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Chaotic Oscillations of Noise Suppressed Solid State Lasers Used in Optical-Wireless Networks

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Abstract–Diode pumped solid-state lasers are commonly used in high quality telecommunication applications, however due to their significant noise peak at the relaxation oscillation frequency optoelectronic reduction of intensity noise is required. Nevertheless the noise suppression feedback loop can make the laser operate in its chaotic regime. The paper shows the way to chaos through bifurcations at different levels of the feedback signal. Additionally a limit for chaos free operation is also given. The bifurcation diagram of the controlled laser is calculated as well.

1. Introduction

Close to carrier intensity noise of solid state lasers is a crucial parameter in many systems of the optical communications. Therefore significant effort is being done to investigate and reduce the noise of these lasers. Considering solid-state laser sources Nd:YVO_4 presents a good solution due to its high output power and good phase noise characteristics. However, it also suffers from the intensity noise peak at the relaxation resonance. At the frequency of the relaxation oscillations the noise has a high peak, 30-40 dB higher than that outside of the resonance region. This resonant range is between 100kHz to 2MHz with a resonant frequency defined by the pump power.

There were some publications in the literature on noise suppression and its possible application in optically supported wireless networks [1], Kane [2] has designed electronic feedback for the reduction of intensity noise in a diode pumped Nd:YAG laser. Regarding our previous report, Csörnyei et al. [3], a new design model was used for realization of optoelectronic noise suppression loops for Nd:YVO_4. These innovations made it possible to reduce the overall system noise and to carry out new investigations. Nevertheless the effect of the feedback signal of the optoelectronic noise suppression system on the dynamic behavior of the laser was not investigated so far. However there were some publications in the field of chaotic oscillations in solid state lasers induced by pump or cavity modulation by Luo [4], Pisarchik [5] and others. In this paper a realized approach is presented for the suppression of the RIN (Relative Intensity Noise) peak by active feedbacking, and the limit for possible chaotic oscillations caused by the noise suppression feedback loop is defined as well. The structure of the paper is the following. The second section presents the theory of the suppression and outlines the optical subsystem, exhibits the feedback loop electronics and the noise suppression. The third section presents the dynamic solution of the Nd:YVO_4 diode pumped solid-state laser rate equations and shows the way through period doubling bifurcations to chaos considering population inversion and photon density. The last part of the paper summarizes the results and the further possible efforts in this field.

2. Optoelectronic Noise Suppression

The block diagram of the low frequency feedback loop for noise suppression is depicted in Fig. 1. A fraction of the laser output signal is detected, differentiated, phase shifted, amplified and fed back to the pump laser biasing. The output wavelength of the laser crystal is 1064nm and it needs a 808nm pump signal. The used crystal laser consists of a Nd:YVO_4 lasing material and a MgO:$LiNbO_3$ electro-optic material for the active mode-locking. The geometrical size of the microchip defines the frequency difference between the two active modes of the laser. In our case, these two modes are 60GHz apart matched to the requirement of future wireless applications. By mode locking at the electro-optic material, high quality microwave generation is possible. This scheme is used for optical generation of microwaves in local oscillator remoting networks. The solid-state laser is pumped optically by an 1064nm high power (1.4W) laser diode. The 1064nm output light is focused on an InGaAs photodiode. The relaxation oscillations peak appears at an offset frequency of 100kHz-2MHz off the optical carrier. The measured relative intensity noise peak is shown in Fig. 2. Concerning the measurement circumstances the RIN has a peak value of -70dBc/Hz at the relaxation oscillation.

The experimental arrangement of the electronics used for the intensity noise suppression consisted of a transimpedance amplifier, differentiator circuit, amplifiers and the pump laser biasing circuitry. The by the feed backing achieved noise reduction can be seen in Fig. 3. The measured diagram shows an intensity noise suppression of 17dB at the relaxation oscillation.

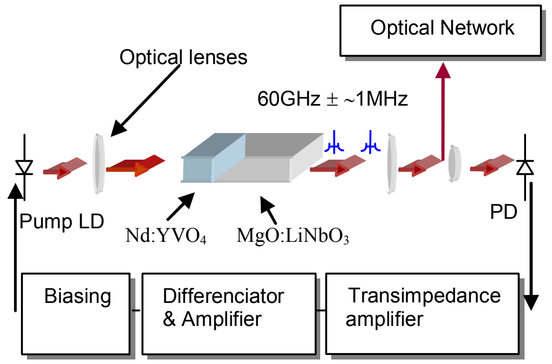

Figure 1. The realized feedback loop for intensity noise suppression. The two modes of the laser are 60GHz apart. A small fraction of the output power is fed back in the control loop.

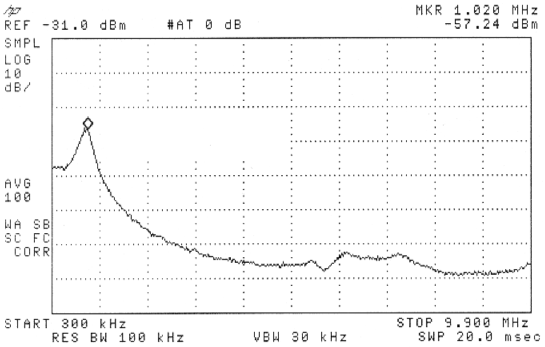

Figure 2. The relative intensity noise peak at a pump power of 350mW The relaxation oscillation frequency was at 1020kHz. The frequency of the relaxation oscillation is tunable by the pump power between 300kHz and 1.5MHz.

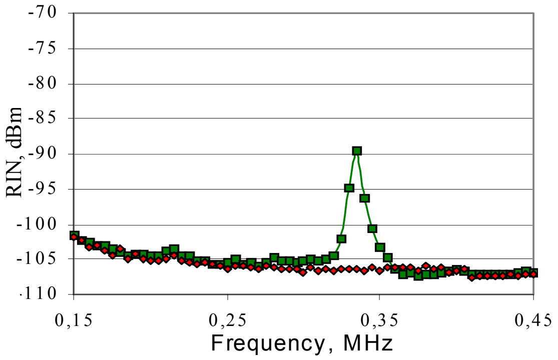

Figure 3. The observed noise suppression at a pump power of 80mW (300mA). The green curve is the noise spectrum of the free running laser, red curve presents the suppressed noise spectrum.. The measurement conditions were the following: Resolution BW=10 kHz, Video BW=10 kHz, 100 point Video Averaging, feedbacking modulation depth: 10%.

3. Dynamic Behavour of the Noise Suppressed Microchip Laser

In order to be able to investigate the dynamic operation of the laser one have to solve rate equations which present a system of nonlinear differential equations. Without input modulation the rate equations of the Nd:YVO$_4$ SSL are described in (1)-(2)

$$\frac{dn}{dt} = -n_2\sigma\varphi c - \frac{n_2}{\tau_f} + W_p(n_0 - n_2) \tag{1}$$

$$\frac{d\varphi}{dt} = c\varphi\sigma n \quad - \frac{\varphi}{\tau_c} + S \tag{2}$$

where n stands for upper level electron density, φ for photon density in the laser cavity, c for velocity of the light, n_0 is the total electron density ($4*10^{26}$), W is the pumping rate, τ_f the fluorescent lifetime (10^{-4}), τ_c is the cavity decay time (10^{-8}) and σ the emission cross section ($15*10^{-23}$) [9]. In (2) S represents the spontaneous emission of the laser (10^{17}) [11]. If one applies an optoelectronic feedback loop the feedback signal acts as an input modulation which turns the equation into a non-autonomous system of nonlinear differential equations. The rate equations in presence of external modulation are as written in (3)-(4). In (3) *a* and *ω* are the amplitude and the angular frequency of the pump modulation respectively.

$$\frac{dn}{dt} = -n_2\sigma\varphi c - \frac{n_2}{\tau_f} + W_p(1 + a.\cos(\omega t))(n_0 - n_2) \tag{3}$$

$$\frac{d\varphi}{dt} = c\varphi\sigma n \quad - \frac{\varphi}{\tau_c} + S \tag{4}$$

The solution of the equation system of (3)-(4) is feasible by numeric methods. Depending on the amplitude and frequency of the modulated signal the laser can operate in different modes. Without input modulation the electron and photon density are changing according to the relaxation oscillations, which gives a damped output. Applying a modulation signal the solution of the system equations will be a limit cycle with a frequency of the pump modulating signal. In case of optoelectronic feedbacking the modulating signal is a phase shifted version of the relaxation oscillation which causes a resonance suppression. At special frequencies, increasing the modulation amplitude the laser describing functions and the laser output goes through period doubling bifurcations. Further increasing the feedback signal level at same frequencies the laser behaves chaotic.

Fig. 4. depicts the electron density solution of (3)-(4) at very low modulation depth. The pumping rate (W) was 0.7, modulation depth 1%, at the relaxation oscillations frequency of 330kHz. Increasing the pump modulation the solution for the electron density shows a large amplitude sinusoidal periodic signal. At amplitude levels approaching the level of the DC pump rate (W) the output signal indicates period doubling bifurcations, as it can be seen in Fig. 5. This means that electron density is not altering periodically between two states like in case of the limit cycle but the signal splits for four levels. The phase portrait of the period doubling bifurcation is depicted in Fig. 6. Further increasing the feedback signal amplitude into the order of magnitude of the pumping rate the laser

behavior will be chaotic as it is demonstrated in Fig. 7. In the chaotic signal of Fig. 7 there is no periodicity.

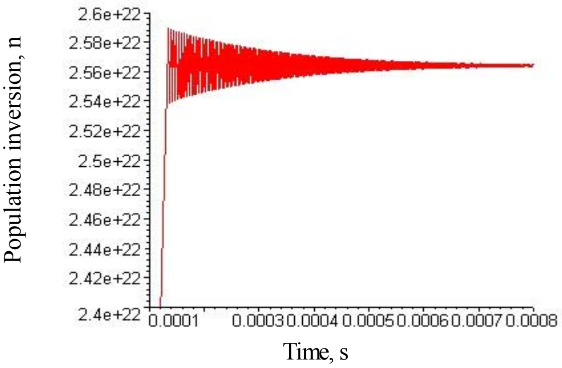

Figure 4. The upper level electron density of the Nd:YVO$_4$ laser at very low modulation depth (1%). The modulation frequency was 330kHz. The curve represents the relaxation oscillations decay and a very small amplitude limit cycle.

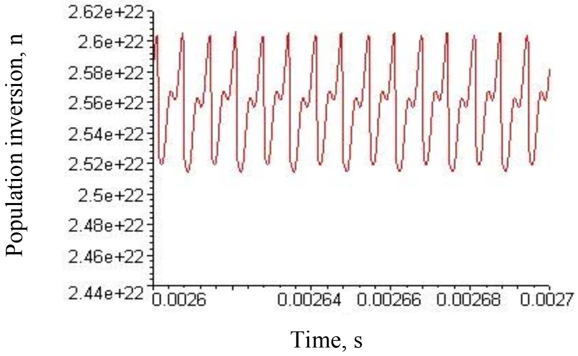

Figure 5. Period doubling bifurcation of the upper level electron population is depicted. The turn on on transient is not shown here (time window: 0.0026s..0.0027s). The electron density function alter around four possible levels. (Modulation depth:35%)

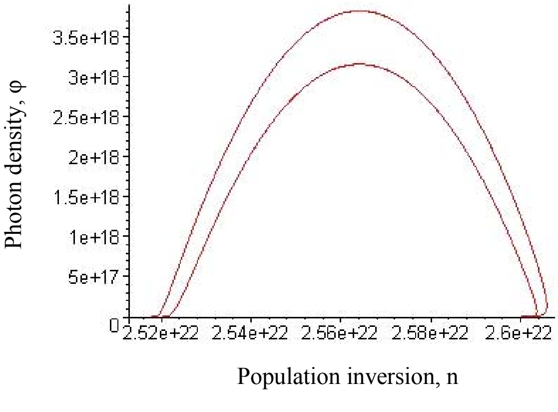

Figure 6. Phase portrait of the noise suppression modulated laser system in the state of the period doubling bifurcations. Noticeable is the four different level in the electron density. The population inversion of Fig. 5. is extended here with the information about the photon density (Modulation depth:35%)

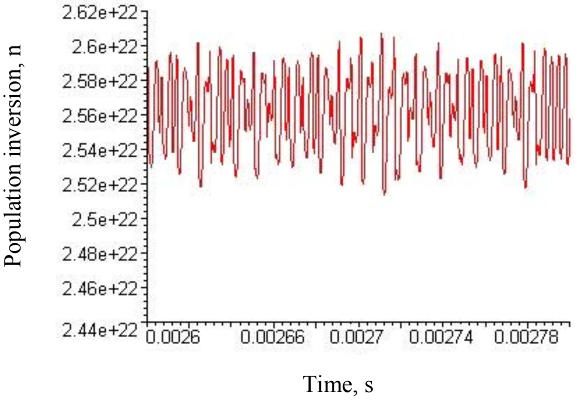

Figure 7. Chaotic oscillations of the Nd:YVO4 solid state laser. The pump is periodically modulated at the relaxation oscillation. (Modulation depth: 40%)

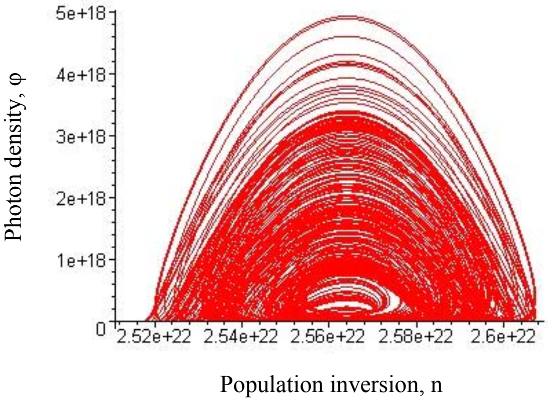

Figure 8. Phase portrait of the two variables, electron density and photon density of the modulated rate-equations of the Nd:YVO$_4$ SSL. Modulation depth: 91%.

The chaotic phase portrait containing the simultaneous representation of the electron density and the photon number is depicted in Fig. 8.

Comparing Fig. 5 to Fig. 7 and Fig. 6 to Fig. 8 very big difference can be seen between the period doubling bifurcation and the chaotic operation. In the latter case there is no more periodicity in the output oscillation.

In order to avoid the generation of unwanted periodic and chaotic oscillations in this nonlinear system special attention has to be given to the amplifier gain in the feedback loop. The optimal solution is if the feedback signal has an amplitude less than 10% of the pump diode biasing, which is a safe modulation depth ensuring noise suppression. If the modulation depth is higher than 10% limit cycle occurs. Further increasing the control signal amplitude the system behavior starts to turn to chaotic.

Summarizing the solutions of the time variant rate-equations which is presented by a non-autonomous, nonlinear differential equation system the bifurcation diagram of the photon density can be drawn which is depicted in Fig. 9. Similar diagram can be captured for the changes of the population inversion too. This figure gives an overview of the dynamic operation of the laser at the

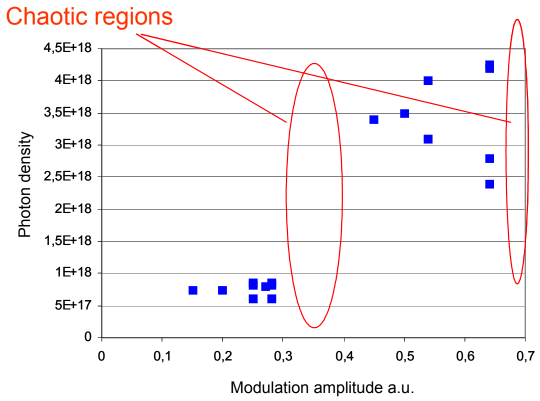

Figure 9. Bifurcation diagram of a Nd:YVO4 laser.

pumping rate of W=0.7. For other pump powers the bifurcations and the chaotic behavior will appear at different modulation depth but the overall form of the diagram will be the same.

3. Conclusions

In this paper we have demonstrated possible chaotic oscillations in noise suppressed Nd:YVO_4 solid state lasers. Microchip lasers are excellent potential sources for radio-over-fiber networks, because their high output power and outstanding phase noise characteristics. However they suffer from high peak of the the low frequency relaxation oscillation, which needs to be suppressed. In the first part of this paper we have presented a way of optoelectronic noise suppression which bases on a very simple RIN model of the laser [3].

In the second part of the paper we have analyzed the effect of the optoelectronic noise suppression loop on to the dynamic behavior of the laser diode pumped microchip laser. The chaotic behavior of the system is depending on the laser nonlinearity and gain of the amplifier in the optoelectronic feedback loop. The optically detected, amplified and phase shifted signal is fed back to the bias circuit of the pump diode. This signal can be looked as a modulation on the laser input. If the modulation depth is less than 10% the noise cancellation can be successful. However increasing the modulation signal strength first period doubling bifurcations and in case of further increasing the signal amplitude chaotic behavior can occur at harmonics of the relaxation oscillations frequency. This means that in special cases the feedback signal of optoelectronic noise suppression system can force the laser in its chaotic region of operation.

Acknowledgments

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