2023 International Symposium on Nonlinear Theory and Its Applications
NOLTA2023, September 26-29, 2023, Catania and Online

Complex behavior of two-dimensional optical bistable device with external feedback

Takashi ISOSHIMA

Nano Medical Engineering Laboratory, Cluster for Pioneering Research (CPR), RIKEN
2-1 Hirosawa, Wako, Saitama 351-0198, Japan

Email: isoshima@riken.jp

Abstract– Bistable system with a spatial expanse can support a traveling wavefront, an interface between two stable states. We investigate a two-dimensional optical bistable device (2DOBD) for natural computing including maze exploration. In our device, wavefront propagation is flexibly controllable by light intensity and other parameters. In this paper, we describe complex dynamics in wavefront propagation due to an external feedback. Refractory feedback can provide not only periodic but also unstably fluctuating pulse trains that may be chaotic.

1. Introduction

In a bistable system with a spatial expanse, the two stable states - in this paper they are called as "on" and "off" states - can coexist occupying different locations in the system. The interface between the "on" and "off" areas can travel like a wavefront [1]. In our research, we investigate properties and applications of such wavefront propagation in a two-dimensional bistable device for information processing, as a kind of natural computation. Natural computation is a scheme for information processing that utilizes dynamics of various natural phenomena. It is attracting attentions as an alternative to the digital computing on the silicon-based integrated circuits. Maze exploration is a popular application of natural computation, and various reports to solve a maze by natural computation have been published. Some examples are: propagating chemical reaction waves in an excitable Belousov-Zhabotinsky (BZ) medium [2], a self-propelling oil droplet in aqueous solution of amphiphilic reagent driven by pH gradation [3], expansion and shrinkage of a true slime mold [4], and so on. In our research, we have employed a two-dimensional optical bistable device (2DOBD) as the medium for wavefront propagation. In our previous works [5-12], we have reported that the wavefront of "on"-state area in a 2DOBD presents extension and reduction (retreat) behavior controllable by the irradiated light intensity, that Steinbock maze can be explored, that the wavefront velocity is also controllable geometrically and thermally.

In addition, we introduced an external feedback to 2DOBD to realize more complex behavior in the wavefront propagation. We have demonstrate external refractory feedback inspired by a nerve cell enables 2DOBD to generate pulses [6,12]. In this report, we will present that external refractive feedback can provide various interesting dynamics such as annihilation of colliding pulses and unstable pulse generation that might be chaotic.

2. Two-dimensional optical bistable device (2DOBD): principle of operation and device structure

In general, bistability is achieved with a positive feedback and nonlinearity. Optical bistability can be categolized into two: all-optical type and thermo-optical type. All-optical bistability utilizes third-order optical nonlinearity, mostly optical Kerr effect that varies refractive index with the light intensity. Bistability can be realized using an optical resonator such as a Fabry-Perot cavity filled with an optical Kerr medium. The optical resonator provides feedback and the optical Kerr medium provides nonlinearity. The device is irradiated with laser light at a wavelength slightly off-resonant to the resonator. At this moment, only a small fraction of the light enters into the resonator. If we increase the incident light intensity, the refractive index of the medium is modulated by the optical Kerr effect, inducing resonance wavelength shift, and thus the light wavelength becomes more on-resonant. This increases the fraction of incident light going in the resonator, that causes further shift of resonance. This positive feedback provides fast shift of resonance to the "on" state. If we decrease the light intensity, the device keeps its "on" state until the intensity is reduced to much lower than that for turn-on, resulting into hysteresis property. Thus optical bistability is realized. Such an all-optical bistable device operates very fast, but requires extremely large intensity of light for operation that usually requires a femtosecond pulse laser. On the other hand, thermo-optical bistability is achieved through interaction between temperature, temperature-dependent optical property change (usually this presents nonlinearity), and heat generated by light. More detailed description will be provided in the next paragraph. Although this device operates much slower than an all-optical one, the light intensity required for operation is significantly smaller than the other. These features makes experiment much easier: slow operation of the device can be easily observed with an ordinary video camera, and

---

ORCID iD First Author: 0000-0002-3421-036X low light intensity for operation can be realized by a video projector. Considering these advantages, we have employed the thermo-optical bistability in this work.

To realize thermo-optical bistability, we have designed a device composed of a liquid crystal and an optical absorber. The liquid crystal presents phase transition between isotropic and nematic phases at a certain temperature. For example, 4-Cyano-4'-pentylbiphenyl (5CB) presents the phase transition at about T_{PT} = 35 °C. Below T_{PT}, 5CB is in the nematic phase and looks opaque, thus the optical transmission is low because of significant light scattering. Above T_{PT}, 5CB is transparent in the isotropic phase. Therefore 5CB presents nonlinear behavior in the optical transmission as a function of temperature. Our device has a layered structure composed of a top cover, the 5CB layer, and a black bottom substrate, as shown in Fig.1. The bottom substrate absorbs light transmitted through the 5CB layer, generating heat from light. Assuming that the temperature of the whole device is below T_{PT}, 5CB is in nematic phase and opaque. Light irradiated from the top side is scattered in 5CB layer and only a small fraction of the light is absorbed by the bottom black layer. If the incident light intensity is increased, the heat by photoabsorption also increases, and the temperature raises. When the local temperature reaches T_{PT}, 5CB starts phase transition to isotropic phase, becoming transparent. This further increases heat generation and temperature, functioning as the positive feedback to cause transition to the high transmission "on" state. Once turned on, even if the light intensity is decreased, the device stays in "on" state until the light intensity becomes significantly lower than the turn-on threshold, presenting the hysteresis property. In this hysteresis region of light intensity, the device presents bistability.

If the device has a spatial expanse, "on" (high transmission) and "off" (low transmission) areas can coexist at different locations in the device, and thus it can provide wavefront propagation. Under irradiation of bias light at the bistable intensity on the whole device area, the whole device can stay at the "off" state at the beginning. If sufficiently strong light is irradiated to a small area on the device, it is locally triggered to "on" state. The turn-on wavefront can propagate two-dimensionally due to thermal diffusion. In maze exploration, a maze pattern is irradiated on the device, the path area irradiated at a higher intensity and the wall area at a lower intensity. Exploration starts by irradiating strong trigger light at the start point of the maze, and the wavefront propagates through the paths in the maze. The propagation velocity can be controlled by light intensity, and even negative velocity - retreat of on area - can be realized. Utilizing these extension and reduction (retreat) modes, a maze can be explored to present the start-to-goal path(s). [5-10]

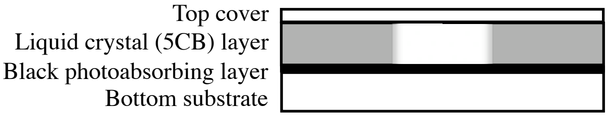

Figure 1: Schematic cross-sectional diagram of a 2DOBD. The top cover and the bottom substrate compose a cell structure to encapsulate the liquid crystal 5CB. The light irradiated from the top side transmits through the 5CB layer and is absorbed in the black layer to generate heat. 5CB presents lower optical transmission due to optical scattering at a temperature lower than 35°C (in nematic phase, shown in grey), while it shows higher transmission at a temperature higher than 35°C (in isotropic phase, shown in white). The wavefront (interface between the grey and white area) can propagate laterally.

3. Numerical Simulation Method

The operation of a 2DOBD can be modeled using a thermal diffusion equation including heat source by photoabsorption terms, as follows: [5-12]

$$\frac{\partial T}{\partial t} = \Delta T + \sigma - \rho \qquad (1)$$

$$\sigma = A(T)\, I \qquad (2)$$

$$\rho = \alpha\, T \qquad (3)$$

Here t stands for the time. T the temperature, σ the generated heat, and ρ the heat dissipation at the surface to surrounding air, and these are functions of spatial coordinates and time $(x,y,z;t)$. A is the optical absorption (0: no absorption ~ 1: complete absorption) depending on the temperature T, and I is the incident light intensity as a function of x, y, and t. α is the heat dissipation constant (heat resistance). Nonlinearity essential for the bistability is included in the term $A(T)$.

Time-dependent three-dimensional (3D) numerical simulation was performed by means of the finite element method using FreeFEM++ Ver.4.10 software [13]. Device structure shown in Fig.1 was used in the simulation. Using this device structure, we are also trying experimental demonstration, which will be reported separately in future. The materials used here are same as our experimental device. Acrylic polymer is used for the the top cover and bottom substrate. Bottom surface of the substrate is contacted to a temperature-controlled hot plate kept constant at 30 °C. The top cover and side wall of the device is exposed to air at 25 °C. Thicknesses of the top cover, 5CB layer, black layer, and bottom substrate were 0.2, 0.1, 0.02, and 1.0 mm, respectively. Optical absorption of the black layer is 100 %, converting all light reaching to this layer to heat. Thermal diffusion constant of acrylic polymer and 5CB is 0.92 and 1.0 [10^{-3} cm^2 s^{-1}], respectively. Heat dissipation constant (heat resistance) at the boundary of the device is 0.92×10^{-2} [W °C^{-1}]. The finite element size in the simulation was 0.5 mm and the time step for sequentially solving the differential equation was 2 sec. Temperature-dependent transmission of 5CB layer was approximated to be 0.5 at 35 °C or lower, 1.0 at 37 °C or higher, and linearly increasing from 0.5 to 1.0 in the temperature range of 35 to 37 °C. Light intensity was 0.250 and 0.130 W/cm^2 at the path and the wall area of the maze, respectively, if not described explicitly. Trigger light intensity was 0.450 W/cm^2.

External feedback system is assumed to be experimentally composed of a 2DOBD, external video camera, a personal computer and a video projector, as

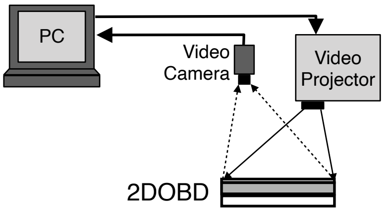

Figure 2: Schematic diagram of an experimental setup for 2DOBD with external feedback. Video camera observes the state of the 2DOBD. PC, a personal computer, analyzes distribution of the state of 2DOBD obtained by the camera, and generates the light pattern based on the feedback feedback algorhithm. Video projector irradiates patterned light onto the 2DOBD.

schematically illustrated in Fig.2. The state of 2DOBD is observed by the camera and transferred to the PC to analyze the state distribution and generate an irradiation pattern based on the feedback algorithm, and the projector irradiates the patterned light to the 2DOBD. Refractory feedback was designed to mimic the refraction period of a nerve cell: once turn-on of a (small) area of the 2OOBD is detected, the light intensity irradiated to the area is reduced for a fixed time (refraction time) to force the area turned off. To reduce the amount of calculation, the test path was divided into pixels of 3x3 mm as the units for detection of the state and feedback control of the light intensity. (Note that the width of the path and wall is 3 mm.) Refraction time was set to 120 sec for the simulation of counter-propagating pulses, and set to various value between 60 and 120 sec to examine dependence to refraction time. During the refraction period, the light intensity was reduced by 44 %. [12]

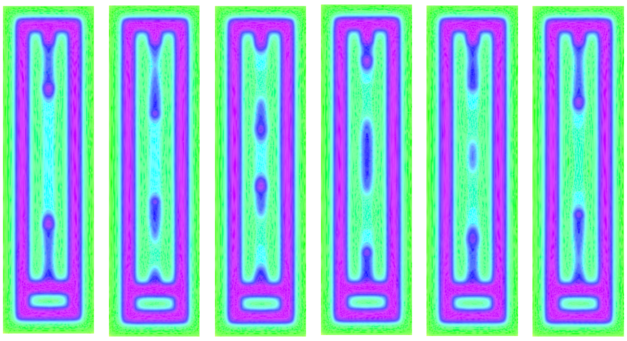
80 sec 100 sec 120 sec 140 sec 160 sec 180 sec

Figure 3: Temperature distribution at 5CB layer of 2DOBD, obtained by 3D finite element method simulation. In this simulation, the external double rectangular path is always “on”, and refractory feedback is applied only to the central path. Therefore, the connections of the central path and the “always-on” path (top and bottom ends of the central path) trigger the central path at the top and bottom ends. "On" areas propagates through the center path (80 sec), and refractory feedback turns the backward area off, providing pulses (100-120 sec). The counter-propagating pulses collide to each other (140 sec), and cannot propagate further due to the refraction area of the other pulse, resulting into annihilation (160-180 sec). Color coding: green stands for 30 °C (wall area), light blue for 34 °C (path area in "off" state), dark blue for 38 °C and purple for 41 °C (both path area in "on" state). Irradiated light intensity $I_{\mathrm{path}} = 0.250$ W/cm^2 and $I_{\mathrm{wall}} = 0.130$ W/cm^2, refractory time $t_{\mathrm{ref}} = 120$ sec.

4. Results and Discussions

As described in our previous literature [12], refractory feedback provides excitability to 2DOBD, and pulse train can be generated in the path if continuously triggered at one end of the path. If a path is triggered at both ends, counter-propagating pulse trains are generated. The counter-propagating pulses collide to each other and disappears at the center of the path, as shown in Fig.3. This "collision-annihilation" is due to the fact that each pulse is traced by refractory area and thus a pulse cannot proceed forward when colliding with another counter-propagating pulse having a refractory area in "front" of the first pulse. Such a collision-annihilation is an interesting phenomenon also observable in other nonlinear pulse-propagating system like solitons.

Another interesting dynamics can be observed if the refraction time is in a suitable intermediate range. Dynamic behavior of 2DOBD with refractory feedback was investigated in terms of dependence to the refraction time.. At a shorter refraction time (60 sec or less), the on-area cannot be disconnected and no pulse was generated. At a sufficiently long refraction time (110 sec or more), stable periodic pulse train was generated, since the detected on-area is completely "reset" to the off-state during the sufficient refraction time.

At an intermediate refraction time (70-100 sec), the behavior is complex and unstable: both length and period of the pulses are fluctuating. Fig.4 shows snapshots of the state distribution of 2DOBDs at the refraction time of 80 sec. In this intermediate range of refraction time,

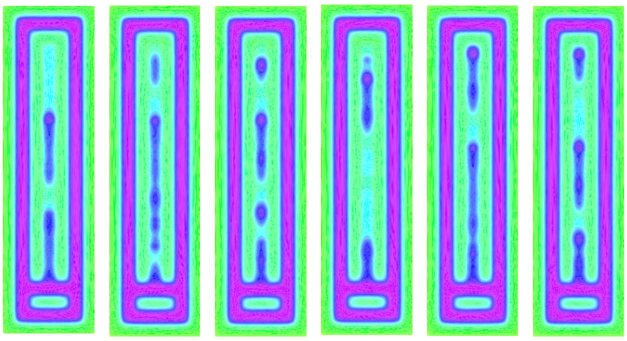
1000 sec 1200 sec 1400 sec 1600 sec 1800 sec 2000 sec

Figure 4: Temperature distribution at 5CB layer of 2DOBD, obtained by 3D finite element method simulation. In this simulation, the external double rectangular path is always “on”, and refractory feedback is applied only to the central path. The central path is connected to the surrounding rectangular path only at the bottom, and the top of the central path is a "dead end". Therefore, the central path is triggered only at bottom end. "On" areas propagates through the center path from bottom to top, but due to intermediate refraction time of 80 sec, the behavior is quite unstable (possibly chaotic) as shown in the snapshots at 1000 - 2000 sec. Color coding and conditions are same as in Fig.3, except for the refractory time $t_{\mathrm{ref}} = 80$ sec.

refractory feedback can turn off the "on" area, but at the end of refractory time the state (temperature distribution) is not completely "reset" yet, and therefore next turn-on behavior fluctuates affected by the previous state. In some cases, the "on" state cannot be turned off, resulting in relatively long pulse. Such a complex and unstable state dynamics results into inhomogeneous pule length and pulse distance, and fluctuating period. This pulse behavior might be chaotic. Indeed refractory feedback can be regarded as a kind of delayed feedback, and therefore it is highly possible that the complex behavior shown here is chaotic. Evaluation of Lyapunov exponent or other chaos exponent is not completed yet, and will be performed in near future.

4. Concluding Remarks

In this paper, we presented various complex dymamic behavior in 2DOBD with an external refractory feedback, obtained by numerical simulation using finite element method. At a sufficiently long refraction time (110 sec or longer), stable pulse train can be generated. If counter-propagating pulses collides to each other, annihilation of the colliding pulses occurs. At a short refraction time (60 sec or less), the on area cannot be disconnected. Interestingly, at an intermediate length of refraction time (70-100 sec), the pulsing behavior becomes very complex, showing inhomogeneous pulse length and distance The complex dynamics might be chaotic, possibly because refractory feedback is a kind of delayed feedback.

If the behavior is chaotic, we can generate chaos in any area of 2DOBD by applying refractory feedback with an intermediate refraction time. Generated chaotic pulse train can be guided through designed paths, and collision or other kind of interaction can be introduced easily. Using such a flexible configuration, we may design a kind of chaotic optical neural network with 2DOBD. External feedback will extend the horizon of 2DOBD applications.

Acknowledgments

Part of this work was financially supported by Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research No.26540129).

References

[1] J. D. Murray, "*Mathematical Biology I: An Introduction - Third Edition*," Chapter 13, Springer Science+Business Media, New York, 2001.

[2] O. Steinbock, Á. Tóth, K. Showalter, "Navigating Complex Labyrinths: Optimal Paths from Chemical Waves", *Science*, Vol.267, p.868, 1995.

[3] I. Lagzi, S. Soh, P. J. Wesson, K. P. Browne, B. A. Grzybowski, "Maze Solving by Chemotactic Droplets", *J. Am. Chem. Soc.*, Vol.132, p.1198, 2010.

[4] T. Nakagaki, H. Yamada, Á Tóth, "Maze-solving by an amoeboid organism", *Nature*, Vol.407, p.470, 2000.

[5] Y. Okabayashi, T. Isoshima, E. Nameda, S. Kim, M. Hara, "Two-Dimensional Nonlinear Fabry-Perot Interferometer: An Unconventional Computing Substrate for Maze Exploration and Logic Gate Operation.," *Int. J. Nanotech. Mol. Comp.*, Vol.3, p.13, 2011.

[6] T. Isoshima and Y. Ito, "Two-dimensional optical bistable device with external feedback for pulse propagation and spatio-temporal instability", *Proc.NOLTA2014*, pp.625-629, 2014.

[7] T. Isoshima, "Control of wavefront propagation in two-dimensional bistable system", *Proc.NOLTA2017*, pp.271-274, 2017.

[8] T. Isoshima, "Operation mode of wavefront propagation for maze exploration in two-dimensional optical bistable device", *Proc.NOLTA2018*, pp.97-100, 2018.

[9] T. Isoshima, "Geometrical control of propagation properties in two-dimensional optical bistable devices", *Proc.NOLTA2019*, pp.550-553, 2019.

[10]T. Isoshima, "Multiple control of two-dimensional optical bistable device for encoding of complex maze", *Proc.NOLTA2020*, pp.430-433, 2020.

[11]T. Isoshima, "Wavefront propagation in two-dimensional optical bistable device and its application to maze exploration", *Nonlinear Theory and Its Applications, IEICE*, Vol.13, pp,53-59, 2022.

[12]T. Isoshima, "Destabilizing two-dimensional optical bistable device by external feedback", *Proc. NOLTA2022*, pp.73-76, 2022.

[13]Université Pierre et Marie Curie and Laboratoire Jacques-Louis Lions, *FreeFEM++*, http://www.freefem.org/ff++/