BER Performance Comparison Between OFDM/TDM and MC-CDMA with MMSE-FDE

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) combined with time division multiplexing (TDM) (called OFDM/TDM) using minimum mean square error (MMSE) frequency-domain equalization (FDE) can bridge the single carrier (SC) and the conventional OFDM transmissions while reducing the peak-to-average power ratio (PAPR) and maintaining the same data rate. Multi-carrier code division multiple access (MC-CDMA) with MMSE-FDE is one of the promising signaling techniques for future mobile communication systems. In this paper we present the performance comparison between OFDM/TDM and multi-code MC-CDMA with MMSE-FDE in a frequency-selective Rayleigh fading channel. The performance is evaluated in terms of average bit error rate (BER) by computer simulation. It is shown that uncoded OFDM/TDM achieves a lower BER performance in comparison to multi-code MC-CDMA while keeping the same data rate transmission and significantly reducing the PAPR. However, the coded BER performance of these two is almost the same.**

***Index Terms*—OFDM/TDM, multi-code MC-CDMA, turbo code, FDE.**

I. Introduction

Recently, a combination of orthogonal frequency division multiplexing (OFDM) and code-division multiple-access (i.e., single-code MC-CDMA) with frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criteria has been attracting much attention. This was due to its robustness against frequency-selective fading, efficient spectrum utilization and simple one-tap FDE [1], [2]. Extreme cases of MC-CDMA are OFDM and single carrier (SC) signaling techniques. Single-code MC-CDMA with MMSE-FDE achieves lower bit error rate (BER) performance in comparison to OFDM owing to frequency diversity effect when spreading factor (SF) increases. However, in the case of single-code MC-CDMA, two issues arise; (*i*) transmission data rate decreases when SF increases, and (*ii*) the problem of high peak-to-average-power ratio (PAPR). To alleviate the former a multi-code MC-CDMA was proposed. However, for the same transmission data rate as in OFDM, the BER performance of multi-code degrades in comparison to single-code MC-CDMA case due to high inter-code interference (ICI). Thus, the high PAPR of multi-code MC-CDMA cannot be avoided. It is worth mentioning that the single-code is a special case of multi-code MC-CDMA for $C = 1$, where C denotes the number of codes. In order to achieve the OFDM data rate transmission, multi-code case requires that $C = SF$. For multi-code MC-CDMA, data-modulated sequence is serial-to-parallel converted into $C = SF$ parallel streams and for each stream, the signal is serial-to-parallel (S/P) converted to N_c/SF streams; each symbol is then copied SF times and spread by multiplying with an orthogonal code having spreading factor SF. The C different streams are then added and further multiplied by a long scramble sequence [1], [2].

OFDM combined with time domain multiplexing (OFDM/TDM) with MMSE-FDE has been proposed [3], [4] to bridge the conventional OFDM and SC transmissions while keeping the same data rate transmission and significantly reducing the PAPR. In OFDM/TDM, the N_c-sample block for inverse fast Fourier transform (IFFT) is divided into K slots; during each slot one OFDM signal with $N_m = N_c/K$ subcarriers is transmitted and no guard interval (GI) is inserted between them. Thus, OFDM/TDM system achieves a lower PAPR in comparison with MC-CDMA. To the best of the authors' knowledge a performance comparison between the OFDM/TDM and MC-CDMA with MMSE-FDE has not yet been addressed in the literature.

In this paper, we present the performance comparison between OFDM/TDM with MMSE-FDE and multi-code MC-CDMA with MMSE-FDE under same transmission and propagation conditions. We discuss the achievable BER performance of these two systems for both coded and uncoded cases. Our results have indicated that in the case of uncoded systems, OFDM/TDM with MMSE-FDE outperforms multi-code MC-CDMA with MMSE-FDE due to lower inter-carrier interference. However, in the case of coded systems the achievable BER performance is the same.

Remainder of this paper is organized as follows. In Sect. 2, brief overview of OFDM/TDM and multi-code MC-CDMA transmission systems model is given. In Sect. 3, we evaluate the BER performance of OFDM/TDM and MC-CDMA in a frequency-selective fading channel. Conclusion is set out in Sect. 4.

II. Transmission System Model

The OFDM/TDM and MC-CDMA transmission system model is illustrated in Fig. 1. In an OFDM/TDM system,

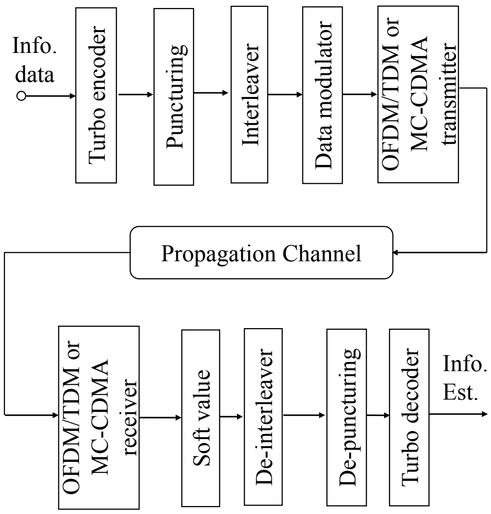
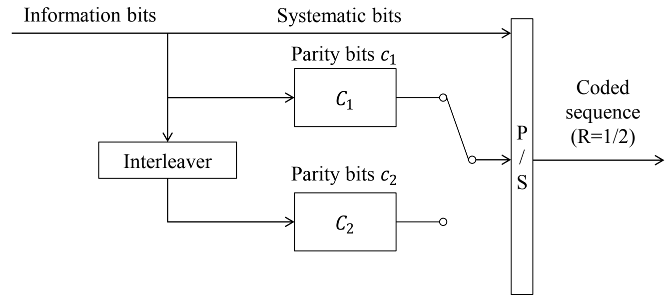
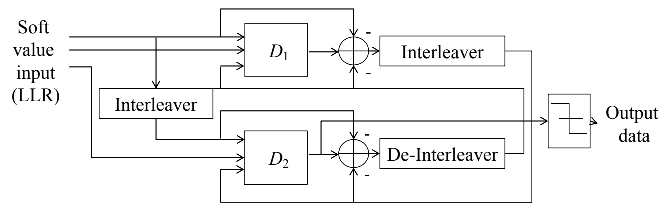


Fig. 1: Transmission system model.



(a) Encoder



(b) Decoder

Fig. 2: Turbo encoder and decoder structures.

N_c data-modulated symbols $\{d(i); i = 0 \sim N_c - 1\}$ with $E[|d(i)|^2] = 1$, where $E[\cdot]$ denotes the ensemble average operation, are transmitted during one OFDM/TDM frame. For comparison we use multi-code MC-CDMA with code-multiplexing order C. We note here that $C = 1$ represents the case of single-code MC-CDMA and the data rate degrades in comparison to OFDM. In this paper we use $C = SF$ in order to transmit the same number (i.e., N_c) data-modulated symbols $\{d(i); i = 0 \sim N_c - 1\}$ with $E[|d(i)|^2] = 1$ over a duration of N_cT_c as with conventional OFDM. In MC-CDMA, each data-modulated symbol is spread over a number of subcarriers using an orthogonal spreading code. The orthogonal property between subcarriers degrades, due to a frequency selective channel. This distraction of orthogonal property between subcarriers produces a large inter-symbol interference (ISI), which can be partially restored while achieving the frequency diversity effect by using the MMSE-FDE. Note that OFDM/TDM (MC-CDMA) becomes SC when $K(SF) = N_c$ and becomes conventional OFDM when $K(SF) = 1$. Because of this property, the OFDM/TDM and MC-CDMA provide flexibility in designing the OFDM-based transmission systems.

A. Transmit Signal Representation

The structure of turbo encoder and iterative decoder is shown in Fig. 2. In this paper, we use turbo encoder that is consisted of two parallel-concatenated recursive systematic convolutional (RSC) encoders C_1 and C_2, connected by a internal interleaver of size N. Overall coding rate is $R = 1/3$, but in this paper, puncturing is used to achieve a coding rate of $R = 1/2$.

OFDM/TDM data-modulated sequence $\{d(i)\}$ is divided into K blocks each of which has $N_m(= N_c/K)$ symbols. The kth OFDM/TDM block symbol sequence is denoted as $\{d(k, i); i = 0 \sim N_m - 1\}$, where $d(k, i) = d(kN_m + i)$. Then, N_m-point IFFT is applied to each data block to generate a sequence of K OFDM signals with $N_m = N_c/K$ subcarriers as shown in Fig. 3.

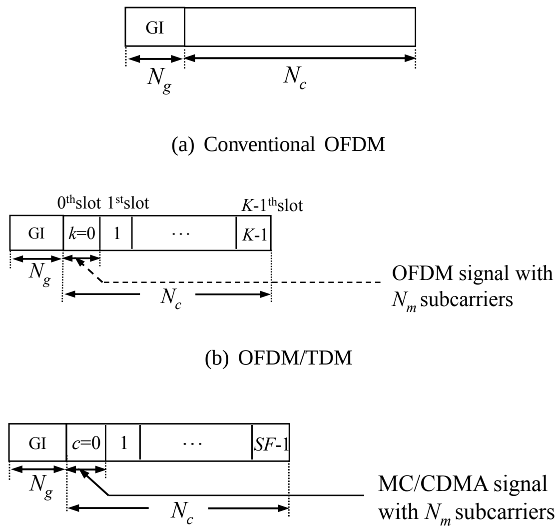
For multi-code MC-CDMA data-modulated sequence is serial-to-parallel converted into C parallel streams $\{d(c, i); c = 0 \sim C-1\}$. For each stream, the signal is serial-to-parallel (S/P) converted to N_c/SF streams; each symbol is then copied SF times and spread by multiplying with an orthogonal code $\{c_{oc,c}(n); c = 0 \sim C - 1, n = 0 \sim SF - 1\}$ with spreading factor SF. The C different streams are then added and further multiplied by a long scramble sequence $c_{scr}(n)$.

The resulting OFDM/TDM (MC-CDMA) signal can be expressed using the equivalent lowpass representation as [3], [5]

$$s(t) = \begin{cases} \sqrt{\frac{2E_c}{T_c}} \sum_{i=kN_m}^{(k+1)N_m-1} d(k,i) \exp\left(j2\pi t \frac{i}{N_m}\right) \\ \\ \sqrt{\frac{2E_c}{T_c}} \sum_{c=0}^{C-1} c_{oc,c}(t \bmod SF) c_{scr} d\left(c, \lfloor \frac{t}{SF} \rfloor\right) \end{cases} \quad (1)$$

for $t = 0\ N_c - 1$ and $k = \lfloor t/N_m \rfloor$, where $\lfloor x \rfloor$, E_c and T_c, respectively, denote the largest integer smaller or equal to x, the energy per subcarrier and the sampling period. In (1), the first (second) expression denotes the OFDM/TDM (MC-CDMA) signal. Before transmission, the last N_g samples are copied as a cyclic prefix and inserted into GI as illustrated in Fig. 3. Finally, the signal is transmitted over a frequency-selective fading channel.

We assume a frequency-selective fading channel having a discrete-time channel impulse response $h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l)$, where L, h_l, τ_l and $\delta(t)$, respectively, denote the number of channel paths, the path gain, the lth path normal-



(a) Conventional OFDM

(b) OFDM/TDM

(c) MC-CDMA

Fig. 3: Frame structure.

ized by IFFT sampling period and the Dirac delta function. $\{h_l;\, l = 0, ..., L-1\}$ are zero-mean independent complex variables with variance $E[|h_l|^2] = 1/L$.

B. Receive Signal Representation and FDE

The received OFDM/TDM (MC-CDMA) signal can be expressed as [3], [5]

$$r(t) = \sum_{l=0}^{L-1} h_l s(t-\tau_l) + n(t) \tag{2}$$

for $t = -N_g \sim N_c - 1$, where $n(t)$ is the zero-mean complex Gaussian noise process with a variance of $2N_0/T_c$ due to the additive white Gaussian noise (AWGN) with single-sided power spectrum density N_0.

After the removal of GI, the received time-domain signal $\{r(t);\, t = 0 \sim N_c - 1\}$ is decomposed into N_c frequency components $\{R(n);\, n = 0 \sim N_c - 1\}$ by applying N_c-point FFT as

$$\begin{aligned} R(n) &= \frac{1}{N_c}\sum_{t=0}^{N_c-1} r(t)\exp\left(-j2\pi n\frac{t}{N_c}\right) \\ &= d(n)H(n) + N(n), \end{aligned} \tag{3}$$

where $d(n)$, $H(n)$ and $N(n)$, respectively, denote the signal component, propagation channel gain and noise component at the nth frequency given by

$$\begin{cases} d(n) = \frac{1}{N_c}\sum_{t=0}^{N_c-1} s(t)\exp\left(-j2\pi n\frac{t}{N_c}\right) \\ H(n) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi n\frac{\tau_l}{N_c}\right) \\ N(n) = \frac{1}{N_c}\sum_{t=0}^{N_c-1} n(t)\exp\left(-j2\pi n\frac{t}{N_c}\right) \end{cases}$$

Then, one-tap MMSE-FDE is carried out as

$$\hat{d}(n) = w(n)R(n), \tag{4}$$

where $w(n)$ is the equalization weight for the nth frequency given by [8]

$$w(n) = \begin{cases} \dfrac{H^*(n)}{|H(n)|^2 + \left(\frac{E_s}{N_0}\right)^{-1}} & \text{for OFDM/TDM} \\[2ex] \dfrac{H^*(n)}{|H(n)|^2 + \left(\frac{CE_s}{SFN_0}\right)^{-1}} & \text{for MC-CDMA} \end{cases} \tag{5}$$

where $(\cdot)^*$ denotes the complex conjugate operation.

C. Data Demodulation

Finally, the OFDM/TDM (MC-CDMA) demodulation is performed to recover the transmitted data symbol sequence as [3], [5]

$$\hat{d}(k,i) = \begin{cases} \frac{1}{N_c}\sum_{n=0}^{N_c-1} \hat{d}(n)\Psi(n;i,k) \\[2ex] \frac{1}{SF}\sum_{n=iSF}^{(i+1)SF-1} \hat{d}(n)c^*_{oc,c}(n \bmod SF)c^*_{scr}(n) \end{cases} \tag{6}$$

where frequency-domain filter $\Psi(n;i,k)$ is given by [3]

$$\begin{aligned} &\Psi(n;i,k) = \\ &= \frac{\sin\left(\pi N_m \frac{iK-n}{N_c}\right)}{N_m \sin\left(\pi\frac{iK-n}{N_c}\right)} \exp\left[-j\pi((2k+1)N_m - 1)\frac{iK-n}{N_c}\right] \end{aligned} \tag{7}$$

for $i = 0 \sim N_m - 1$ (or $i = 0 \sim N_c/SF - 1$ for MC-CDMA) and $c = 0 \sim C - 1$.

D. Turbo Decoding

Turbo decoding principle is based on iterative algorithm that requires soft decision values as input. Log-likelihood ratio (LLR) approximation is used for the generation of the soft values. The decision variable given by (7) includes the ISI and noise due to AWGN. Assuming that the ISI can be approximated as a zero-mean complex-valued Gaussian variable, the sum of ISI and AWGN can be treated as a new zero-mean complex-valued Gaussian noise having the variance $2\sigma^2$. The LLR approximation for the bth ($b \in \{0,1\}$) bit in the nth symbol is given by [5], [7]

$$L(b) \approx \frac{\left|\hat{d}_c(n) - \hat{H}(n)\hat{s}_0\right|^2}{2\sigma^2} - \frac{\left|\hat{d}_c(n) - \hat{H}(n)\hat{s}_1\right|^2}{2\sigma^2}. \tag{8}$$

Here, $\hat{s}_0$ or $\hat{s}_1$ is the candidate symbol, with 0 (or 1) in the bth bit position, for which the Euclidian distance from $\hat{d}_c(n)$ is minimum. The LLR values are computed for all the bits in the symbol and turbo decoding is performed using these LLR values as soft input. The iterative decoding process is shown in Fig. 2(b).

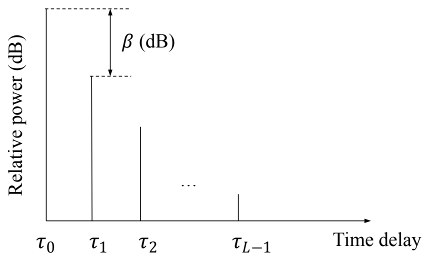


Fig. 4: Channel power delay profile.

TABLE I: Simulation parameters.

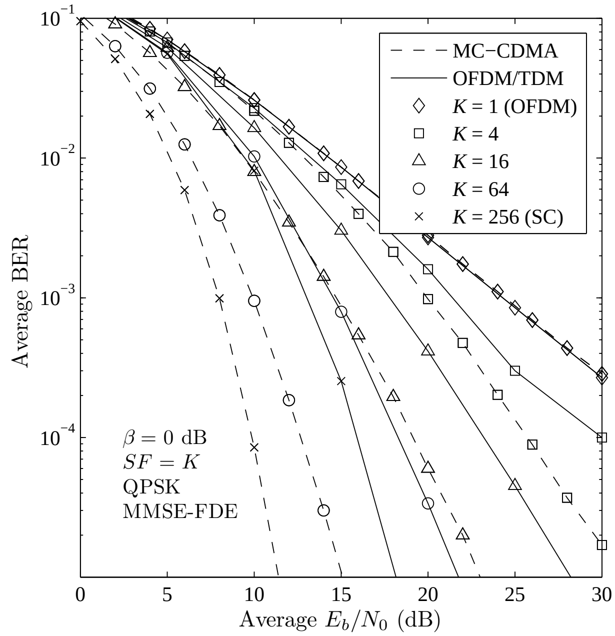
OFDM	No. of subcarriers	N_c
	GI	N_g
OFDM/TDM	No. of subcarriers	$N_m = N_c/K$
	GI	N_g
	Frame length	$N_c + N_g$
MC-CDMA	No. of subcarriers	N_c
	Code-multiplexing	$C = SF$
	GI	N_g
	Frame length	$N_c + N_g$

III. Simulation Results and Discussions

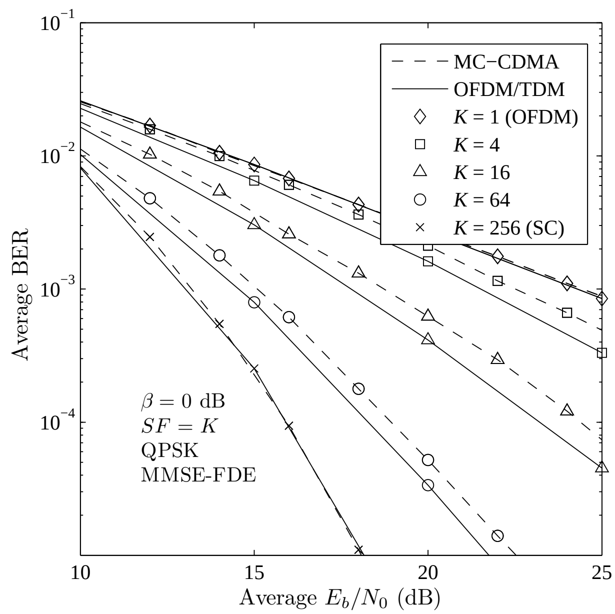
The OFDM/TDM and multi-code MC-CDMA parameters, compared to conventional OFDM are shown in Table 1. The BER performance of OFDM/TDM and multi-code MC-CDMA is evaluated by computer simulation. We assume QPSK data modulation with $|d(i)| = 1$, $N_c = 256$ and $N_g = 32$. The channel is assumed to be an L=16-path frequency-selective Rayleigh fading channel having exponential power delay profile with decay factor β as shown in Fig. 4. Without loss of generality, we assume $\tau_0 = 0 < \tau_1 < ... < \tau_{L-1}$, where the lth path delay is given by $\tau_l = l\triangle$, with $\triangle$ (≥ 1) being the time delay separation between adjacent paths. A block fading is assumed, where path gains stay constant over the one OFDM/TDM frame that corresponds to multi-code MC-CDMA and conventional OFDM signaling interval. We assume ideal channel estimation.

A. Uncoded BER Performance

The average BER performance comparison of OFDM/TDM and MC-CDMA using MMSE-FDE is illustrated in Fig. 5 as a function of the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 for various values of K and SF $(1 \sim 256)$, where $E_b/N_0 = 0.5(E_s/N_0) \times (1 + N_g/N_c)$, in which the power loss due to GI insertion is taken into account. We observe from Fig. 5a that the single-code MC-CDMA outperforms OFDM/TDM with MMSE-FDE which becomes more evident as the spreading factor increases. It can be seen from Fig. 5b that OFDM/TDM achieves a better BER performance than multi-code MC-CDMA due to less interference in the dispreading process. The target BER performance is achieved for lower required E_b/N_0 when using OFDM/TDM instead of multi-code MC-CDMA. In particular, for the target



(a) OFDM/TDM vs. single-code MC-CDMA.



(b) OFDM/TDM vs. multi-code MC-CDMA.

Fig. 5: Uncoded BER.

$\text{BER} = 10^{-4}$ required E_b/N_0 reduces for 1, 1.25 and 1.2 dB with OFDM/TDM in comparison to multi-code MC-CDMA which corresponds to the $K(SF) = 4$, 16 and 64.

B. Coded BER Performance

Fig. 6 shows the coded BER performance comparison between OFDM/TDM and multi-code MC-CDMA as a function

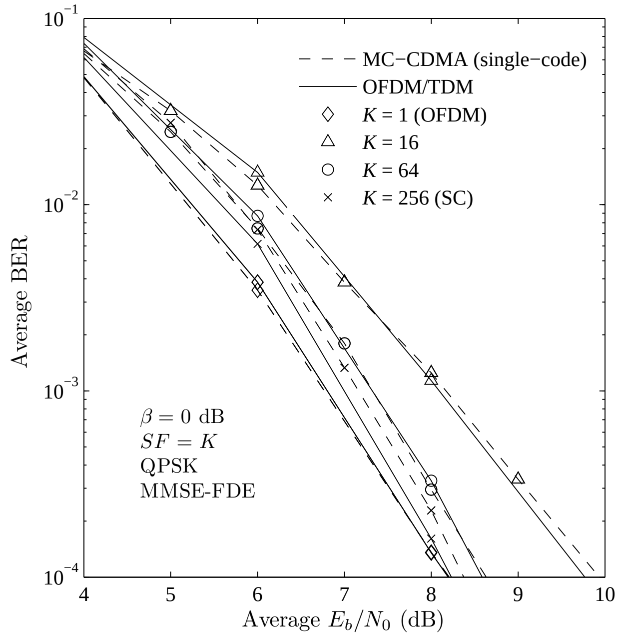


Fig. 6: Coded BER (OFDM/TDM vs. multi-code MC-CDMA).

of the E_b/N_0 for various values of K and SF $(1 \sim 256)$, where E_b/N_0. From the figure it can be seen that both OFDM/TDM and multi-code MC-CDMA achieves almost the same BER performance and that $K = SF = 1$ shows the best performance.

IV. Conclusion

In this paper, the BER performance comparison between OFDM/TDM and MC-CDMA with MMSE-FDE was evaluated by computer simulation. It is shown that for uncoded systems OFDM/TDM achieves a lower BER performance in comparison to multi-code MC-CDMA due to less ICI. However, when the systems are coded the BER performance becomes almost the same. The transmission data rate is kept the same for both systems while the OFDM/TDM reduces the PAPR when K increases. For uncoded case single-code MC-CDMA give better BER performance than OFDM/TDM due to less ISI. The cost paid for this performance improvement for single-code MC-CDMA is much lower data rate transmission and high PAPR problem.

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