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Oscillatory Phenomena in Cellular Neural Network Using Two Kinds of Templates

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Abstract—In this study, oscillatory phenomena in a cellular neural network using two kinds of templates are investigated. The number of cells is limited to 3×3. The relationship among oscillatory phenomena and two parameters which are corresponding to coupling factors and biases. We consider that investigating this system is important to understanding coupled oscillatory systems. Because this system is a new class of coupled oscillatory systems.

1. Introduction

Cellular Neural Networks (CNNs) [1]-[3] is one kind of mutually coupled neural networks. The main characteristics are the local connection and the parallel signal processing. There have been many studies on CNNs and many kinds of CNNs have been proposed. One of them is two-layer CNN [4]. Two-layer CNN can generate many interesting phenomena. For instance, self-organizing pattern [4], active wave propagation [5] and so on are observed. Like this, some kinds of CNNs generate interesting phenomena. Investigating these system contributes to understand complex systems and to apply them to engineering systems.

In our earlier study [6], we proposed a CNN using two kinds of templates. This system was proposed in order to investigate a new class of coupled oscillatory system. Namely, by using two kinds of templates, oscillatory factors are coupled checkered. It is not so easy to implement this structure by using oscillatory circuit. On the proposed system, self-organizing pattern, active wave propagation and clustering phenomena were observed. Especially, clustering phenomena were investigated in detail. It was confirmed that clusters were classified into four kinds of patterns. Relationship among four patterns were also investigated.

In this system, we observed oscillatory phenomena by changing parameters. This phenomena means that this system is a new class of coupled oscillatory system. In this study, we investigate this phenomena. Especially, we pay attention to the relationship among values of templates and observed phenomena.

2. CNN Using Two Kinds of Template

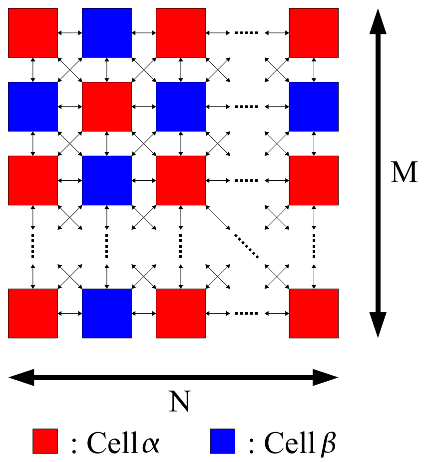


Figure 1: System model of CNN using two kinds of templates.

Figure 1 shows a system model of our proposed system. We assume that the system has a two-dimensional M by N array structure. Each cell in the array is denoted as $c(i, j)$, where (i, j) is the position of the cell, $1 \leq i \leq M$ and $1 \leq j \leq N$. The coupling radius is assumed to be one. In this proposed CNN, two kinds of templates are used. Cells having one template are called as Cell α and the other are called as Cell β. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

1: The case that $i + j$ is an even number.

$$\begin{aligned}\frac{dx_{ij}}{dt} &= -x_{ij} + I_\alpha \\ &+ \sum_{c(k,l)} A_\alpha(i, j; k, l) y_{kl} \\ &+ \sum_{c(k,l)} B_\alpha(i, j; k, l) u_{kl}\end{aligned} \tag{1}$$

2: The case that $i + j$ is an odd number.

$$\begin{aligned}\frac{dx_{ij}}{dt} &= -x_{ij} + I_\beta \\ &\quad + \sum_{c(k,l)} A_\beta(i,j;k,l)y_{kl} \\ &\quad + \sum_{c(k,l)} B_\beta(i,j;k,l)u_{kl}\end{aligned} \tag{2}$$

$A_{\{\alpha\beta\}}(i,j;k,l)y_{kl}$, $B_{\{\alpha\beta\}}(i,j;k,l)u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively. The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}). \tag{3}$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \tag{4}$$

The variables u and y are the input and output variables of the cell, respectively. $\boldsymbol{A}_\alpha$, $\boldsymbol{B}_\alpha$, $\boldsymbol{A}_\beta$ and $\boldsymbol{B}_\beta$ are 3 times 3 matrices, which can be described to have a similar form to Eq. (5).

$$\begin{pmatrix} A_\alpha(i,j;i-1,j-1) & A_\alpha(i,j;i-1,j) & A_\alpha(i,j;i-1,j+1) \\ A_\alpha(i,j;i,j-1) & A_\alpha(i,j;i,j) & A_\alpha(i,j;i,j+1) \\ A_\alpha(i,j;i+1,j-1) & A_\alpha(i,j;i+1,j) & A_\alpha(i,j;i+1,j+1) \end{pmatrix} \tag{5}$$

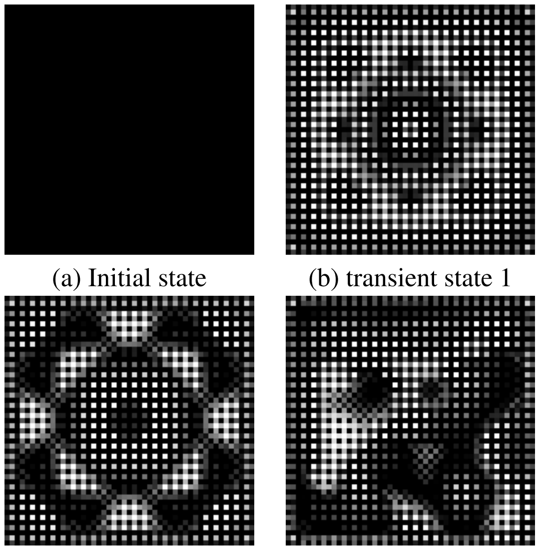
This proposed system is more complex than the normal CNNs. This system has a unique characteristic. Namely, a pair of cell α and cell β are needed for a simple oscillation. Additionally, one cell α connects with four neighbor cells β and one cell β also connects with four neighbor cells α. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators. Hence, we consider that this system is a new class of coupled oscillatory systems.

3. Relationship Between Oscillatory Phenomena and Parameters

In our earlier study, clustering phenomena observed in the system have been investigated. The initial conditions and parameters are set as follows. The values of boundary cells are set zero. The template is set as

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}, \ \boldsymbol{A}_\beta = \begin{pmatrix} 1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \tag{6}$$
$$\boldsymbol{B}_\alpha = 0, \ \boldsymbol{B}_\beta = 0, \ I_\alpha = 0, \ I_\beta = 0,$$

The number of cells are set as 49×49. In this clustering phenomena, the cells between clusters oscillate and the inner cell of clusters don't oscillate. Namely, most of the cells don't oscillate in spite of our purpose which is proposing a new class of coupling oscillatory systems. However, by changing only I_α and I_β of Eq. (6), oscillatory phenomena are observed shown in Fig. 2. In this case, parameters



(a) Initial state (b) transient state 1

(c) transient state 2 (d) transient state 3

Figure 2: Oscillatory phenomena for the CNN array consists of 49×49.

I_α and I_β are set as 1. Initial states of cells are set as 1 shown in Fig. 2 (a). By starting the computer simulation, oscillatory phenomena are observed as shown in Figs. 2 (b)-(d) in series. At first, the phenomena look like symmetry shown in Figs. 2 (b) and (c). Then, symmetric phenomena is broken shown in Fig. 2 (d), suddenly. After that all cells keep oscillating. In order to investigate this phenomena, the number of cells is limited to 3×3 in this study. The initial conditions and parameters are set as follows. Initial state values of boundary cells are set as 0.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -p & p & -p \\ p & p & p \\ -p & p & -p \end{pmatrix}, \ \boldsymbol{A}_\beta = \begin{pmatrix} p & -p & p \\ -p & -p & -p \\ p & -p & p \end{pmatrix}, \tag{7}$$
$$\boldsymbol{B}_\alpha = 0, \ \boldsymbol{B}_\beta = 0, \ I_\alpha = q, \ I_\beta = q,$$

where p and q are corresponding to the coupling factors and biases I_α and I_β, respectively. By using this conditions and parameters, computer calculations are carried out. Here, state variables of each cells are defined as shown in Fig. 3. Figure 4 shows the computer simulation results in the case of cell array 3×3. Vertical axis is the value of state variables. Horizontal axis is time. Cell A, C, G and I are the same values each other. In the same way, cell B, D, F and H are also the same values each other. Therefore, only three waveforms are observed. The case of $p = 1$ and $q = 1$ is corresponding to Fig. 2. In this case, oscillatory phenomena are not observed as shown in Fig. 4 (a). However, oscillatory phenomena are observed in the cases of some combinations of p and q. Some of these results are shown in Figs. 4 (b)-(d). Parameters of Fig. 4 (b) differ from parameters of Fig. 2 in I_α and I_β. As a result of this investigation, we can confirm that the waveform frequency and amplitude are influenced by changing variables p and q.

Next, we investigate the relationship among oscillatory phenomena, variables p and q. Figure 5 shows the relationship among oscillatory phenomena, variables p and q. Horizontal axis is the value of p. Vertical axis is the value of q. "O" means that the oscillatory phenomena are observed. "N" means that the oscillatory phenomena aren't observed. Oscillatory phenomena are observed in two regions. In lower side region, similar waveforms to Fig.4 (b) are observed. Increasing the value p increases the amplitude and decreases the frequency. In upper side region, similar phenomena to Fig.4 (c) are observed. Increasing the values p and q increases the amplitude and the frequency. Please note that the oscillatory phenomena are not observed in the case of $q = 1.0$. This line separates two region. Additionally, in the case of 49×49, oscillatory phenomena are observed. It can be considered that border cells influence the oscillatory phenomena.

4. Conclusions

In this study, oscillatory phenomena in cellular neural networks using two kinds of templates were investigated. Especially, the case that the number of cells are limited to 3×3 were investigated. As a result, the relationship among oscillatory phenomena, values p and q were shown. Additionally, it could be confirmed that the border of CNNs influences the oscillatory phenomena.

In our future works, we will investigate the case of the large numbers of cells.

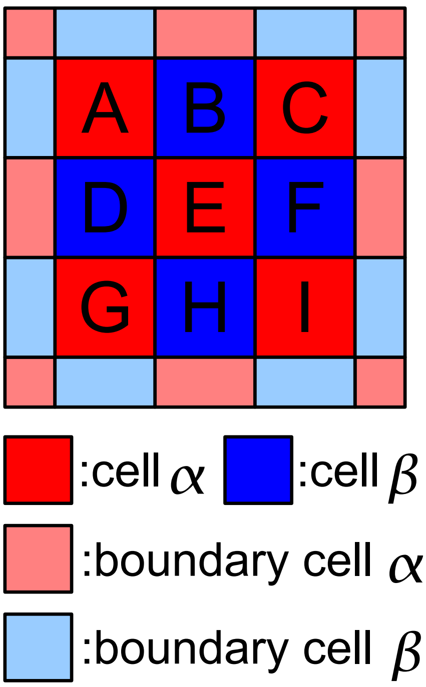
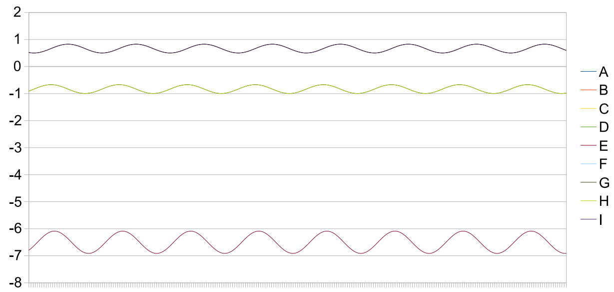
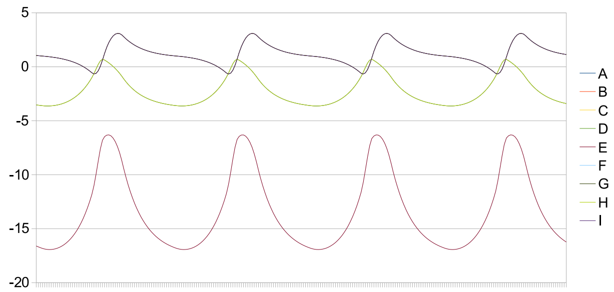


Figure 3: The CNN array consists of 3×3.

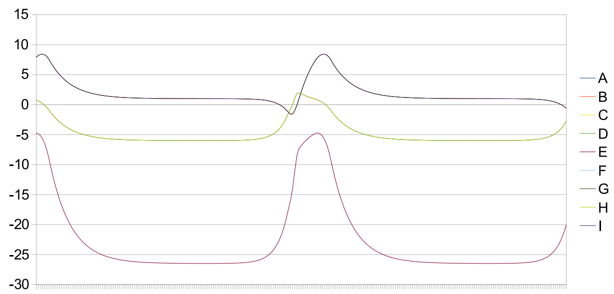
(a) $p = 1$ and $q = 1$.



(b) $p = 1$ and $q = 0.5$.



(c) $p = 2$ and $q = 0.5$.



(d) $p = 3$ and $q = 0.5$.

Figure 4: Waveforms of state values of each cells. The number of cells is 3×3.

q \ p	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
9.5	N	N	N	N	N	N	N	N	O	O	N	N	N
9.0	N	N	N	N	N	N	N	N	O	O	N	N	N
8.5	N	N	N	N	N	N	N	O	O	N	N	N	N
8.0	N	N	N	N	N	N	N	O	O	N	N	N	N
7.5	N	N	N	N	N	N	O	O	N	N	N	N	N
7.0	N	N	N	N	N	N	O	O	N	N	N	N	N
6.5	N	N	N	N	N	N	O	N	N	N	N	N	N
6.0	N	N	N	N	N	O	O	N	N	N	N	N	N
5.5	N	N	N	N	N	O	O	N	N	N	N	N	N
5.0	N	N	N	N	O	O	N	N	N	N	N	N	N
4.5	N	N	N	N	O	O	N	N	N	N	N	N	N
4.0	N	N	N	O	O	N	N	N	N	N	N	N	N
3.5	N	N	N	O	O	N	N	N	N	N	N	N	N
3.0	N	N	O	O	N	N	N	N	N	N	N	N	N
2.5	N	N	O	O	N	N	N	N	N	N	N	N	N
2.0	N	N	O	O	N	N	N	N	N	N	N	N	N
1.5	N	N	O	N	N	N	N	N	N	N	N	N	N
1.0	N	N	N	N	N	N	N	N	N	N	N	N	N
0.5	N	O	O	O	O	O	O	N	N	N	N	N	N
0.0	N	O	O	O	O	O	O	O	O	O	O	O	O

Figure 5: Relationship among the oscillatory phenomena, value of p and q. "O" means that oscillatory phenomena are observed. "N" means that oscillatory phenomena are not observed.

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