2022 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2022, Virtual, December 12-15, 2022

Global Stabilization for Nonlinear Two-Port Characteristics of Bidirectional DC/DC Converter

Kenta Yamamoto† Takashi Hisakado† Mahfuzul Islam† and Osami Wada†

†Department of Electrical Engineering , Kyoto University
Kyotodaigakukatsura, Nishikyo-ku, Kyoto 616-8510, Japan
Email: kenta@cct.kuee.kyoto-u.ac.jp, hisakado@kuee.kyoto-u.ac.jp

Abstract—This paper describes the nonlinear two-port characteristics of bidirectional DC/DC converters and its two different operating points. The nonlinearity of the two-port characteristics affects global stability. We show that global stabilization of the bidirectional DC/DC converter can be achieved by a feedback method based on the nonlinear two-port characteristics. Experiment confirms the proposed feedback method to realize global stability.

1. Introduction

In recent years, power electronics technology has been progressing with the advent of near-lossless switching devices. Ideally, the conversion characteristics of power conversion circuits are modeled as a two-port circuit network element called the time-variable transformer (TVT) [1][2].The TVT is a power-conservative two-port networks element [1]. Due to the energy's nonlinearity, there are two duty ratios that give the same output operating points. Related to this phenomenon, there exist studies on the two equilibrium points and the global stability that occur in power conversion circuits with a PV connected [3][4]. These two duty ratios inevitably occurs regardless of the circuit connected to the ports. This nonlinearity is formulated as nonlinear two-port characteristics.

In this paper, we discuss the nonlinearity of bidirectional DC/DC converter from the viewpoint of the two-port characteristics. Commonly used feedback to a target stabilizes the one equilibrium and destabilize other equilibrium. However the feedback does not ensure global stability. So, we propose a feedback control that eliminate one of the two equilibrium and show that this feedback method can realize global stability. We verify the global stability of the feedback from both simulation and experiment.

2. Nonlinear Two-port Characteristics of Bidirectional DC/DC Converter

2.1. Bidirectional DC/DC Converter as TVT

A bidirectional DC/DC converter is the basic power conversion circuit shown in Fig. 1. The two switches Q1 and Q2 operate in complementary. We drive switches by pulse width modulation (PWM) with duty ratio α. The duty ratio can be time-variable within the range $0 \leq \alpha \leq 1$. When ignoring the dynamics of C_1, C_2 and L, the two-port characteristics is expressed as

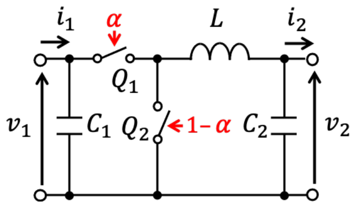

Figure 1: Bidirectional DC/DC converter

$$\begin{bmatrix} v_1(k) \\ i_1(k) \end{bmatrix} = \begin{bmatrix} 1/\alpha(k) & 0 \\ 0 & \alpha(k) \end{bmatrix} \begin{bmatrix} v_2(k) \\ i_2(k) \end{bmatrix}. \tag{1}$$

Eq. (1) is a TVT with a transformation ratio of $\alpha(k)$ and k is discrete time [1] [2]. TVT is a lossless two-port circuit element, so the port-to-port power relationship is $v_1 i_1 = v_2 i_2$.

2.2. Nonlinear Two-port Characteristics

Bidirectional DC/DC converter is controlled by feedback. The Control of two-port characteristics is realized by feedback on the secondary-side operating points i_2 and v_2 to the duty ratio α. The two-port characteristics generated by the feedback are formulated in the follow

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 1/\alpha(i_2, v_2) & 0 \\ 0 & \alpha(i_2, v_2) \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}. \tag{2}$$

Feedback of general DC/DC converters are provided to α for keep in v_2 or i_2 constant. In this case, the power conversion circuits are used as one-port circuits. It only uses the local conversion characteristics of the two-port circuits. However, the essence of the bidirectional DC/DC converter is a nonlinear 2-port circuits that maps i_1-v_1 to i_2-v_2 characteristics. Therefore, we determine the target i_2-v_2 characteristics and provide feedback so that (i_2, v_2) are equilibrium on the target characteristics. Using such feedback makes it possible to maximize the global conversion characteristics of the bidirectional DC/DC converter.

ORCID iDs First Author: 0000-0001-7152-1389, Second Author: 0000-0002-5078-2296, Third Author: 0000-0002-5011-4044, Fourth Author: 0000-0002-8869-4785

2.3. Two Operating Points for Single Output

Assume that the power supply represented by Thevenin's equivalent circuit connects to the primary side of a bidirectional DC/DC converter. Thevenin's equivalent circuit is a typical power supply circuit, and the i_1-v_1 characteristics are expressed below

$$v_1 = e_1 - r_1 i_1 \ . \tag{3}$$

In Eq. (3), r_1 is the internal resistance, and e_1 is the internal electromotive force of the power supply. Substituting i_1-v_1 caracteristics into Eq. (2) and eliminating i_1,

$$v_2 = \alpha(i_2, v_2)e_1 - \alpha(i_2, v_2)^2 r_1 i_2 \tag{4}$$

we get the relationship between the secondary-side operating points (i_2, v_2) and the duty ratio α following equation

$$\alpha(i_2, v_2)_{(\pm)} = \frac{e_1 \pm \sqrt{{e_1}^2 - 4 r_1 i_2 v_2}}{2 r_1 i_2} \tag{5}$$

The graphical representation of Eq. (5) on (i_2, v_2, α) space is shown in Fig. 2.

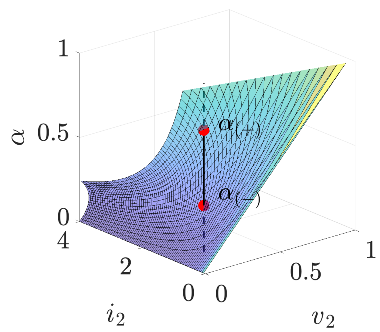

Figure 2: Two operating points $\alpha_{(+)}$,$\alpha_{(-)}$ for single (i_2, v_2)

In Fig. 2, we see that there are two duty ratios, so (i_2, v_2) and α are not uniquely determined. Because there are two combinations of (i_1, v_1) that satisfy the power relationship $i_1 v_1 = i_2 v_2$ when $v_2 i_2 < {e_1}^2/4r_1$.

3. Elimination of Unwanted Equilibrium

3.1. Simple Feedback based on Target Characteristics

We introduce a simple feedback to realize secondary-side characteristics. As the most simple feedback, we assume feedback that is in equilibrium on the secondary-side characteristics. We realize the secondary characteristics of the follow

$$v_2 = e_2 - r_2 i_2 \ . \tag{6}$$

As a function that takes zero at (i_2, v_2) on the secondary-side characteristics of Eq. (6), we have

$$f_{\mathrm{iv}}(i_2, v_2) = e_2 - r_2 i_2 - v_2 \ . \tag{7}$$

A simple the feedback of DC/DC converter based on Eq. (6), we obtain

$$\alpha(k+1) = \alpha(k) + K f_{\mathrm{iv}}(i_2(k), v_2(k)) \ . \tag{8}$$

In the equilibrium of feedback, $\alpha(k+1) = \alpha(k)$. Therefore, (i_2, v_2) satisfies Eq. (6) in the equilibrium state. From Eq. (5), there are two duty ratios corresponding to the same (i_2, v_2), so this feedback has two equilibrium points $\alpha(i_2, v_2)_{(+)}$ and $\alpha(i_2, v_2)_{(-)}$.

3.2. Feedback to Realize a Unique Equilibrium

We propose to realize feedback with unique equilibrium by eliminating the unwanted equilibrium. When moving the operating points (i_2, v_2) to $(\Delta i_2 + i_2, \Delta v_2 + v_2)$, there are two different variations of duty ratio $\Delta\alpha$ that correspond to Δi_2 and Δv_2. We propose feedback targeting the duty ratio by selecting the one corresponding to the target duty ratio one of the two $\Delta\alpha$. Substitute $\Delta i_2 + i_2$, $\Delta v_2 + v_2$, $\Delta\alpha + \alpha$ for i_2, v_2, α respectively in Eq. (4), and rearrange as

$$\begin{aligned} r_1(i_2 + \Delta i_2)\Delta\alpha^2 - \{e_1 - 2\alpha r_1(i_2 + \Delta i_2)\}\Delta\alpha \\ + \alpha^2 r_1 \Delta i_2 + \Delta v_2 = 0 \ . \end{aligned} \tag{9}$$

We solve Eq. (9) for $\Delta\alpha$, we obtain

$$\begin{aligned} \Delta\alpha_{(\pm)} = \frac{e_1}{2r_1(i_2 + \Delta i_2)} - \alpha \\ \pm \frac{\sqrt{[e_1 - 2\alpha r_1(i_2 + \Delta i_2)]^2 - 4r_1(\alpha^2 r_1 \Delta i_2 + \Delta v_2)(i_2 + \Delta i_2)}}{2r_1(i_2 + \Delta i_2)} \ . \end{aligned} \tag{10}$$

From Eq. (10), there exist two $\Delta\alpha$ corresponding to Δi_2 and Δv_2. We analyze the duty ratios at equilibrium for each of $\Delta\alpha_{(+)}$ and $\Delta\alpha_{(-)}$. By substituting Eq. (5) into α of equation Eq. (10), the conditions for (i_2, v_2) under which $\Delta\alpha = 0$ holds are investigated. The results are shown in Table 1.

Table 1: Equilibrium state and two duty ratios in Eq(10)

Eq. (10)	$\alpha_{(+)}$	$\alpha_{(-)}$
$\Delta\alpha_{(+)}$	equilibrium	disequilibrium
$\Delta\alpha_{(-)}$	disequilibrium	equilibrium

From Table 1, by selecting the sign of $\Delta\alpha$, we can eliminate the duty ratio at unwanted equilibrium state. In feedback based on Eq. (10), we need to determine δi_2 and δv_2 based on (i_2, v_2). From simple feedback concept, we assume the following equations to give Δi_2 and Δv_2.

$$\begin{cases} \Delta i_2 = K_i f_{\mathrm{iv}}(i_2, v_2) \\ \Delta v_2 = K_v f_{\mathrm{iv}}(i_2, v_2) \end{cases} . \tag{11}$$

K_i and K_v are gain of Δi_2 and Δv_2. Substitute Eq. (10) and Eq. (11) for $\alpha(k+1) = \alpha(k) + \Delta\alpha_{(-)}$, we rearrange to obtain the follow

$$\begin{aligned} \alpha(k+1) &= A(k) - \mathrm{sgn}(A(k))\sqrt{(A(k)-\alpha(k))^2 - B(k)} \\ A(k) &= \frac{e_1}{2r_1\{(1-K_i r_2)i_2(k) + K_i(e_2 - v_2(k))\}} \\ B(k) &= \frac{(\alpha(k)^2 r_1 K_i + K_v)(e_2 - v_2(k) - r_2 i_2(k))}{r_1\{(1-K_i r_2)i_2(k) + K_i(e_2 - v_2(k))\}} \,. \end{aligned} \quad (12)$$

By using Eq. (12), we get feedback to realize a unique mapping (i_2, v_2) to $\alpha_{(-)}$. However, except $i_2 = 0$.

4. Gain for Global Stabilization

4.1. System with Load

The operating point of the secondary-side is determined by connecting a load to the bidirectional DC/DC converter. We connect the load represented by Thevenin's equivalent circuit. When connecting the load, the circuit system is shown in Fig. 3.

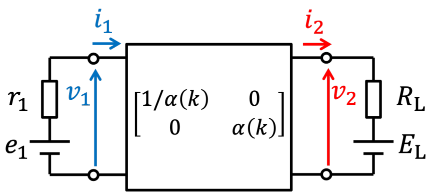

Figure 3: Bidirectional DC/DC converter with load and power supply

From Fig. 3, i_2 and v_2 at $\alpha(k)$ appear as following equations.

$$i_2(\alpha(k)) = \frac{\alpha(k)e_1 - E_L}{r_1\alpha(k)^2 + R_L} \,. \quad (13)$$

$$v_2(\alpha(k)) = E_L + R_L\frac{\alpha(k)e_1 - E_L}{r_1\alpha(k)^2 + R_L} \,. \quad (14)$$

4.2. Derivation of Gain for Global Stabilization by Contraction Mapping

We define the return-map f as the mapping from $\alpha(k)$ to $\alpha(k+1)$, as shown

$$\alpha(k+1) = f(\alpha(k)) \,, \quad \alpha(k) \in [0,1] \,. \quad (15)$$

The return-map f is a nonlinear mapping representing the transients of the duty ratio α. The return-map of simple feedback is obtained by substituting Eq. (13),(14) into Eq. (8). Similarly, the return-map of the proposed feedback is obtained by substituting Eq. (13),(14) into Eq. (12).

We use the concept of contraction mapping to explain the global stability of feedback system. f is contraction mapping when the holds

$$|\, f(x) - f(y)\,| \le \mu|\, x - y\,| \,, \quad x, y \in [0,1] \quad (16)$$

under $0 \le \mu < 1$. From the Banach fixed-point theorem, contraction mapping has only one fixed-point and iterations of the mappping converge to a fixed-point. Therefore, if the return-map f is contraction mapping, the feedback of f realize global stabilty.

The proposed feedback return-map has only one equilibrium point, i.e., fixed-point. Therefore, global stability is guaranteed by setting the gains K_i and K_v so that Eq. (16) holds. When $\mu = 0$ in Eq. (16), return-map corresponds to the feedback that reaches equilibrium in one discrete step. The gain satisfying $\mu = 0$ is calculated from the proposed feedback return-map as

$$K_i = K_v = \frac{(E_L r_1 \alpha_{(-)} + e_1 R_L)\alpha_{(-)}}{(e_2 R_L + E_L r_2)(1 + r_1 \alpha_{(-)}{}^2)} \,. \quad (17)$$

4.3. Example of return-map

Return-maps of simple feedback and the proposed feedback are shown in Fig. 4 and Fig. 5, respectively. In both return-maps, the circuit parameter are $e_1 = 100, r_1 = 20, e_2 = 50, r_2 = 7, E_L = 0, R_L = 3$, and the gain of each map are given in the figures.

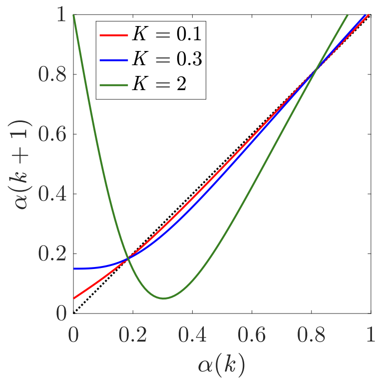

Figure 4: Return-map of simple feedback

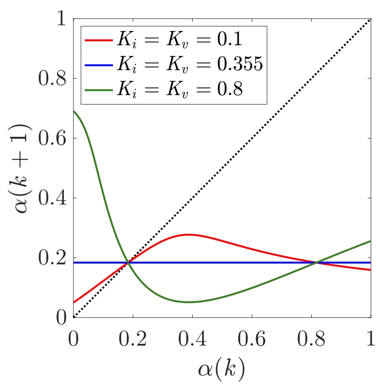

Figure 5: Return-map of proposeed method

From Fig. 4, the simple feedback have two fixed-points $\alpha_{(-)}$

and $\alpha_{(+)}$. So the map of the simple feedback is not a contraction mapping and can't realize global stability. From Fig. 5, the propsed feedback has only one fixed-points $\alpha_{(-)}$. When $K_i = K_v = 0.1$ and $K_i = K_v = 0.355$, the return-map is contractive, and global stability is achieved. On the other hand, when the gain is set to $K_i = K_v = 0.8$, the return-map changes to the non-contractive mapping, and the equilibrium becomes unstable.

5. Experimental Demonstration

We verifed the feedback control using bidirectional DC/DC converter. Fig. 6 shows the SiC bidirectional converter module used in the experiments. We used 100 V voltage source and 20 Ω electronic load for generate i_1-v_1 characteristics, and 3 Ω register was used as the R_L. This parameter is similar to PV, where the entire i_1-v_1 area is in the operable. The gains in each feedback were set to $K = 0.3$ and $K_i = K_v = 0.355$. Feedback control was implemented on a dsPIC and controlled in 33 ms cycles.

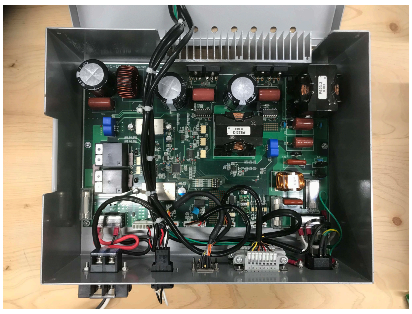

Figure 6: SiC bidirectional converter module

Global stability was verified by setting the initial values of $\alpha(k)$ to 0.2, 0.5, and 0.8, and by providing feedback. The transient responses of the simple feedback acquired from experiments are shown in Fig. 7, and transient of propsed feedback are Fig. 8.

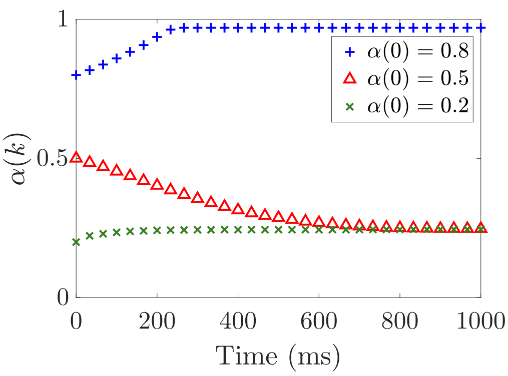

Figure 7: Experimental result of simple feedback

From Fig. 7, the simple feedback has divergent transient response of $\alpha(0) = 0.8$. Therefore, it does not have global

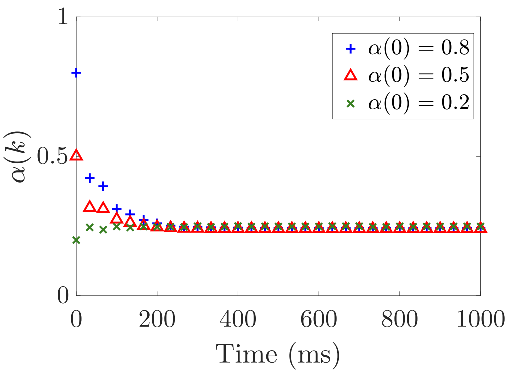

Figure 8: Experimental result of proposed method

stability. On the other hand, Fig. 8 shows that in the proposed feedback, all operating points converge to a fixed point and realize global stability. Theoretically, the transient response converges in one discrete step, but in Fig. 8, it takes several steps to converge. One possible cause is the loss of the bidirectional DC/DC Converter.

6. Conclusion

We formulated the two-port characteristics of bidirectional DC/DC converter as nonlinear two-port characteristics and clarified it's nonlinearity. Due to these nonlinearities, there are two duty ratios corresponding to a single secondary-side operating point. We analyze two types of feedback methods: simple feedback and feedback that eliminate one of two duty ratios, and it is shown that global stability can be achieved by using the latter type of feedback. We verified the dynamics and two-port characteristics by experimental demonstration, showing that the proposed feedback has global stability. The global stability obtained in this study provides the basis for constructing converter network systems.

References

[1] S.Singer, S.Efrati, Me.Alon, and D.Shmilovitz, "Maximum Electrical Power Extraction from Sources by Load Matching," Energies, vol.20, No.8025, 2021.

[2] D.Kiss, T.Hisakado, T.Matsushima, and O.Wada, "Peer-to-Peer Energy Transfer by Power Gyrators Based on Time-Variable-Transfomer Concept," IEEE Trans. on PE, vol.34, pp.8230–8240, 2019.

[3] L.Guo, L.Xu, k.Sun, and Y.Li, "Stability Analysis of Stand-alone Photovoltaic System Considering Controller Time Delay," IEEE Conference on EI^2, 2020.

[4] H.Bae, J.Lee, S.Park, and B.H.Cho, "Large-signal stability analysis of solar array power system," IEEE Trans. Aerosp. Electron. Syst., vol.44, pp.538–547, 2008.