Closed Form Formulas of Correlation Coefficient for Ultra Wideband Impulse Radio Systems

#Pichaya Supanakoon†, Sathaporn Promwong† and Jun-ichi Takada††

† Department of Telecommunication Engineering, Faculty of Engineering,
King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand.
Email: {kspichay, kpsathap}@kmitl.ac.th
†† Graduate School of Engineering, Tokyo Institute of Technology,
2-12-1-S6-4, O-okayama, Meguro-ku, Tokyo, 152-8550, Japan.
Email: takada@ide.titech.ac.jp

Abstract

Correlation coefficient is a important parameter that indicates the efficiency of correlation receiver for Ultra Wideband Impulse Radio (UWB-IR) systems. The correlation coefficient is also used to analyze the waveform distortion. In this paper, the closed form formulas of correlation coefficient are derived using the rectangular passband waveform and the extension of Friss' transmission formula. The free space and ground reflection channels are considered. The results obtained from proposed formulas are compared with measurement. For ground reflection channel, the proposed formula of correlation efficient corresponds with measurement.

Keywords : ultra wideband impulse radio (UWB-IR), correlation coefficient

1 Introduction

Ultra wideband Impulse Radio (UWB-IR) is promising technique to achieve very high capacity, low cost and low power consumption properties for wireless systems. The Federal Communications Commission (FCC) in United States of America (USA) specified that UWB-IR signal has frequency spectrum ranging from 3.1 GHz to 10.6 GHz. The FCC defined that UWB-IR signal has fractional bandwidth equal to or greater than 0.20, or occupied bandwidth equal to ro greater than 500 MHz [1].

For UWB-IR correlation receiver with transmitted template signal, the correlation coefficient used to identify the receiver efficiency. The correlation coefficient is equal to 1 means the efficiency is 100%. In other word, that is the special case of correlation receiver, which is called as matched filter receiver. The transmitted template signal is identical with received signal. the correlation coefficient is decreased when the template signal is more differ. If the correlation coefficient is equal to 0, that means the template signal is orthogonal with received signal. There is no signal that can detect from the receiver. The correlation coefficient is also applied to analyze the waveform distortion [2].

In this paper, the closed form formulas of correlation coefficient are derived using the rectangular passband waveform [3] and the extension of Friss' transmission formula [4]-[5]. The free space and ground reflection channels are considered. The measurement was done for the ground reflection channel. The results obtained from proposed formula are shown and compared with measurement.

The rest of this paper is organized as follows. In section 2, correlation coefficient formulas are derived. Next, measurement and results are explained in sections 3 and 4, respectively. Finally, the conclusions are given in section 5.

2 Derivation of Correlation Coefficient Formulas

For deriving the correlation coefficient, the transmitted waveform is defined as rectangular passband waveform [3]. The expression of this waveform in time domain v_{t} and its spectral density V_{t} are

$$v_{\mathrm{t}}(t) = \frac{1}{f_{\mathrm{b}}}[f_{\mathrm{H}}\mathrm{sinc}(2f_{\mathrm{H}}t) - f_{\mathrm{L}}\mathrm{sinc}(2f_{\mathrm{L}}t)], \tag{1}$$

$$V_{\mathrm{t}}(f) = \begin{cases} \frac{1}{2f_{\mathrm{b}}} & ||f| - f_{\mathrm{c}}| \leq \frac{f_{\mathrm{b}}}{2} \\ 0 & ||f| - f_{\mathrm{c}}| > \frac{f_{\mathrm{b}}}{2} \end{cases}, \tag{2}$$

where t is the time, f is the frequency, f_{c} is the center frequency, f_{b} is the spectral bandwidth and $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. This waveform has 1 V maximum amplitude at time zero.

Subsequently, the received waveform v_{r} in time domain can be evaluated using

$$v_{\mathrm{r}}(t) = \int_{-\infty}^{\infty} H_{\mathrm{c}}(f) \cdot V_{\mathrm{t}}(f) \cdot e^{j2\pi ft} \mathrm{d}f, \tag{3}$$

where H_{c} is frequency transfer function of channel. The transmitter and receiver antennas are assumed to be isotropic antenna.

The correlation coefficient is defined as

$$C = \frac{\max | \int_{-\infty}^{\infty} v_{\mathrm{r}}^{*}(t) \cdot v_{\mathrm{t}}(t+\tau)\mathrm{d}t|}{\sqrt{\int_{-\infty}^{\infty} |v_{\mathrm{r}}(t)|^2 \mathrm{d}t \cdot \int_{-\infty}^{\infty} |v_{\mathrm{t}}(t)|^2 \mathrm{d}t}}, \tag{4}$$

where * is the complex conjugate operator. This definition is used to derive the closed form formula for free space and ground reflection channels.

2.1 Free Space Channel

The extension of Friis' transmission formula [4]-[5] is used to characterize the UWB-IR free space channel. The frequency transfer function of free space channel $H_{\mathrm{c,f}}$ can be written as

$$H_{\mathrm{c,f}}(f) = \frac{c}{4\pi|f|d} e^{-j2\pi fd/c}, \tag{5}$$

where c is the velocity of light and d is the distance.

The closed form formula of correlation coefficient for free space channel C_{f} can be derived as

$$C_{\mathrm{f}} = \frac{\sqrt{f_{\mathrm{L}}f_{\mathrm{H}}}}{f_{\mathrm{b}}} \ln\left(\frac{f_{\mathrm{H}}}{f_{\mathrm{L}}}\right). \tag{6}$$

2.2 Ground Reflection Channel

The frequency transfer function of ground reflection channel $H_{\mathrm{c,g}}$ is the addition between frequency transfer function of direct and reflection paths, and can be written as

$$H_{\mathrm{c,g}}(f) = \frac{c}{4\pi|f|d'} e^{-j2\pi fd'/c} + \Gamma \frac{c}{4\pi|f|d''} e^{-j2\pi fd''/c}, \tag{7}$$

where Γ is the reflection coefficient of ground, $d' = \sqrt{(h_{\mathrm{t}} - h_{\mathrm{r}})^2 + d^2}$ and $d'' = \sqrt{(h_{\mathrm{t}} + h_{\mathrm{r}})^2 + d^2}$ are distances of direct and reflection paths, h_{t} and h_{r} are the heights of Tx and Rx antennas, respectively.

The closed form formula of correlation coefficient for ground reflection channel C_{g} can be derived as

$$C_{\mathrm{g}} = \frac{\frac{1}{f_{\mathrm{b}}}\left\{\frac{1}{d'}\ln\left(\frac{f_{\mathrm{H}}}{f_{\mathrm{L}}}\right) + \frac{\Gamma}{d''}\left[\mathrm{Ci}\left(\frac{2\pi f_{\mathrm{H}}\Delta d}{c}\right) - \mathrm{Ci}\left(\frac{2\pi f_{\mathrm{L}}\Delta d}{c}\right)\right]\right\}}{\sqrt{\frac{1}{f_{\mathrm{L}} f_{\mathrm{H}} f_{\mathrm{b}} d' d''}\left\{\frac{f_{\mathrm{b}}(d''^2+\Gamma^2 d'^2)}{d' d''} - 2\Gamma\mathrm{Cr}\left(\frac{2\pi\Delta d}{c}\right) - \frac{4\pi\Gamma f_{\mathrm{L}} f_{\mathrm{H}}\Delta d}{c}\left[\mathrm{Si}\left(\frac{2\pi f_{\mathrm{H}}\Delta d}{c}\right) - \mathrm{Si}\left(\frac{2\pi f_{\mathrm{L}}\Delta d}{c}\right)\right]\right\}}}, \quad (8)$$

where $\Delta d = d'' - d'$ is the difference between direct and reflection paths, $\mathrm{Cr}(x) = f_{\mathrm{L}}\cos(x f_{\mathrm{H}}) - f_{\mathrm{H}}\cos(x f_{\mathrm{L}})$, $\mathrm{Ci}(x) = -\int_x^\infty \frac{1}{\tau}\cos(\tau)\mathrm{d}\tau$ and $\mathrm{Si}(x) = \int_0^x \frac{1}{\tau}\sin(\tau)\mathrm{d}\tau$ are cosine integral and sine integral functions, respectively.

3 Measurement

The measurement equipment, setup and parameters are the same as in [6]. The frequency transfer function of ground reflection channel was measured using a vector network analyzer (VNA). The measurement was done in the open area that is the soccer field of King Mongkuts Institute of Technology Ladkrabang (KMITL), Thailand. The metallic plate was laid on the ground between Tx and Rx antennas for the perfect reflection condition ($\Gamma = -1$).

The biconical antennas were used as Tx and Rx antennas. Both Tx and Rx antennas were fixed at the height of $h_{\mathrm{t}} = h_{\mathrm{r}} = 0.75$ m with horizontal polarization for focusing the signal from direct and ground reflection paths. The measurement distance d was ranged from 1 to 5 m with 0.25 m distance increment step. The frequency transfer functions of Tx and Rx antennas were evaluated by using three-antenna method [7] and were used to remove from the measurement frequency transfer functions of the ground reflection channels. The center frequency of waveform f_{c} is fixed at 4.1 GHz. The two cases of signal bandwidths are investigated, that are $f_{\mathrm{b}} = 0.5$ and 1.4 GHz.

4 Results

For free space channel, parameters f_{c} is fixed at 6.85 GHz and f_{b} is varied from 500 MHz to 7.5 GHz. Figure 1 shows the correlation coefficient. At narrow bandwidth, the correlation coefficient is approached to 1. The efficiency of correlation receiver is almost to 100%. The minimum correlation coefficient is 0.94 at 7.5 GHz bandwidth. The comparison between transmitted and received waveforms for this case is shown in Fig. 2. The efficiency of receiver is very good for free space channel. That means the waveform is slightly distorted in free space channel.

For ground reflection channel, the analyzed parameters are the same as measurement. Figures 3 and 4 show the comparison between correlation coefficeint from proposed formula and measurement with 500 MHz and 1.4 GHz bandwidths, respectively. The efficiency is significantly decreased compared with free space channel because of multipath fadding. The wide bandwidth has effect from multipath fadding along distance less than narrow bandwidth.

5 Conclusions

In this paper, the closed form formulas of correlation coefficient are derived using the rectangular passband waveform and the extension of Friss' transmission formula. The proposed formula corresponds with measurement. For free space channel, the efficiency of correlation receiver is very good and is more than 94%. On the other hand, the efficiency is significantly decreased for ground reflection channel because of multipath fadding. Averagely, the efficiency of correlation receiver is reduced to 77% and 78% with 500 MHz and 1.4 GHz bandwidths, respectively.

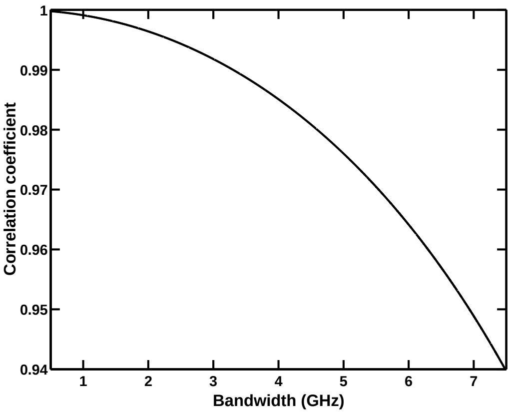

Figure 1: Correlation coefficient along 500 MHz to 7.5 GHz bandwidth for free space channel.

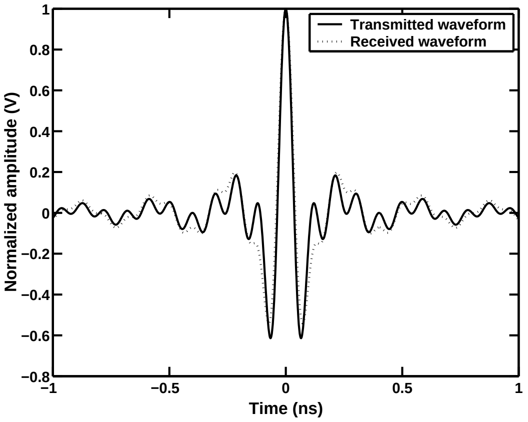

Figure 2: Comparison between transmitted and received waveforms with 7.5 GHz bandwidth.

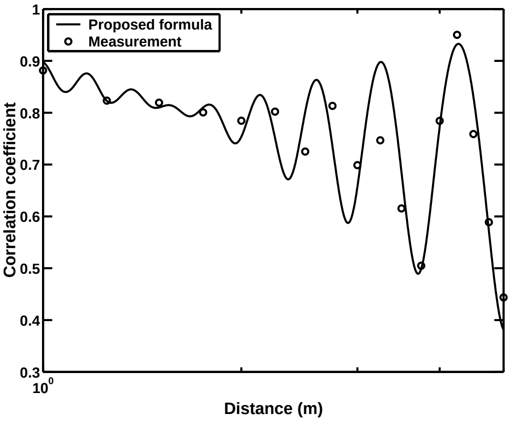

Figure 3: Comparison between correlation coefficient from proposed formula and measurement with 500 MHz bandwidth.

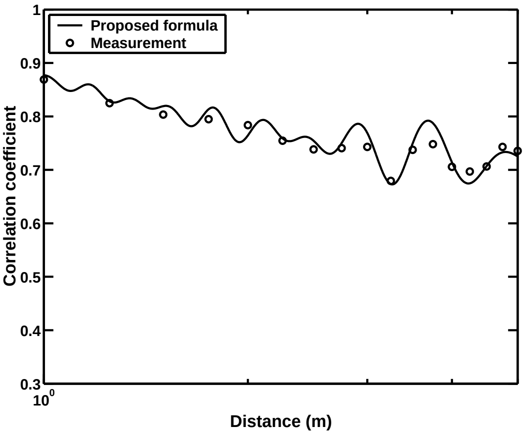

Figure 4: Comparison between correlation coefficient from proposed formula and measurement with 1.4 GHz bandwidth.

References

[1] Federal Communications Commission, "Revision of part 15 of the commission's rules regarding UWB transmission systems," First Report, FCC 02-48, Apr. 2002.

[2] S. Promwong, P. Supanakoon, J. Takada, "Waveform distortion and transmission gain on ultra wideband impulse radio," *IEICE Transactions on Communications*, vol. E93-B, no. 10, pp. 2644-2650, Oct. 2010.

[3] P. Supanakoon, K. Wansiang, S. Promwong and J. Takada, "Simple waveform for UWB communication," *Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON)*, pp. 626-629, 12-13 May 2005.

[4] J. Takada, S. Promwong and W. Hachitani, "Extension of Friis' transmission formula for ultra wideband systems," *Technical Report of IEICE*, WBS2003-8/MW2003-20, May 2003.

[5] S. Promwong and J. Takada, "Free space link budget estimation scheme for ultra wideband impulse radio with imperfect antennas," *IEICE Electronics Express*, vol. 1, no. 7, pp. 182-192, July 2004.

[6] S. Promwong, P. Supanakoon and J. Takada, "Ground refection transmission loss evaluation scheme for ultra wideband impulse radio," *ECTI-EEC Transactions on Electrical Engineering, Electronics, and Communications*, vol. 5, no. 1, pp. 18-24, Jan. 2007.

[7] S. Promwong, W. Hachitani and J. Takada, "Free space link budget evaluation of UWB-IR systems," *International Workshop on Ultra Wideband System Joint with Conference on Ultra Wideband Systems and Technologies (UWBST & IWUWBS)*, pp. 312-316, May 2004.