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Error-Correcting Method Based on Chaotic Dynamics for Noncoherent Chaos Communications

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Abstract—This paper proposes the error-correcting method based on the chaotic dynamics for noncoherent chaos communications. We generate successive chaotic sequences from the identical chaotic map. And for the next sequence we set the initial value to the end value of the former sequence. By such way we can create the successive chaotic sequences having the same chaotic dynamics. This feature gives the receiver additional information to correctly recover the received noisy signal. Therefore, by analyzing the chaotic dynamics at the receiver, it is possible to improve the error performance. In other words, error performance of the receiver can be improved by the utilization of the suboptimal receiver with the analysis of chaotic dynamics of the successive received sequence. As results of computer simulations, we confirm about 3 dB gain in BER performance compared to the conventional suboptimal receiver.

1. Introduction

Recently, research on digital communications systems using chaos becomes a hot topic [1]− [8]. Especially, it is attracted to develop noncoherent detection systems which do not need to recover basis signals (unmodulated carries) at the receiver. The differential chaos shift keying (DCSK) [1] and the optimal receiver [2] are well known as a typical noncoherent system. In this study, we focus attention on the optimal receiver.

The optimal receiver performs an optimal detection by using the probability density function (PDF) between the received signals and the same chaotic map of the transmitting side. Thus the receiver can detect the information symbol by comparing the PDFs. However, the optimal receiver suffers from a computational complexity due to the large chaotic sequence length. Also, the large chaotic sequence gives rise to a decrease in the performance of the optimal receiver. Thus, it is important to develop a receiver with performance equivalent to the optimal receiver using different algorithms, i.e., a suboptimal receiver.

In our previous research, we proposed the suboptimal receiver using the shortest distance approximation [9]. Instead of calculating the PDF, the proposed suboptimal receiver approximates the PDFs by calculating the shortest distance between the received signals and the chaotic map. As results of the computer simulations, we confirmed the validity of the proposed suboptimal receiver as an approximation method of the optimal receiver.

Note that the suboptimal receiver shows a slight performance loss to the optimal receiver. In other words, the detection characteristic of the suboptimal receiver is not superior to the optimal receiver. However, we expect to improve the performance of the suboptimal receiver by focusing features of chaos. Because we investigated and analyzed the influence on chaos communications systems by the chaotic dynamics and its behavior previously [10]− [12] and confirmed that the chaotic dynamics affect the performance of chaos communication systems greatly. Therefore, we consider that it is important to develop communication systems based on features of chaos.

The purpose of this is to propose an error-correcting method based on the chaotic dynamics for the suboptimal receiver. Let us consider two successive chaotic sequences that are generated from the identical chaotic map. We generate the second sequence whose initial value is the end value of the former sequence. In this case, the chaotic dynamics of these two successive chaotic sequences are the same. This feature gives the receiver additional information to correctly recover the received noisy signal. Therefore, by analyzing the chaotic dynamics at the receiver, it is possible to improve the error performance. In other words, error performance of the receiver can be improved by the utilization of the suboptimal receiver with the analysis of chaotic dynamics of the successive received sequence.

As results of computer simulations, we confirm about 3 dB gain in BER performance compared to the conventional suboptimal receiver.

2. System Overview

We consider the discrete-time binary CSK communication system, as shown in Fig. 1. Detail of each block is described below.

2.1. Transmitter

In the transmitter, a chaotic sequence is generated by a chaotic map. In this study, we use a skew tent map which is one of simple chaotic maps, as shown in Fig. 2(a), and it is described by Eq. (1)

$$x_{i+1} = \begin{cases} \dfrac{2x_i + 1 - a}{1 + a} & (-1 \le x_i \le a) \\[2ex] \dfrac{-2x_i + 1 + a}{1 - a} & (a < x_i \le 1) \end{cases} \quad (1)$$

where a denotes a position of the top of the skew tent map. The information symbol is modulated by Chaos Shift Keying (CSK) which is a digital modulation system using

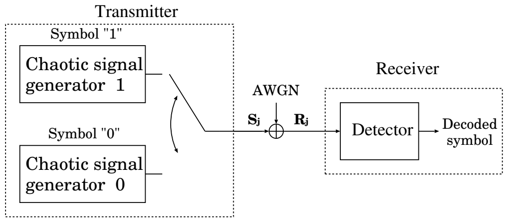
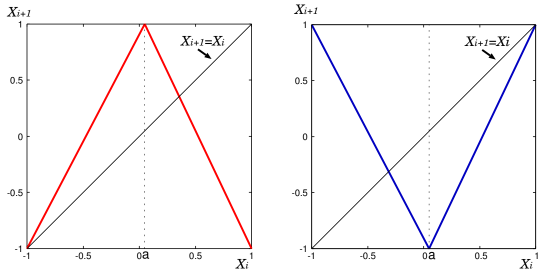


Figure 1: Block diagram of discrete-time binary CSK communication system.



(a) Symbol "1" (b) Symbol "0"

Figure 2: Chaos Shift Keying.

chaos. When the transmitter generates the signals, we use chaotic sequences generated by different chaotic maps depending on the value of an information symbol. If the information symbol "1" is sent, Eq. (1) is used (Fig. 2(a)), and if "0" is sent, the reversed function of Eq. (1) is used (Fig. 2(b)). To transmit a 1-bit information, N chaotic signals are generated, where N is the chaotic sequence length.

In this study, to perform the error-correcting by the receiver, information are transmitted by a packet which consists of a data part and a redundancy part, as shown in Fig. 3, where $\mathbf{S_j}$ is the transmitted signal vector, $j = (0, 1, \cdots, K-1)$, K is total bit per packet, **Seq "1"** and **Seq "0"** are the chaotic sequence generated by Fig. 2(a) and (b), respectively. Note that the transmitted signal vector $\mathbf{S_j}$ is different for each symbol. [**When the symbol "1" is sent**]

$$\mathbf{S_j} = (x_j \; f^{(1)}(x_j) \cdots f^{(i)}(x_j) \cdots f^{(N-1)}(x_j)) \qquad (2)$$

[**When the symbol "0" is sent**]

$$\mathbf{S_j} = (y_j \; g^{(1)}(y_j) \cdots g^{(i)}(y_j) \cdots g^{(N-1)}(y_j)) \qquad (3)$$

where $f^{(i)}$ and $g^{(i)}$ are the function of the skew tent map for symbol "1" and "0" (Figs. 2(a) and (b)) respectively, i is the iteration of f or g, x_j or y_j denotes the initial value of the jth symbol = "1" or "0" respectively. When K bits data are transmitted, the length of the data part becomes $K \times N$. The redundancy is the sequence which has the known information symbol, and its length is different depending on the error-correcting method. In addition, the initial value of the chaotic sequence is also different in each chaotic signal generator and is updated each packet, namely, the chaotic sequence generated by each generator in the packet is a successive sequence based on the chaotic dynamics. As an example, let us assume the jth and $(j+3)$th symbols are "1" (i.e. the $(j+1)$th and $(j+2)$th symbols are "0"). In this case, x_{j+3} becomes $f^{(N)}(x_j)$. In the same way, y_{j+2} becomes $g^{(N)}(y_{j+1})$. Moreover, **Seq "1"** and **Seq "0"** are also a successive sequence based on the chaotic dynamics. These reasons are described by Sec. 3.

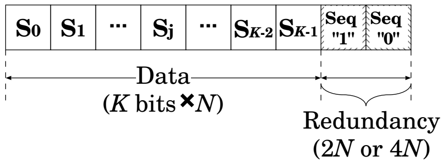


Figure 3: Packet Format.

2.2. Channel and Noise

The channel distorts the signal and corrupts it by noise. In this study, noise of the channel is assumed to be the additive white Gaussian noise (AWGN). Thus, the received signals block is given by $\mathbf{R_j} = (R_{j,0} \; R_{j,1} \cdots R_{j,N-1}) = \mathbf{S_j} + AWGN$.

2.3. Receiver

The receiver recovers the transmitted signals from the received signals and demodulates the information symbol. Also, the receiver performs the error-correcting in this study. Since we consider a noncoherent receiver, the receiver memorizes the chaotic map used for the modulation at the transmitter. However, the receiver never knows the initial value of chaos and the information symbol in the transmitter. Before explaining the error-correcting, we introduce the operation of our suboptimal receiver to be the basis for the proposed error-correcting.

[Operation of Our Suboptimal Receiver

∼Shortest Distance Approximation Method∼]

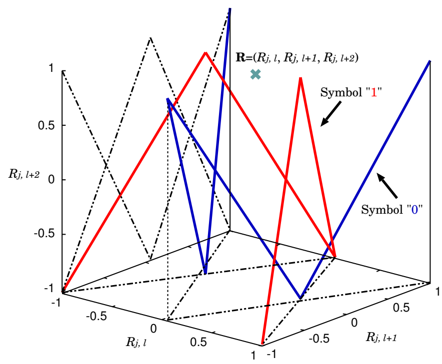


Figure 4: Operation of Our Suboptimal Receiver ($N_d = 3$).

Our suboptimal receiver calculates the shortest distance between the received signals and the chaotic map in the N_d-dimensional space using N_d successive received signals (N_d : $2, 3, \cdots$). As an example, we explain the case of $N_d = 3$. Figure 4 shows the 3-dimensional space of the skew tent map whose coordinates correspond to the three successive received signals $\mathbf{R} = (R_{j,l}, R_{j,l+1}, R_{j,l+2})$, where $R_{j,l}$ denotes the jth symbol and lth signal of the received signals ($l = 0 \; 1 \cdots N-3$). We can calculate the shortest distance between $\mathbf{R}$ and the chaotic map using the scalar product of the vector. Any two points of $\mathbf{P_0} = (x_0, y_0, z_0)$ and $\mathbf{P_1} = (x_1, y_1, z_1)$ are chosen from each straight line in the space of Fig. 4, as shown in Fig. 5.

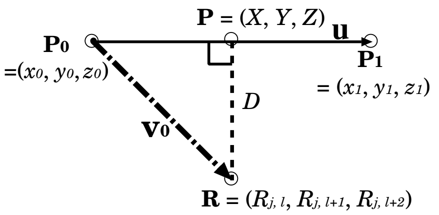


Figure 5: Calculation of the shortest distance.

Using Fig. 5, we can calculate $\mathbf{P} = (X, Y, Z)$ and the shortest distance D by the following equations.

$$\mathbf{P} = (X, Y, Z) = (\mathbf{u} \cdot \mathbf{v_0})\,\mathbf{u} + \mathbf{P_0} \tag{4}$$

$$D \quad = \quad ||\mathbf{P} - \mathbf{R}|| \tag{5}$$

where

$$\text{Unit vector } \mathbf{u} \quad = \quad \frac{\mathbf{P_1} - \mathbf{P_0}}{||\mathbf{P_1} - \mathbf{P_0}||} \tag{6}$$

$$\mathbf{v_0} \quad = \quad \mathbf{R} - \mathbf{P_0}\ . \tag{7}$$

Note that if the point is outside the cube, we calculate the distance between the point and the nearest edges of the maps.

For the 3-dimensional case, there are four straight lines in the space. Therefore, the minimum value in four distances is chosen as the shortest distance D_1 for symbol "1". In the same way, D of symbol "0" is chosen as D_0. We calculate both of D_1 and D_0 for all l and find their summations $\sum D_1$ and $\sum D_0$. Finally, we decide the decoded symbol as 1 (or 0) for $\sum D_1 < \sum D_0$ (or $\sum D_1 > \sum D_0$).

The calculation of the shortest distance can be extended to N_d-dimensional space for $N_d \geq 4$.

In our previous research, we observed that the performance of our method corresponds that of the optimal receiver when $N = N_d$-dimension [9]. Therefore, we confirmed the validity of the proposed suboptimal receiver as the approximation method of the optimal receiver.

3. Proposed Error-Correcting Method

First of all, we explain a motivation of the proposed method. To develop the error-correcting method, we focus attention on successive chaotic sequences generated from the identical chaotic map based on the chaotic dynamics. Let us consider two successive chaotic sequences that are generated from the identical chaotic map. Here, we assume that the initial value of the second sequence is the end value of the former sequence. In this case, the chaotic dynamics of these two successive chaotic sequences are the same. This feature gives the receiver additional information to correctly recover the received noisy signal. Therefore, we consider that it is possible to improve the error performance by analyzing the chaotic dynamics at the receiver.

To take advantage of these characteristics, we propose the error-correcting method based on the chaotic dynamics. Our proposed error-correcting method calculates the shortest distance by the double or triple of the sequence length N which used for the modulation in the transmitter, i.e. the double or triple dimension ($N_d = 2N$ or $3N$). Figure 6(a) shows the image of the operation of the error-correcting. To simplify the explanation, we assume the redundancy length to be $2N$. The redundancy is the sequence which has the known information, "1" and "0". Here, let us expect that the $j = (K-1)$th bit is "1" or "0". Since the $(K-1)$th bit is "1" or "0" and the redundancy has "1" or "0" certainly, the $(K-1)$th bit sequence also has the relation with either of two types of the sequence based on the chaotic dynamics. Moreover the $(K-2)$th sequence also has the relation with either of three types of the sequence. Thus we can consider the j and $j-1$th bit as the four pattern, as shown in Fig 6(b), where $D_1(.)$ and $D_0(.)$ mean the shortest distance between the chaotic map of Symbol "1" and "0", respectively. Since the sequence connected two types becomes the double sequence length, we can calculate the shortest distance using $N_d = 2N$-dimensional space. We calculate each distance like the pattern of Fig 6(b). Finally, we choose the pattern of the shortest distance out of the four patterns, and the $(K-1)$th bit is decided accurately. By using this determined symbol for detection of the next bit, these operation can be shifted and it can demodulate to all information. Therefore, the likelihood of information can improve by expecting the combination of the sequence based on the chaotic dynamics, and we can detect information in high accuracy. In this case, the coding rate becomes $\frac{K}{K+2}$.

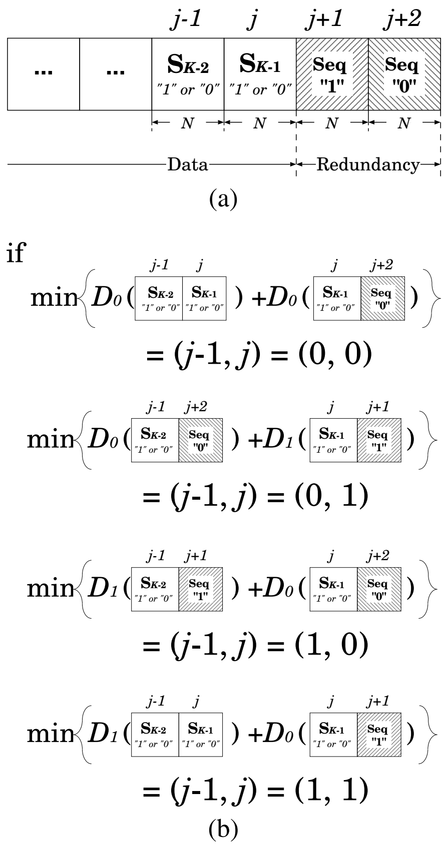
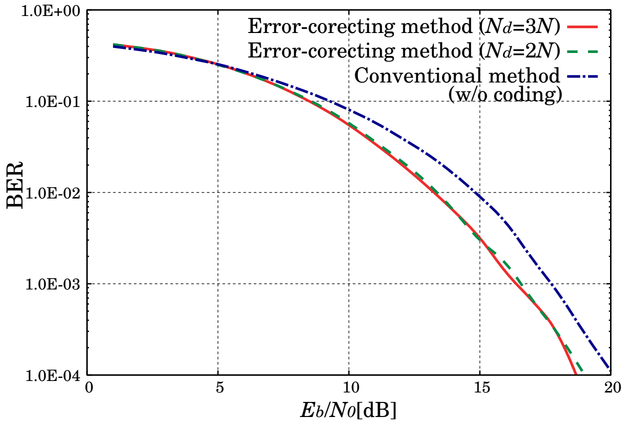


Figure 6: Proposed Error-Correcting Method ($N_d = 2N$).

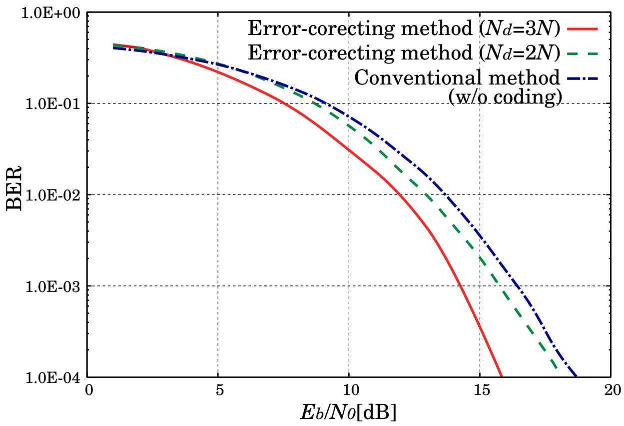
In the same way, we can calculate the shortest distance using $N_d = 3N$-dimensional space to perform the error-correcting. In this case, the redundancy length is $4N$, the combination of the sequence becomes the eight patterns and the coding rate becomes $\frac{K}{K+4}$.

4. Simulation Results and Discussion

To evaluate the proposed error-correcting method, we carry out computer simulations with following simulation conditions. In the transmitting side, the packet consists of the data part with $K = 64$ bits and the redundancy part with $2N$ or $4N$. In this case, each coding rate becomes 0.96 or 0.94. Here, the parameter of the skew tent map is fixed as $a = 0.05$, and we use the chaotic sequence length $N = 3$ and 4. To compare the performance of N_d-dimensional space to calculate the shortest distance, we use $N_d = 2N$-dimensional space, and $N_d = 3N$-dimensional space. Based on these conditions, we iterate the simulation 10,000 times and calculate the average of BERs.



(a) $N = 3$.



(b) $N = 4$.

Figure 7: Simulation results.

Figures 7(a) and (b) show the BERs versus E_b/N_0 for $N = 3$ and $N = 4$, respectively. To compare the performance of the error-correcting method, Figs. 7(a) and (b) also show the performance of our suboptimal receiver, namely, the performance without the coding. From these results, we observe that the both BERs of the error-correcting method show gain over the system without the coding. Significant improvements are observed for $N = 4$ with $N_d = 3N$-dimensional. In this case, since the receiver calculate the shortest distance using $N_d = 12$-dimensional space, the computation complexity also increase. As our receiver calculates the likelihood of the received symbols not only by the shortest distance between the received signals and the chaotic map but also analyze the chaotic dynamics of the successive symbols, the likelihood of the received symbol increase and the performance improvement is achieved. Moreover, although the cording rate is very high, the proposed method can well correct the error. Therefore, we can say that the efficiency of the proposed error-coding is excellent.

However, we also confirm that the curve of the proposed method of $N = 3$ with $N_d = 3N$-dimensional space corresponds that with $N_d = 3N$-dimensional, as shown in Fig. 7(a). We have not clearly understood the reason why two curves correspond, yet. We would like to investigate and analyze the chaotic sequence and its behavior in detail when our error-correcting method is used.

5. Conclusions

In this study, we have proposed the error-correcting method based on the chaotic dynamics for the suboptimal receiver. We generate the chaotic sequence from the same chaotic map and make the successive sequences to have the same chaotic dynamics. We perform the detection by analyzing the chaotic dynamics of the received sequences. As results, we have confirmed that the likelihood of the received symbol increases and obtained the 3dB gain in error performance.

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