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The Implementability Analysis of Hybrid Controlled Behavior with Discrete Event Controllers in the 2D System Theoretic Framework

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Abstract—The implementability of a given hybrid specification with discrete event controllers is presented. Particularly, by regarding hybrid dynamical systems as a class of systems whose dynamics evolve with two different types of independent variables, that is, 2-dimensional (2D) dynamical systems, we give a necessary and sufficient condition for a specification to be implemented with discrete event controllers within a behavioral framework, where a set of trajectories plays a central roles in the system synthesis and analysis.

1. Introduction

The problem whether a given specification (controlled behavior) can be achieved or not is one of the most fundamental issues in control system synthesis. Since this is equivalent to investigate of the existence of a certain controller to be implemented so as to achieve the specification, this property is referred to as the *implementability* of a specification. The implementability is deeply related to the dynamics of a system, so it is natural and rational to consider that it is to be addressed with *representation-free*, which corresponds to that synthesis and analysis of a dynamical system is performed without using mathematical models of a system.

Recently, there are some important studies on this issue with respect to the behavioral approach, where the set of trajectories along which the dynamics of a system obey plays a central role for analysis and synthesis of the dynamical system. Particularly, in [2], [3] and [9], the implementability of the controlled behavior in general systems were discussed. In discrete event system theory, the references [4] and [5] provided conditions under which a given specification can be achieved for the $l-$ complete approximated system that is described by the finite automaton. Since in the behavioral approach, as written above, the central role for synthesis and analysis is played with the set of trajectory directly, it is to be expected that this approach provides generalized and relevant theoretical results for the implementability.

By the way, the study of a *hybrid system*, that consists of not-only continuous-valued dynamics but also event driven dynamics heterogeneously, has attracted many researchers from theoretical and practical points of view (e.g. cf. [10] and so on). The mathematical languages used for describing these dynamics are completely different, it is difficult to view hybrid system from broader perspective. Thus, it is rational to see hybrid systems with focusing the set of trajectories directly. In this sense, the behavioral approach is expected to provide unified standpoints to such a class of systems. Moreover, for a plant with the above hybrid dynamics, the rational method of hybrid controller has not been well-established yet.

From these backgrounds, this paper considers the implementability of a given hybrid specification with discrete event controllers. The reason why we focus on discrete event controllers instead of using hybrid controllers is tha fact that the design method of hybrid controller is not so well-established while the design of the discrete event controllers have been provided by many researchers. Here, by regarding hybrid dynamical systems as a class of systems whose dynamics evolve with two different types of independent variables, that is, 2-dimensional (2D) dynamical systems, we give a necessary and sufficient condition for a specification to be implemented with discrete event controllers within a behavioral framework, where a set of trajectories plays a central role in the system synthesis and analysis.

2. The implmentability and the canonical controllers

Here, we give brief reviews required in this paper on the implementability and the canonical controllers in a behevioral framework (See [9] for more details). First, we view a dynamical system (with the manifest behavior) Σ is defined as a triple

$$\Sigma = (\mathbb{T}, \mathbb{W}, \mathcal{B}_{\mathrm{f}}) \tag{1}$$

where $\mathbb{T}$ is the *time axis*, $\mathbb{W}$ is the *signal space* and $\mathfrak{B} \subseteq \mathbb{W}^{\mathbb{T}}$ is the manifest *behavior*, where $\mathbb{W}^{\mathbb{T}}$ is the set of the maps from $\mathbb{T}$ to $\mathbb{W}$. in the behavioral framework (cf.[11],[12]). We also introduce a dynamical system defined by

$$\Sigma = (\mathbb{T}, \mathbb{W} \times \mathbb{C}, \mathcal{B}_{\mathrm{f}}) \tag{2}$$

where $\mathcal{B}_{\mathrm{f}} \subseteq (\mathbb{W} \times \mathbb{C})^{\mathbb{T}}$ is referred to as the full behavior. For $(w, c) \in \mathcal{B}_{\mathrm{f}}$, w is the variable to be controlled, c is the variable to be interconnected with a controller illustrated in Fig 1.

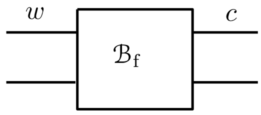

Figure 1: A plant $\mathcal{B}_{\mathrm{f}}$ with interconnected varibles

A control in the behavioral approach is viewed as follows (cf.[9]). We are given a plant $\Sigma = (\mathbb{T}, \mathbb{W} \times \mathbb{C}, \mathcal{B}_{\mathrm{f}})$ and a specification (a controlled behavior) $\mathcal{K} \in \mathbb{W}^{\mathbb{T}}$. Let $\Sigma_c = (\mathbb{T}, \mathbb{C}, \mathcal{C})$ be a certain system. The (partial) interconnection of Σ and Σ_c is defined as

$$\Sigma \wedge \Sigma_c = (\mathbb{T}, \mathbb{W}, \mathcal{B}_{\mathrm{f}} \|_c \mathcal{C}) \tag{3}$$

and the behavior of this interconnected system is defined by

$$\mathcal{B}_{\mathrm{f}} \|_c \mathcal{C} = \{ w \in \mathbb{W}^{\mathbb{T}} |^{\exists} c \in \mathcal{C} \text{ s.t. } (w, c) \in \mathcal{B}_{\mathrm{f}} \}. \tag{4}$$

This is illustrated in Fig. 2.

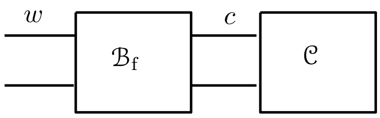

Figure 2: Interconnection of systems

The purpose of the interconnection is to find $\mathcal{C}$ such that

$$\mathcal{B}_{\mathrm{f}} \|_c \mathcal{C} = \mathcal{K}. \tag{5}$$

And if there exists a controller $\mathcal{C}$ satisfying Eq.(5), $\mathcal{K}$ is said to be implementable . Together with the implementability of $\mathcal{K}$, the following controller

$$\mathcal{C}^{\mathrm{can}} = \{ c \in \mathbb{C}^{\mathbb{T}} |^{\exists} \tilde{w} \in \mathbb{W}^{\mathbb{T}} \text{s.t. } (\tilde{w}, c) \in \mathcal{B}_{\mathrm{f}}, \tilde{w} \in \mathcal{K} \} \tag{6}$$

which was introduced in [9] also plays a central role. The controller $\mathcal{C}^{\mathrm{can}}$ yields the specification $\mathcal{K}$ exactly after the interconnection with $\mathcal{B}_{\mathrm{f}}$(see Fig.3). See also Fig 4 illustrating the interconnection of a plant $\mathcal{B}_{\mathrm{f}}$ and a canonical controller $\mathcal{C}_d^{\mathrm{can}}$. In the linear case, a canonical controller corresponds to well-known Youla parameterization.

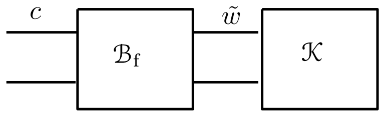

Figure 3: A canonical controller $\mathcal{C}^{\mathrm{can}}$ for $\mathcal{K}$

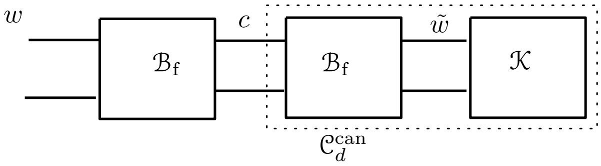

Figure 4: The interconnection of $\mathcal{B}_{\mathrm{f}}$ and $\mathcal{C}^{\mathrm{can}}$ for $\mathcal{K}$

3. Problem Formulation

3.1. Hybrid system as 2D system

Before going to the problem formulation we attack here, we now introduce a new concept for hybrid dynamical systems. Generally, the dynamics of a hybrid system evolves with not only the (conventional) time but also the event occurrence, so *a hybrid system can also regarded as a dynamical system with two independent variables*, one is the time, the other is the event occurrence. This is a new view points of this paper.

Let $\mathbb{T}$ denote the time axis of a hybrid system. We also denote $\mathbb{T}_c$ and $\mathbb{T}_d$ with the time and the event occurrence, respectively. For example, if the continuous valued part of the hybrid system is described by differential (difference) equations, we view $\mathbb{T}_c = \mathbb{R}$ ($\mathbb{Z}$, respectively). if the discrete event part is described by finite automata with n states, we regard $\mathbb{T}_d = \mathbb{Z}/n\mathbb{Z}$. In the sequel, we divide the time axis $\mathbb{T}$ into

$$\mathbb{T} = \mathbb{T}_d \times \mathbb{T}_c. \tag{7}$$

as Cartesian product of two different time axis.

3.2. Projection operators on the behaviors

Here, we prepare some projection maps required to formulate the problems and state our main result of this paper. The first type of the projection maps is used for restricting the dynamics of the full behavior onto the manifest behavior or the interconnecting variables behavior. Consider a dynamical system described by Eq.(2). We then define the map $\pi_w(\bullet)$ as the subset of the maps $\mathbb{W} \times \mathbb{C} \to \mathbb{W}$ described by

$$\pi_w(\mathcal{B}_{\mathrm{f}}) = \{ w \in \mathbb{W} |^{\exists} c \in \mathbb{C}^{\mathbb{T}} \text{ s.t.} (w, c) \in \mathcal{B}_{\mathrm{f}} \}. \tag{8}$$

This map corresponds to the restriction of $(w, c) \in \mathcal{B}_{\mathrm{f}}$ into the manifest variable w. Similarly, we define the map $\pi_c(\bullet)$ as the subset of the maps $\mathbb{W} \times \mathbb{C} \to \mathbb{C}$ as

$$\pi_c(\mathcal{B}_{\mathrm{f}}) = \{ c \in \mathbb{W} |^{\exists} w \in \mathbb{W}^{\mathbb{T}} \text{ s.t.} (w, c) \in \mathcal{B}_{\mathrm{f}} \}. \tag{9}$$

The second type of the projection maps is used for the restriction of the hybrid dynamics evolving with the two-different time axises onto the dynamics with one of them. We define the map $\gamma(\bullet)$ as the subset of the maps $\mathbb{W}^{\mathbb{T}} \to \mathbb{W}^{\mathbb{T}_d}$ described by

$$\gamma(\mathcal{B}_{\rm f}) = \{w(\bullet, i) \in W^{\mathbb{T}_d} \text{ s.t.}(w,c) \in \mathcal{B}_{\rm f}\}. \tag{10}$$

That is, the role of γ is to pick up the discrete event dynamics only of $w \in \mathbb{W}^{\mathbb{T}}$.

It should be noted that these above maps are surjective. and not bijective. Thus, we define $\pi_w(\bullet)^{-1}$, $\pi_c(\bullet)^{-1}$ and $\gamma(\bullet)^{-1}$ as the inverse of the images.

3.3. The Problem Formulations

Now we are ready to formulate the problems we attack here.

Firstly, a hybrid plant we consider here is assumed to be described by

$$\Sigma = (\mathbb{T}, \mathbb{W} \times \mathbb{C}, \mathcal{B}_{\rm f}) \tag{11}$$

where, the time axis $\mathbb{T} = \mathbb{T}_d \times \mathbb{T}_c$.

Next, we focus on a specification to be achieved. We assume that the specification $\mathcal{K}$ is with the time axis $\mathbb{T}_d$, that is, $\mathcal{K} \subseteq \mathbb{W}^{\mathbb{T}_d}$. We assume that a controller to be implemented with $\mathcal{B}_{\rm f}$ is a discrete event driven system $\mathbb{T}_d$, which is denoted with $\mathcal{C} \subseteq \mathbb{C}^{\mathbb{T}_d}$. We denote the interconnection of $\mathcal{B}_{\rm f}$ and $\mathbb{C}$ thorugh the variavle $c \in \mathbb{C}$ with $\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C}$, and is described by

$$\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} = \{w \in \mathbb{W}^{\mathbb{T}}| \\ {}^{\exists}c \in \mathbb{C}^{\mathbb{T}} \text{ s.t.}\{\gamma(c) \in \mathcal{C} \text{ and } (w,c) \in \mathcal{B}_{\rm f}\}\}. \tag{12}$$

In the discrete event systems, it is important to forbit that an undesirable event occur. From this stand point, the first problem we address here is formulated as follows.

Problem 3.1 *For a given specification $\mathcal{K} \subseteq \mathbb{W}^{\mathbb{T}}$, find a condition under which there exists a controller $\Sigma_c = (\mathbb{T}_d, \mathbb{C}, \mathcal{C})$ such that satisfies*

$$\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} \subseteq \mathcal{K}. \tag{13}$$

Since $\mathcal{K}$ is given so as to contain no undesirable event, the behavior after the interconnection does not also contains undesirable event as long as the designed controller satisfies the above condition.

The second problem is more strict than Problem 3.2, in the sense that it requires that the interconnection should achieve a given specification exactly. That is to say, it is also less conservative than the first problem.

Problem 3.2 *For a given specification $\mathcal{K} \subseteq \mathbb{W}^{\mathbb{T}}$, find a condition for $\mathcal{K}$ to be implementable with $\mathcal{B}_{\rm f}$, or equivalently, for the existence of a controller $\Sigma_c = (\mathbb{T}_d, \mathbb{C}, \mathcal{C})$ such that satisfies*

$$\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} = \mathcal{K} \tag{14}$$

In the end of this section, we provide the useful lemma reauired in the following main results.

Lemma 3.1 *For Eq.(12), the following relation holds;*

$$\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} = \{w \in \mathbb{W}^{\mathbb{T}}| \\ {}^{\exists}c_d \in \mathcal{C}\text{s.t.}\{{}^{\exists}c \in \gamma^{-1}(c_d)\text{s.t. } (w,c) \in \mathcal{B}_{\rm f}\}\} \tag{15}$$

(Proof): It is enough to show that the sets of the right hand side of Eq.(12) and Eq.(15). Assume that $w \in \mathbb{W}^{\mathbb{T}}$ is included in the right hand side of Eq.(12). Then, or equivalently, there exists a $c \in \mathbb{C}^{\mathbb{T}}$ such that $(w,c) \in \mathcal{B}_{\rm f}$ and $\gamma(c) \in \mathcal{C}$. Define $c_d := \gamma(c) \in \mathcal{C}$, which implies $c \in \gamma^{-1}(c_d)$. Together with the condition $(w,c) \in \mathcal{B}_{\rm f}$, we see that for c_d there exists c such that $c \in \gamma^{-1}(c_d)$ and $(w,c) \in \mathcal{B}_{\rm f}$. Thus, $w \in \mathbb{W}^{\mathbb{T}}$ is included in the right hand side of Eq.(15).

Conversely, assume that $w \in \mathbb{W}^{\mathbb{T}}$ is included in the right hand side of Eq.(15). Then, or equivalently, there exists a $c_d \in \mathcal{C}$ such that $(w,c) \in \mathcal{B}_{\rm f}$ and $c \in \gamma^{-1}(c_d)$. This implies that $c_d \in \gamma(c) \in \mathcal{C}$, which also implies that for this w there exists $c \in \mathbb{C}^{\mathbb{T}}$ such that $\gamma(c) \in \mathcal{C}$ and $(w,c) \in \mathcal{B}_{\rm f}$. Thus, $w \in \mathbb{W}^{\mathbb{T}}$ is included in the right hand side of Eq.(12). (Q.E.D)

4. Main results

4.1. A solution to Problem 3.1

Theorem 4.1 *For $\mathcal{S} \subseteq \mathbb{W}^{\mathbb{T}}$, if for any (w,c) and $(\tilde{w},\tilde{c}) \in \mathcal{B}_{\rm f}$ such that $\gamma(c) = \gamma(\tilde{c})$, $w \in \mathcal{K}$ implies $\tilde{w} \in \mathcal{S}$, then there exists a controller $\Sigma_c = (\mathbb{T}_d, \mathbb{C}, \mathcal{C})$ such that $\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} \subseteq \mathcal{S}$.*

(Proof): Here we prepare a canonical controller defined by

$$\mathcal{C}_d^{\text{can}\prime} := \{c_d \in \mathbb{C}^{\mathbb{T}_d}|{}^{\forall}c \in \gamma^{-1}(c_d) \cap \pi_c(\mathcal{B}_{\rm f}) \text{ s.t.} \\ \{{}^{\exists}w \text{ s.t.}(w,c) \in \mathcal{B}_{\rm f} \text{ and } w \in \mathcal{K}\}\} \tag{16}$$

and show $\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C}_d^{\text{can}} \cap \mathcal{K}$. Assume that $w \in \mathcal{B}_{\rm f}\|_{c_d}\mathcal{C}_d^{\text{can}}$. Then, define $c_d := \gamma(c)$ for c such that $(w,c) \in \mathcal{B}_{\rm f}$, $c_d \in \mathcal{C}_d^{\text{can}}$ holds. This implies that $(\tilde{w},\tilde{c}) \in \mathcal{B}_{\rm f}$ such that $\tilde{c} \in \gamma^{-1}(c_d)$ and $\tilde{w} \in \mathcal{K}$. Thus, the above condition in the theorem yields that $w \in \mathcal{K}$ (Q.E.D)

4.2. A solution to Problem 3.2

It should be noted that the conditions of Theorem 4.1 are also sufficient ones. The reason is similar to the case in which a specification is with only one time axis stated in the previous subsection. Thus, here, we also impose the following characterization in order to derive the necessity condition for the achievability of $\mathcal{K}$.

Property 4.1 *For any (w,c) and $(\tilde{w},\tilde{c}) \in \mathcal{B}_{\rm f}$ such that $\gamma(c) = \gamma(\tilde{c})$, $\gamma(w) \in \mathcal{K}$, if there exists a c' such that$(w,c') \in \mathcal{B}_{\rm f}$, then there exists a c^* such that $(\tilde{w},c^*) \in \mathcal{B}_{\rm f}$ such that $\gamma(c^*) = \gamma(c')$.*

By using this property, we obtain the following necessary and sufficient condition as follows.

Theorem 4.2 *Assume that* $\mathcal{B}_{\rm f}$ *is with Property 4.1. For* $\mathcal{K} \subseteq \mathbb{W}^{\mathbb{T}}$*, there exists a controller* $\Sigma_c = (\mathbb{T}_d, \mathbb{C}, \mathcal{C})$ *such that* $\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} = \mathcal{K}$*. if and only if the following two conditions holds:*

1. $^{\exists}\mathcal{C}_0 \subseteq \mathbb{W}^{\mathbb{T}}$ *s.t.* $\mathcal{K} = \mathcal{B}_{\rm f}\|_c\mathcal{C}_0$.
2. *For any* (w, c) *and* $(\tilde{w}, \tilde{c}) \in \mathcal{B}_{\rm ff}$ *such that* $\gamma(c) = \gamma(\tilde{c})$*,* $w \in \mathcal{K}$ *implies* $\tilde{w} \in \mathcal{K}$.

(Proof): Here we also prepare a canonical controller defined by Eq.(16). We first show the 'if' part. Assume $w \in \mathcal{K}$. From the condition 1, there exists a c such that $(w, c) \in \mathcal{B}_{\rm f}$ and $c \in \mathcal{C}_0$. Define $c_d := \gamma(c)$. For $\tilde{c} \in \gamma^{-1}(c_d)$ such that $c \neq \tilde{c}$ if there exists a $\tilde{w}$ such that $(\tilde{w}, \tilde{c}) \in \mathcal{B}_{\rm f}$, then the condition 2 yields that $\tilde{w} \in \mathcal{K}$. This also implies $c_d \in \mathcal{C}_d^{\rm can}$. From these observations, we see that

$$\{c \in \mathcal{C}_0 |^{\exists} w \in \mathcal{K} \text{ s.t. } (w, c) \in \mathcal{B}_{\rm f}\} \in \mathcal{C}_d^{\rm can} \qquad (17)$$

so we obtain

$$\begin{aligned} \mathcal{K} &= \mathcal{B}_{\rm f}\|_{\rm c}\mathcal{C}_0 \\ &= \pi_w(\mathcal{B}_{\rm f} \cap \pi_c^{-1}(\mathcal{C}_0)) \subseteq \pi_w(\mathcal{B}_{\rm f} \cap \pi_c^{-1}\gamma^{-1}(\mathcal{C}_d^{\rm can})) \\ &= \mathcal{B}_{\rm f}\|_{\rm c_d}\mathcal{C}_d^{\rm can}. \qquad (18) \end{aligned}$$

Next, we focus on the 'only if' part as follows. Let $\mathcal{C}$ be a controller satisfying $\mathcal{B}_{\rm f}\|_{c_d}\mathcal{C} = \mathcal{K}$. Firstly we see that the necessity of the condition 1 can be shown defining $\mathcal{C}_0 = \gamma^{-1}\mathcal{C}$. Next, for any (w, c) and $(\tilde{w}, \tilde{c}) \in \mathcal{B}_{\rm f}$ such that $\gamma(c) = \gamma(\tilde{c})$, assume that $w \in \mathcal{K}$. Since $\mathcal{C}$ achieves the specification $\mathcal{K}$, there exists a $c' \in \gamma^{-1}(\mathcal{C})$ such that $(w, c') \in \mathcal{B}_{\rm f}$. From Property 4.1, this implies that there exists a c^* such that $(\tilde{w}, c^*) \in \mathcal{B}_{\rm f}$ and $\gamma(c^*) = \gamma(c') \in \mathcal{C}$. Again since $\mathcal{C}$ achieves $\mathcal{K}$, we see that $\tilde{w} \in \mathcal{K}$. Thus, the condition also holds. (Q.E,D)

5. Concluding Remarks

In this paper, we have addressed an implementability of hybrid dynamical systems in a behavioral framework. by using discrete event driven controller. Here, we have regarded hybrid dynamical systems as a class of systems whose dynamics evolves with two different types of independent variables, one is a "time" with which a solution of differential or difference equations evolve, other is "the event time" or "discrete state index" corresponding to the occurrence of a discrete event or a switching time between one and another states. We have also seen that such a stand point enables us to see that a hybrid system is viewed as a 2-dimensional (which is abbreviated to as 2D in the following) dynamical system . In this new setting, we have provided a necessary and sufficient condition of the existence of a event-driven controller for a given specification to be controlled.

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