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Chaotic Cyclic Attractors Shift Keying for Low-Rate UWB Communications

Yanjun XU, Pascal CHARGE, Danièle FOURNIER-PRUNARET

LATTIS/Toulouse University, 135 Avenue de Rangueil, 31077 Toulouse CEDEX 4, France
Email: yanjun.xu@insa-toulouse.fr,pascal.charge@insa-toulouse.fr,Daniele.Fournier@insa-toulouse.fr

Abstract—In this paper, a chaotic modulation scheme using chaotic cyclic attractor (CCA) is proposed, with a non-coherent receiver. The demodulation relies on the frequency property of CCA that the majority energy of the iteration is approximately distributed in a deal of main frequencies; and these frequencies are determined by the order of CCA. The comparison of performance with DCSK is given, and also the multipath prospect.

1. Introduction

There are several ways to classify the digital modulation schemes in communication systems, among which there is an approach which uses the types of modulation carriers to distinguish[1]: (1) fixed waveforms as carriers, or (2) continuously varying waveforms as carriers. It's known that the conventional modulation schemes use the fixed waveforms, usually sinusoidal-based carrier waves; and chaotic modulation schemes, in the contrast, take the continuously varying waveforms, i.e., chaotic waveforms, as reference waves. Normally, different symbols are mapped to different chaotic reference waveforms, which vary even for the same symbol value.

As long as demodulation is concerned, generally we can use coherent or non-coherent techniques. In a coherent receiver, it requires to regenerate the basis functions, independently of the modulation, which is difficult to realize, especially when the poor propagation conditions are concerned in the wireless communications. If the basis functions are impossible to recover, a non-coherent receiver needs to be considered, which includes: (1) Transmitted Reference (TR) -based auto-correlation receiver, e.g., Differential Chaos Shift Keying (DCSK)[2][3]; (2) differentially encoded a prior to modulation plus a non-coherent receiver, e.g, amplitude detection receiver for ASK, Pulse Polarity Modulation (PPM) and Chaos On-Off Keying (COOK), phase detection receiver for PSK, frequency detection receiver for FSK, etc.

By now, there doesn't exist any literature dealing with the use of phase or frequency detection non-coherent receiver for chaotic modulation schemes. Usually we focus on changing the principles of modulation or demodulation in order to increase the performance, but neglect the possibility of replacing the basis functions by special chaotic attractors, e.g., the iterations of attractors which have some special frequency or phase properties, thereby can be used to realize the demodulation more easily, even with better performance.

In this article, we are going to introduce one of these special chaotic attractors, chaotic cyclic attractors (CCA), which have some special frequency properties. And based on these frequency properties, we propose another chaotic modulation scheme, using CCA, and demodulated by a non-coherent frequencies detection receiver.

2. Chaotic Cyclic Attractor

Several chaotic systems generate iterations as special attractors, i.e., chaotic cyclic attractors (CCA), under certain conditions, e.g., given parameter values and initial conditions. CCAs are one type of chaotic attractors which are composed by several cyclic and disjointed chaotic areas, and the iterations jump from one area to another regularly, with the position in each area chaotic. We use a simple sinusoidal filter system to generate this phenomenon, which is given by the following equation:

$$\begin{cases} x_{n+1} = sin\left(a\pi x_n + b\pi y_n\right) \\ y_{n+1} = x_n \end{cases} \quad (1)$$

with the block diagram shown in Fig.1.

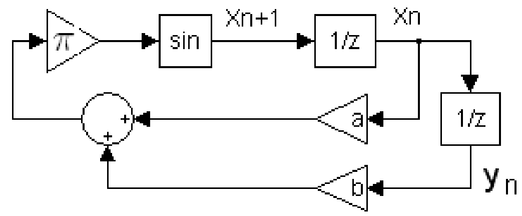


Figure 1: Block diagram of the non-linear system

As a non-linear differential system, system(1) exhibits a quasi-periodic solution via Neimark-Sacker bifurcation, which is often locked into various periodic motions by changing parameter values, and the Arnold's tongue structure[4] can also be found in the two-parameter bifurcation diagram (a, b). Through calculating the isoclines of the fixed point with different specified arguments of complex multipliers [5] and the period-doubling and tangent bifurcation of (1), we can obtain the parameter values around which exist the different orders of cycles or CCAs. Fig.2 shows the simulated result of the bifurcation diagram.

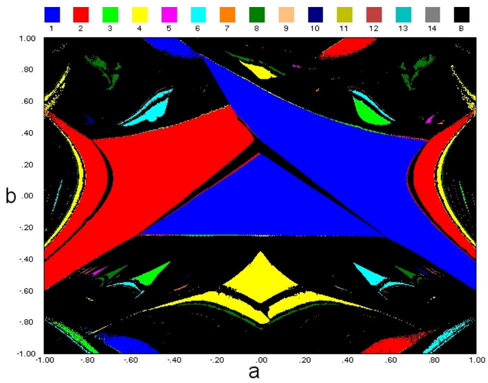
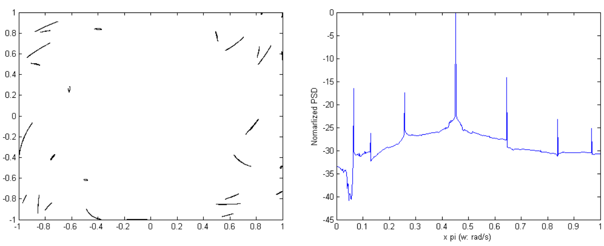


Figure 2: Bifurcation diagram of (1), colored areas denote the cycles of order $K \leq 14$, with K differed by the colors, and black areas represent cycles of order $K > 14$ or chaos.

It is known that the parameter values in the bifurcation diagram plane (a, b) for generating CCA of order K situates around the parameter zone corresponding to the cycle of order K, usually under the corresponding Arnold's tongue. For example, through calculating, we found that (1) with the parameters $(a = 0.3590, b = -0.6184)$ generates the CCA of order 31, with the iteration in phase plane shown in Fig.2.a.



(a) (x_n, x_{n+1}) (b) Normalized PSD of x_n

Figure 3: The iteration of CCA order 31 of system (1) in phase plane (a) and the corresponding PSD (b).

As the iteration of CCA of order q has the characteristics of both non-periodicity and hopping regularly among q different zones of the attractor, it's obvious that the frequency distribution of this iteration should be represented by lots of frequency peaks, among which the main one locates at $\frac{2\pi p}{q}$ in the $[o, \pi]$ lower half frequency band, and several secondary ones locate at $\frac{2\pi p'}{q}$. Here $\frac{p}{q}$ is related to the complex multiplier of the isocline[5], where p, q are coprime, $p, p' < q$, $p' \neq p$, and the possibilities of p' are determined by the attribution of the CCA. The corresponding power spectral density (PSD) of the iteration of CCA of order 31 is shown in Fig.2.b.

The PSD of x_n shown in Fig.2.b. indicates that the magnitude of the main frequency is at least $15dB$ greater than the other frequencies, definitely it's more a narrow band waveform than a large band one. The performance of this kind of carrier will degrade quickly, even entirely lost if it's transmitted through a multipath propagation where exists a deep frequency-selective fading effect.

In order to keep the property of large frequency band, we need to add some function modules in the chaotic generator in Fig.1 to lower the magnitude of the main frequency $\frac{2\pi p}{q}$ in PSD, or proportionally enhance the other secondary important frequencies as $\frac{2\pi p'}{q}$, hence widen the frequency band which the chaotic iteration takes. Through simulation, we found that the two modules added to the CCA generator in Fig.4 can help us to reach this object, which are:
(i) *Folding* module: a function to fold up the attractor on itself towards the origin of the phase plane, by using $x_{f_n} = sign(x_n)\,(1 - |x_n|)$, hence the dominance of the main frequency is considerably reduced, meanwhile many other comparable frequency peaks appear. The iteration of x_{f_n} in phase plane and frequency domain are shown correspondingly in Fig.5.a. and Fig.5.b.
(ii) *Cubing* module: a function to lower the amplitude of each iteration in an exponential way, by using $x_{c_n} = x_{f_n}^3$. The iteration of x_{c_n} in phase plane and frequency domain are shown accordingly in Fig.5.c. and Fig.5.d.

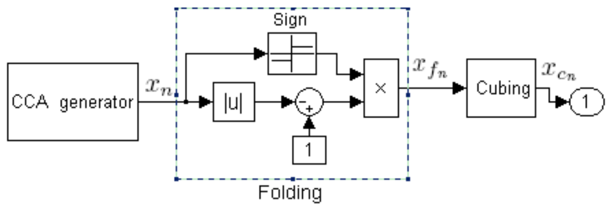
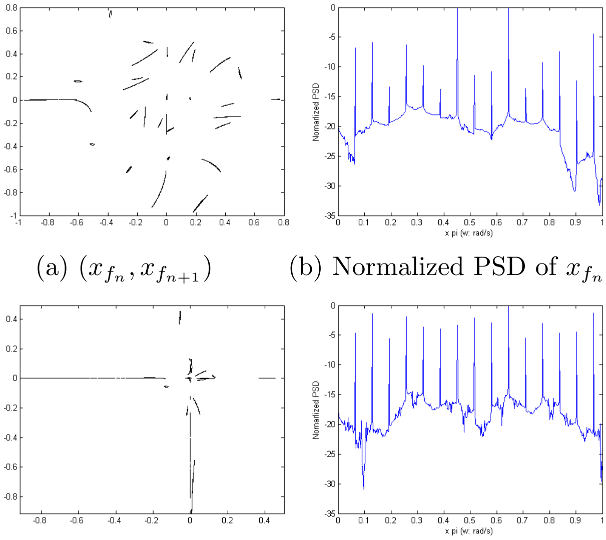


Figure 4: *Folding* and *Cubing* are added to form the new chaotic generator

Through observing the PSD results, we can see that all the main frequencies of iteration x_{c_n} locate in $\frac{2\pi k}{31}$, with $k = 1, 2, ...30$, and the differences among their magnitudes are at most $5dB$. This waveform takes a really large band in the frequency domain, from 0 to Fs, with Fs equals to the oscillation frequency of the chaotic generator. After analysing several different orders of CCA, we can come to a general conclusion



(a) $(x_{f_n}, x_{f_{n+1}})$ (b) Normalized PSD of x_{f_n}

(c) $(x_{c_n}, x_{c_{n+1}})$ (d) Normalized PSD of x_{c_n}

Figure 5: The frequency distribution of the iterations with *Folding* and *Cubing* modules

that, for the iteration of the CCA generator of order q in Fig.4, PSD has a special distribution: it has q main frequencies which occupy most of the power, and locate correspondingly in $\frac{2\pi k}{q}, k = 1, 2, ...q-1$.

As a result of this frequency attribution, the chaotic iteration has the following attractive properties:
(1) Because the iteration is represented by lots of main frequencies distributed in the large band, the energy of the iteration is shared by them, as a result, each main frequency occupies less than $\frac{1}{p}Eb$, with the reduction proportional to the order of CCA p. Ultra large-band communication systems usually reuse the already occupied radio band, and in order to avoid the interference to the conventional system, the PSD should be as low as possible. So talking about applying CCASK in UWB, we should use low symbol energy, and take the bigger p to get even lower PSD.
(2) Because the probability that multipath completely cancels all the signal frequencies is opposite to the frequency bandwidth, the large band lowers the sensitivity to multipath effects.
(3) Because the frequencies are distributed almost averagely in the whole angular period 2π, the Fourier Transform module can be used directly in frequency detection, in order to reduce the complication of the receiver.

Based on these properties, we propose here another chaotic modulation scheme, which uses the iterations as shown in Fig.5.c.-d. as basis functions for modulation, and the method of Fourier Transformation as demodulation. The principles will be given in the next section.

3. The principle of CCASK

Chaotic Cyclic Attracotr Shift Keying (CCASK) is concerned with mapping symbols to chaotic waveforms generated by different orders of CCA generators, whose frequency property can be represented by lots of main frequencies distributed almost averagely in the large frequency band $[0, Fs]$, with Fs equals to the oscillation frequency of the chaotic generator. In this section, we will present the introduction of the simplest binary CCASK.

3.1. Binary CCASK Modulation

In the simplest case of binary CCASK, only two different orders of CCA generators should be used, i.e., a CCA generator of order q_0 when a symbol '0' is transmitted, and another CCA generator of order q_1 when '1' is transmitted, and for each symbol, the chip rate is L chips per symbol.

When a symbol data 'i' ($i = 0, 1$) has to be transmitted, a sequence of L elements $\{x_{c_n}^{(i)}\}$ $(n = 0, 1, ..., L-1)$ is generated by the CCA generator with parameters (a_i, b_i), and sent into the propagation channel. Thus the expression of n^{th} chip of m^{th} symbol, $s_m(n)$, with the input binary symbol d_m $(d_m = i)$ is given by:

$$s_m(n) = d_m x_{c_n}^{(1)} + (1 - d_m) x_{c_n}^{(0)} \tag{2}$$

and the general expression of a chip at the output of the modulation system is $s((m-1)L + n) = s_m(n)$.

3.2. Binary CCASK Demodulation

The non-coherent receiver relies on the distribution of the main frequency peaks of each received symbol, which should be possibly filtered and noised by the communication channel, here we denote it as $\tilde{s}_m(n) + \tilde{b}_m(n)$, with $\tilde{b}_m(n)$ the filtered noise. Through application of the Discrete Fourier Transform (DFT) method on the received symbol, we can easily get the magnitudes of the main observating frequencies as

$$A_m^i(k) = \left| \sum_{n=0}^{L-1} \left(\tilde{s}_m(n) + \tilde{b}_m(n) \right) e^{-j2\pi \frac{k}{q_i} n} \right| \tag{3}$$

where $k = 1, 2, ..., q_i - 1$, and $i = 0, 1$. Obviously, the decision variable D_m for the m^{th} received symbol should be the difference between the magnitudes of the above two groups of frequencies, as

$$D_m = \sum_{k=1}^{q_1-1} \frac{A_m^1(k)}{q_1 - 1} - \sum_{k=1}^{q_0-1} \frac{A_m^0(k)}{q_0 - 1} \tag{4}$$

Consequently, the demodulated symbol, $\hat{d}_m$, can be given by

$$\hat{d}_m = \begin{cases} 1, & \text{if } D_m \geq 0 \\ 0, & \text{if } D_m < 0 \end{cases} \tag{5}$$

4. Comparison of Noise Performance

As a result of the non-coherent frequencies demodulation, the bit energy of CCASK doesn't need to keep constant as differentially coherent DCSK[6]. Due to the quasi-periodicity of CCA, the difference among the magnitudes of the observed frequencies is proportional to the chip rate L, i.e., proportional to the symbol duration T. In contrast, actually the observed frequencies are fixed in the unit of angular frequency for each CCA, so the noise performance doesn't depend on the RF bandwidth $2B$, dislike DCSK. This means that if we don't consider the different interferences from the different frequency bands, we can arbitrarily select the bandwidth and obtain the same BER performance.

For the sake of application of CCASK in the scene of Low-Rate UWB, we suppose in our simulation a high chaotic oscillation rate, e.g., $Fs = 2GHz$, so that the corresponding frequency band is $2B = 2GHz$. Fig.6. shows the simulated noise performance of CCASK through an AWGN channel, based on the factors: $q_0 = 29$, $q_1 = 31$ for 10^5 symbols. From right to left, the dashed curves show the results of simulations with $L = 200, 400, 600$, so that the corresponding symbol durations are $T = 0.1\mu s, 0.2\mu s, 0.3\mu s$. For comparison, the theoretical noise performances of DCSK with the same values of BT are given as solid lines from left to right, corresponding to the approximate expression of the noise performance of DCSK using stochastic technique in [3].

We can see that the performance of CCASK augments with the increase of T, in contrast, the performance of DCSK reduces. For $T = 0.3\mu s$, the noise performance of binary CCASK is already more than $3dB$ better than DCSK for $BER = 10^{-4}$, with the data rate about $\frac{10}{3}Mbps$. As the data rate decreases with the increase of T, we believe that CCASK is a good choice for a Low-Rate UWB modulation scheme.

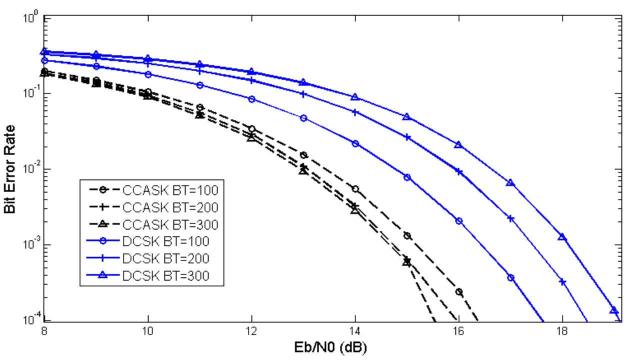


Figure 6: Simulated noise performance of binary CCASK (dashed curves), from right to left, the bit durations are $0.1\mu s, 0.2\mu s, 0.3\mu s$. Comparison of the corresponding noise performance of DCSK (solid curves) from left to right.

5. Conclusion

After decades of research by many researchers in the chaotic communication domain, represented by G. Kolumban, M. P. Kennedy, etc., there exists the theory and performance limits for the chaotic modulation schemes comparing with the conventional ones, especially in the narrow-band applications, and the most promising scene should be the wide-band or even ultra wide-band applications, e.g., WLAN, UWB. [1][6]

As a result of the wide-band carriers and low complexity of transceiver of CCASK, together with the better performance comparing with DCSK in the long symbol duration application (where CCASK can enhance the noise performance in exchange with the transmitting data rate), we propose that CCASK is a good choice for Low Rate UWB application.

The multipath and multi-user performances of CCASK, and also the solution of reducing the interference by the running applications in the occupied frequency bands are being researched.

Acknowledgment

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