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Design of multivalued frequency response filters by using nonlinear feedback - Part II

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Abstract—Even if the research on multivalued frequency response systems is still in progress and more items are to be more deeply approached from a theoretical point of view, in this paper some aspects outlined in a companion contribution to Nolta 2022 [1] will be focused by introducing further examples. Moreover, the discretization of the system is discussed in order to provide a digital implementation of nonlinear filters with multivalued frequency response.

1. Introduction

The linear filters theory, based on electrical and electronic devices, is well known [2] and the huge number of contributions presented during the last century allows us to have a dominant theory for the design of such type of systems. The interest in the future of this topic is until now increasing. Moreover, even if digital filters design opened a new age in signal processing, the techniques for designing linear analog filters is until now a special topic in the VLSI technology. However, the attention has been devoted only to linear networks. The problem of conceiving circuits with more peculiarities with respect to the classic one, characterized only by frequency selectivity, has not been widely studied in the literature. The theory of nonlinear filtering and nonlinear techniques in digital signal processing is well established [3] but is addressed in the area of nonlinear observers or Kalman filters tasks.

In this communication, starting from the results outlined in the companion paper presented to Nolta 2022 [1], some further aspects of the research on multivalued frequency response nonlinear filters will be focused. In particular, the following points will be addressed. The design of the nonlinear part has been approached starting from the choice of the multivalued function at a frequency $\bar{\omega}$, but how this choice can be done? Moreover, may the choice lead to discontinuous frequency responses, that are of course unstable? Moreover, after a numerical selection of the multiple values that lead to continuous solutions, how to establish the set that ensure in a certain sense an optimization in the shape of the filter mask?

The approach presented in the first part of the research is based on shaping a single frequency mask. In this part, the multivalued and multiband frequency response is also discussed. Several examples are included in this part showing also the realization of more multivalued multiband frequency response filters. The realization of the multiband filters is approached in terms of discrete-time filters. Therefore a simple and efficient discretization approach is introduced leading to an implementation with low-cost high performance microcontrollers.

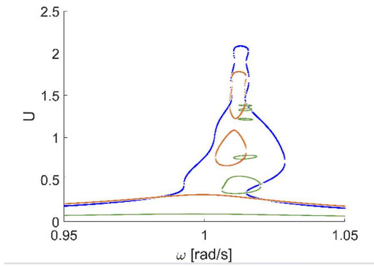


Figure 1: Multivalued frequency responses around $\bar{\omega}$: presence of isolated islands (red and green lines) or without unstable islands (blue line).

2. The islands

Islands in a discontinuous nonlinear frequency response represent unstable regions that may arise from a not admissible set of multiple values at a given frequency $\bar{\omega}$. In fact, in order to start with the multivalued frequency response filter design, a set of U_k with $k = 1, \ldots, n$ must be chosen. This choice is made by the designer in such a way that at the frequency $\bar{\omega}$ the conditions discussed in part I [1] on the imaginary part of $G^{-1}(j\omega)$ are satisfied.

Moreover, as shown in the following example, some pathological choices lead to the response with islands, while other choices lead to the stable solution represented in Fig. 1.

The parameter α_i that are obtained from the pre-fixed choice of U_k lead to the solution with island. In the literature, islands are also called detached resonance curves, i.e. isolated branches of periodic solutions [4], that have been discovered in harmonically excited mechanical systems [5] with nonlinear damping. In those cases, where a limited number of parameters is considered, the set of islands can be analytically defined considering the characteristic equation of the system (see Part I):

$$F(\alpha_i, U, \omega, R) = \left(U^n + \sum_{i=1}^{n-2} \alpha_i U^i \right)^2 + \Im(\omega)^2 U^2 - R^2 \quad (1)$$

from which the following conditions are derived

$$\frac{\partial F}{\partial U} = 0 \quad \frac{\partial F}{\partial \omega} = 0 \quad \frac{\partial^2 F}{\partial U^2} \neq 0 \quad \det d^2 F > 0 \qquad (2)$$

where d^2F is the Hessian matrix of F with respect to U and ω [6].

Moreover, in the considered study, the previous equations cannot lead to a direct locus definition, due to the high number of the parameters α_i, therefore a numerical algorithm is proposed.

Let us consider the algorithm taken into consideration in the Part I of this study to define α_i. Therefore, a first step is to define both the order of the multiband filter and then start with a set of values U_k that are assumed at the main frequency $\bar{\omega}$. The choice of U_k must ensure the stability of the filter, this means that the interest is devoted to the frequency regions around $\bar{\omega}$ that must be a continuous multivalued function. The maximum and the minimum values of U_k can be defined, moreover the aim is to define the set of U_k where no islands occur. This means that the $alpha_i$ must ensure no islands regions. Therefore, we must start by considering a search algorithm based on Montecarlo techniques and to classify the U_k set that leads no island with that will discarded due to the unstable frequency behavior.

The algorithm is organized along the following steps:

1. start with random values of U_k;
2. perform the frequency response after having computed from U_k the nonlinear filter parameters;
3. the maximum U_M and the minimum U_m in the selected frequency range are derived;
4. a step $\Delta U = \frac{U_M - U_m}{n_s}$ with n_s indicating the number of steps, is defined;
5. for each value of U decreasing from U_M to U_m with step ΔU, the function F is evaluated for each frequency monitoring its sign: if the sign remains the same $\forall \omega$ this means that an island occurs, as schematically represented in Fig. 2.

Repeating the algorithm more times, the probability of having island behavior within a statistical distribution of U_k can be established. Therefore, the algorithm gives us the possibility of selecting the proper, i.e. stable, U_k set.

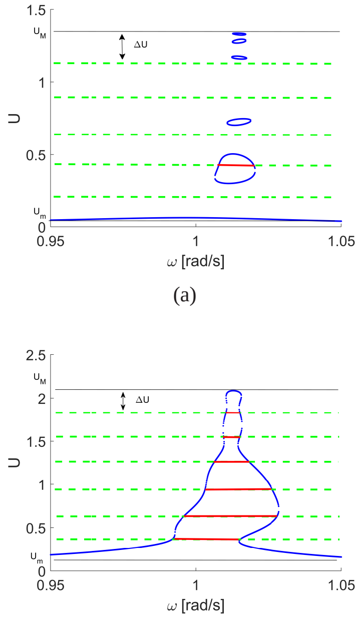


Figure 2: Algorithm to detect islands: (a) the sign of F remains unvaried for all ω at given ΔU: islands detected; (b) the sign of F remains varied for all ω at all ΔU: no islands detected.

The family of suitable U_k contains infinite elements, therefore we have the possibility to select the most suitable set. In particular, the empirical experience said us that around a stable set there are more and more stable sets that can be appropriately selected by matching the desired shape of the frequency response. Moreover, another empirical experience said us that an almost regular distribution of the U_k elements can lead to a no-island frequency response. In the case we are considering, i.e. an initial U_k set drawn from a random positive distribution, smaller is variance more smaller is the probability of islands, moreover more narrow is the multivalued frequency range around $\bar{\omega}$.

3. Optimization

Let be the curve obtained by using one of the U_k set not belonging to the island sets. The shape of the curve can be optimized. This can be done referring on the optimization point on which the desired performance are meant to be met. Let be the example shown in Fig. 3. The frequency f_M, corresponding to the maximum frequency in the multivalued range, can be moved on the right as more as possible. This allows an increasing of the right part of the hysteresis. This can be done by using a random search algorithm, in fact the optimal U_k can be searched around a stable set monitoring the value of f_M. The same approach can be used for obtaining the set uk in order to improve the frequency hysteresis on the left.

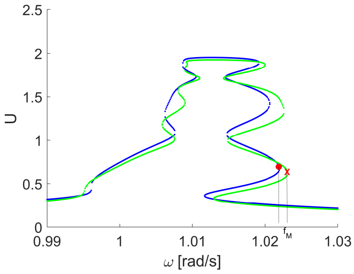


Figure 3: Optimization of the multivalued frequency response range.

4. The multiband case

With the multiband term, we intend the case in which a multivalued frequency range occur around two or more different $\bar{\omega_i}$. This item is mainly based on the design of the linear part of the filter. Therefore, a resonator of the form $G(j\omega) = \sum_{i=1}^{m} \frac{k_i}{(s+\bar{\alpha_i})(s^2+2\xi_i\omega_{n,i}s+\omega_{n,i}^2)}$ is adopted.

After having designed in a first phase the nonlinear part the choice of $G(j\omega)$ parameters must be done in order to obtain $\mathfrak{I} = 0$ and $\mathfrak{R} = \alpha_{n-1}$ in the set of the frequency $\bar{\omega_i}$. This can done by choosing the k_i values. The adopted procedure can start fixing $m-1$ values for k_i and use the free parameter for the tuning. The following example with two $\bar{\omega_i}$ is reported.

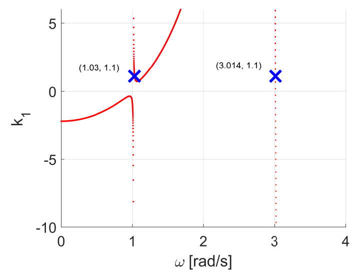
The following two resonators are taken into consideration with the desired $\omega_{n,1} = 1$ and $\omega_{n,2} = 3$, with $\bar{\alpha_1} = \bar{\alpha_2} = 0.3$ and $\xi_1 = \xi_2 = 0.05$, and $k_2 = 20$. The condition under which $\mathfrak{I}(\bar{\omega_1}) = \mathfrak{I}(\bar{\omega_2}) = 0$ is derived by tuning k_1 so that the value of $\mathfrak{R}(\bar{\omega_1}) = \mathfrak{R}(\bar{\omega_2})$. The algorithm must consider at first the condition on the imaginary part, that is, the value of k_1 so that it is null, as reported in Fig. 4(a). It is possible to see that several values can be found so that the imaginary part is nullified with the same k_1 for different ω. Among these possible values, we have to consider those values of k_1 for which $\mathfrak{R}(\bar{\omega_1}) = \mathfrak{R}(\bar{\omega_2})$, therefore the value of $\mathfrak{R}(\omega)$ for each value of k_1 is reported in Fig. 4(b). Therefore, the values of ω to be considered are in the range of multiple solutions for $\mathfrak{R}(\omega)$.

The points highlighted in Fig. 4(b) correspond to $\mathfrak{R}(\bar{\omega_1}) = \mathfrak{R}(\bar{\omega_2})$, from which the points highlighted in Fig. 4(a) can be checking that the in two frequencies $\bar{\omega_1} = 1.03$ rad/s and $\bar{\omega_2} = 3.014$ rad/s, the same value $k_1 = 1.1$ nullifies the imaginary part of $G^{-1}(j\omega)$. This is confirmed by the Nyquist plot of $G^{-1}(j\omega)$ obtained with $k_1 = 1.1$ as reported in the Fig. 5(a).

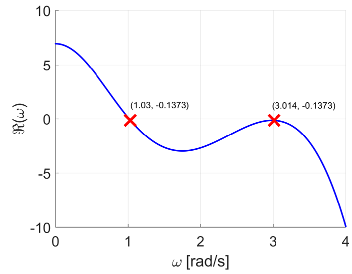
The multiband multivalued frequency response of the designed filter is reported in Fig. 5(b).

4.1. Higher order nonlinearity multivalued frequency response

The generalization to higher order polynomial nonlinearities is also straightforward, as it is necessary to choose

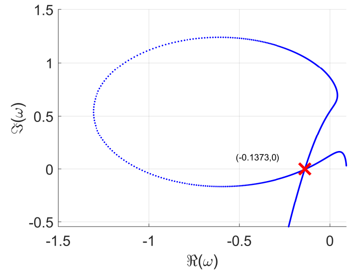


(a)

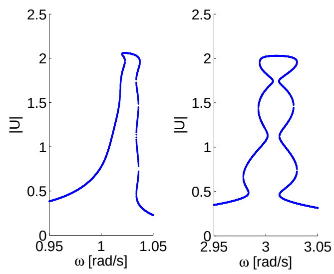


(b)

Figure 4: Design of multiband multivalued frequency response nonlinear filters: (a) k_1 nullifying the imaginary part of $G^{-1}(j\omega)$ at each ω; (b) value of the real part of $G^{-1}(j\omega)$ at each ω for the corresponding gain k_1 nullifying the imaginary part; (c) Nyquist plot of the designed $G^{-1}(j\omega)$.



(a)



(b)

Figure 5: Design of multiband multivalued frequency response nonlinear filters: (a) Nyquist plot of the designed $G^{-1}(j\omega)$; (b) multiband multivalued frequency response of the designed nonlinear filter.

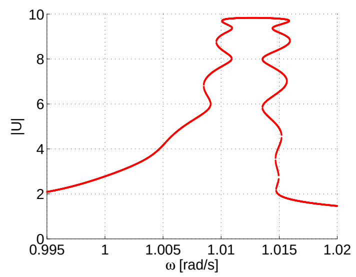


Figure 6: Design of higher order multivalued frequency response nonlinear filters.

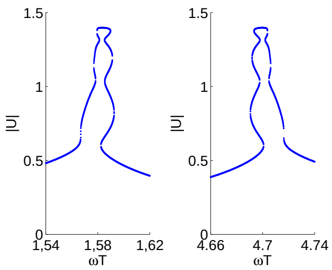


Figure 7: Design of multivalued frequency response nonlinear digital filters.

only the number of multiple values at the frequency $\bar{\omega}$ for which the imaginary part is nullified. In the Fig. 6, the multivalued frequency response of a filter with polynomial nonlinearity of order 21 is reported, ensuring a frequency response with eleven multiple values at $\bar{\omega}$. This scenario allows for the identification of multiple nested paths of increments/decrements in the frequency of the input signal.

5. Discretization of multivalued frequency response systems

In this section a simple discretization of the continuous-time system has been performed. In order to show that the choice of the sampling time duplicates the multivalued bands, a direct bilinear transformation is adopted, obtaining the frequency response reported in Fig. 7.

6. Conclusions

The study is correlated to the behavior of forced nonlinear systems. Moreover, it is essentially a design technique for filters that can detect frequency drifts in more frequency ranges. The paper concerns with the second part of the multivalued frequency response nonlinear filters design. Even if more topics will be object of research, in this contribution, by presenting more examples, various items are remarked. In particular, the discovery of the unstable behavior of these systems is studied by using an appropriate algorithm. Moreover, a random search approach is used to investigate optimization aspects related to the width of the nested hysteresis windows.

A multiband multivalued frequency response example has been proposed. Before this study, it did not exist a general theory for designing this class of nonlinear filters, since the principles of multiple nonlinear resonance have been discussed in the literature only from the analysis point of view and not for design purposes. Even if analytical approaches and numerical simulations should be better established, the design techniques consist in trial and error approaches. An established theory can be obtained when more and more experience is gained. Therefore, the designers must use the main circuit principles and appropriate simulation tools for optimizing their projects.

Acknowledgments

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