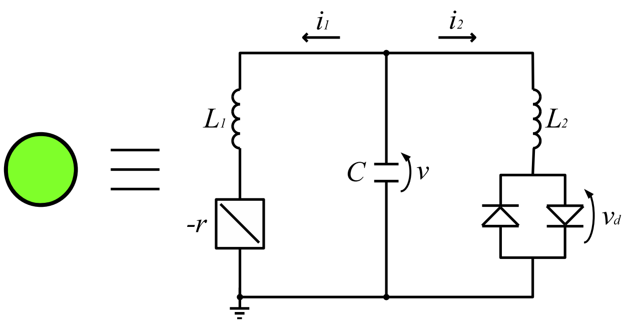


Figure 2: Circuit model.

The circuit equations of this circuit are described as Eq. (1).

$$\begin{cases} L_1 \dfrac{di_1}{dt} = v + ri_1 \\ L_2 \dfrac{di_2}{dt} = v - v_d(i_2) \\ C\dfrac{dv}{dt} = -i_1 - i_2 \end{cases} \qquad (1)$$

The characteristic of nonlinear resistance is described as Eq. (2).

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right) \quad (2)$$

The circuit equations are normalized as Eq. (3) by changing the variables as below.

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1C}}{L_2}Vy;\ v = Vz;$$
$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1C}}{L_2} = \delta;$$
$$t = \sqrt{L_1C}\tau;\ \text{“}\cdot\text{”} = \frac{d}{d\tau};$$

$$\begin{cases} \dot{x} = \alpha x + z \\ \dot{y} = z - f(y) \\ \dot{z} = -x - \beta y \end{cases} \quad (3)$$

The value of $f(y)$ is described as Eq. (4).

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right) \quad (4)$$

Figures 3 and 4 show the chaotic attractor generated from the circuit by using computer simulation (Fig. 3) and circuit experiment (Fig. 4). For the computer simulation, we set the parameters as $\alpha = 0.460, \beta = 3.0$ and $\delta = 470$. For the circuit experiment, the parameters are fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.0153[\mu F]$, and $r_d = 1.46[M\Omega]$.

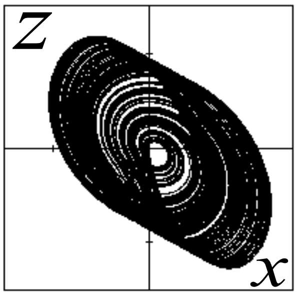


Figure 3: Chaotic attractor (computer simulation).

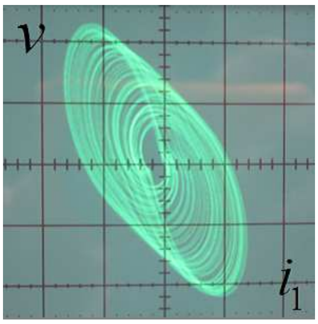


Figure 4: Chaotic attractor (circuit experiment).

In this study, we use the ladder system. In the ladder system, chaotic circuits are connected to only adjacent circuits.

When chaotic circuits are connected to only adjacent circuits, the circuit equations are shown in Eqs. (5) ~ (7).

CC_1:

$$\begin{cases} \dot{x}_1 = \alpha x_1 + z_1 \\ \dot{y}_1 = z_1 - f(y_1) \\ \dot{z}_1 = -x_1 - \beta y_1 - \gamma_{\{1,2\}}(z_1 - z_2) \end{cases} \quad (5)$$

CC_n:

$$\begin{cases} \dot{x}_n = \alpha x_n + z_n \\ \dot{y}_n = z_n - f(y_n) \\ \dot{z}_n = -x_n - \beta y_n - \gamma_{\{n,n-1\}}(z_n - z_{n-1}) \\ \qquad -\gamma_{\{n,n+1\}}(z_n - z_{n+1}) \end{cases} \quad (6)$$

CC_N:

$$\begin{cases} \dot{x}_N = \alpha x_N + z_N \\ \dot{y}_N = z_N - f(y_N) \\ \dot{z}_N = -x_N - \beta y_N \\ \qquad -\gamma_{\{N,N-1\}}(z_N - z_{N-1}) \end{cases} \quad (7)$$

Where the parameter γ_{ij} represents the coupling strength between the circuits. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

$$\gamma_{\{i,j\}} = \frac{g}{(d_{ij})^2}. \quad (8)$$

d_{ij} denotes the Euclidean distance between the i-th circuit and the j-th circuit. The parameter g is coupling coefficient that determines the coupling strengths. In this study, we set the parameter as $g = 1.0 \times 10^{-3}$.

3. Simulation Method

We use the ladder system arranged in one-dimensional coordinate system. We use ten circuits in computer simulations. We divide into the two symmetric groups, and there are five circuits in one side of the group. In the left side and the right side groups, the distances between the circuits are 0.3. The distance between the central circuits is 0.5. The symmetric network structure is shown in Fig. 5(a).

We change the distance between the circuits by changing the coupling strength. We define the distances between the circuits in the left side group as d_1. In the same way, the distances between the circuits in the right side group as d_2. And we define the distance between the central circuits as d_{center}. In this simulation, we fix the values of d_{center} and d_2, and the value of d_1 is changed. The value of d_1 is decreased gradually, and the value of d_1 is changed from 0.3 to 0.1. The asymmetric network structure is shown in Fig. 5(b).

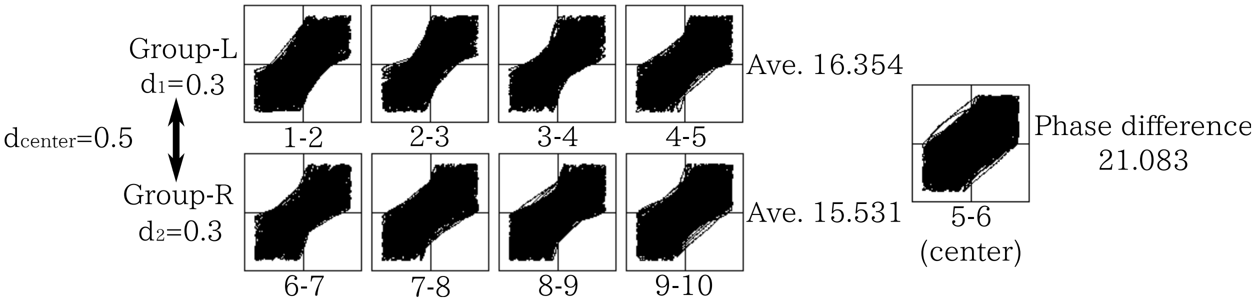


Figure 6: Graphics of the phase difference (computer simulations of symmetric system).

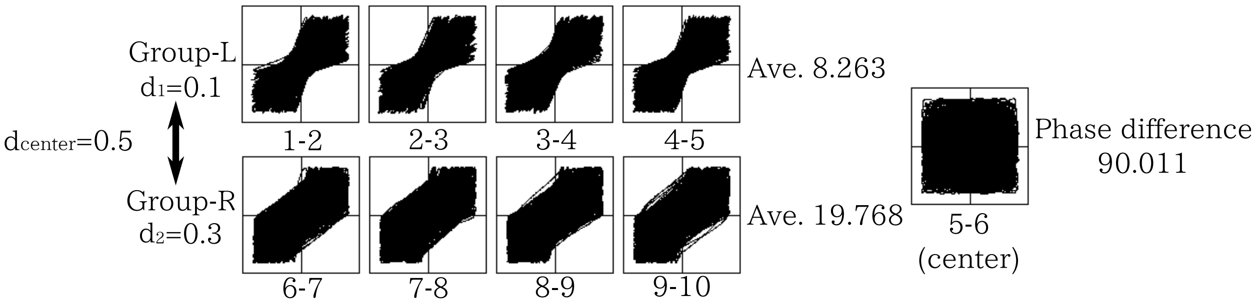
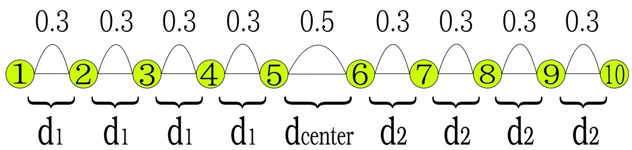
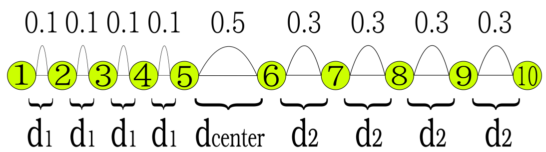


Figure 7: Graphics of the phase difference (computer simulations of asymmetric system).



(a) Symmetric network structure.



(b) Asymmetric network structure.

Figure 5: Network structures.

We measure the phase difference between the circuits using the computer simulation. And we investigate the change in the phase difference when the system is changed from the symmetric system to the asymmetric system.

4. Simulation Result

Figure 6 shows the simulation result of symmetric system. From this figure, we can see that the all circuits are synchronized. Figure 7 shows the simulation result of asymmetric system. From this figure, we can see that the circuits in the groups are synchronized, however the central circuits become asynchronous. And the phase differences in the right side group are larger than the symmetric system. Additionally, the phase differences in the left side group are smaller than the symmetric system. Table 1 shows the averages of the phase differences in each system.

Figure 8 shows the simulation result when d_1 is changed. This result shows the shift of the phase difference. We focus on the phase differences of the central circuits, between the 4*th* and the 5*th* circuits, and between the 6*th* and the 7*th* circuits. From the simulation result, the various results can be obtained.

Table 1: Averages of the phase differences

	Group-L	d_{center}	Group-R
Symmetric system	16.354	21.083	15.531
Asymmetric system	8.263	90.011	19.768

In the case of the central circuits, the phase difference is increasing gradually. And the central circuits become asynchronous around $d_1 = 0.19$. In the case of between the 4*th* and the 5*th* circuits, the phase difference is continued to decrease because the distance between the circuits in the left side group is continued to decrease. In the case of between the 6*th* and the 7*th* circuits, the phase difference is increasing gradually, and the phase difference is decreased

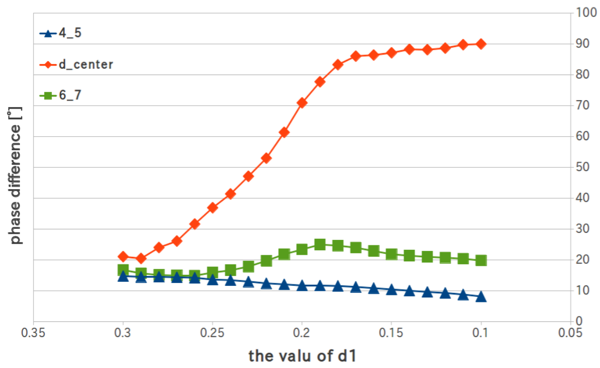


Figure 8: Simulation result.

from around $d_1 = 0.19$.

In the right side group, the phase difference began to decrease with the central circuits became asynchronous. Asynchronous state of the central circuits strengthen the coupling strength of each groups.

5. Conclusions

In this study, we have investigated the synchronization phenomena in coupled chaotic circuits networks. We also investigated the phase difference in the symmetric network system and the asymmetric network system. From the computer simulation, when using the symmetric network system, all circuits are synchronized. And when using the asymmetric network system, only central circuits become asynchronous. From this results, we could make sure of the breakdown of the inter-cluster synchronization.

For the future work, we would like to confirm the same results by using the circuit experiments.

Acknowledgment

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