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Modeling and Functional Estimation of Quantum Wave Lens Frequency Discriminator for Multi-Frequency-Shift Keying Spread Spectrum Communication

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Abstract—Electron wave lens effect can be applied as a function of frequency discrimination in multi-frequency-shift keying terahertz communication systems. Since this lens effect is a quantum mechanical phenomenon, quantum noise causes the discriminator errors. This study examines error reduction by frequency hopping spread spectrum (FHSS) technique. We made a mathematical model of the frequency discriminator based on stochastic quantization for quantum mechanical systems. We confirmed error reduction effect of FHSS by numerical experiments using the model.

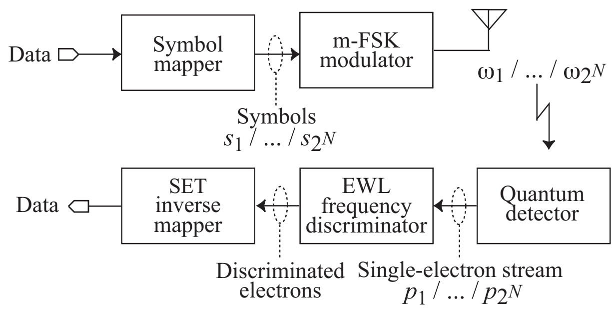


Figure 1: Terahertz communication system using quantum effect devices.

1. Introduction

A receiver built of a quantum terahertz detector, two electron wave guides coupled to each other, and single-electron tunneling (SET) bit-stream circuits has been proposed for two-level frequency shift keying terahertz communication [1]. In this paper, we will attempt to extend the receiver for multi-frequency-shift keying (m-FSK) communication. It is not always a easy task to extend the coupled electron wave guides that function as a frequency discriminator so that they can be applied to m-FSK receiver. Electron wave lenses (EWL) can separate electrons into more than two groups by the difference in their momenta [2]. Then, we will employ EWL as a frequency discriminator of an m-FSK receiver. Figure 1 shows an m-FSK communication system built of the terahertz wave detector, the EWL, and the SET bit-stream processing circuit.

For the devices and the circuits in the receiver, functional error which is due to quantum uncertainty or quantum noise is concerned. In this paper, we will present a procedure of quantitatively estimating the error of the EWL frequency discriminator. We then will attempt to reduce the communication error caused by the discrimination error. Frequency hopping spread spectrum (FHSS) technique will be employed for the error reduction.

The electrons in the detector and the discriminator are described by the wave functions of the Shrödinger equations. On the other hand, electrons in the SET circuits are described as probabilistic particles in their circuit simulator [3]. It is necessary to obtain sample states of probabilistically behaving electrons from the wave function at the output terminals of the discriminator for the future simulation of whole the receiver. Stochastic quantization [4] is useful to obtain the samples. In this paper, we will employ it for the procedure of estimating the discrimination error.

2. M-FSK spread-spectrum system using EWL

Figure 2 shows an m-FSK spread spectrum communication system using the EWL discriminator. N-bit data is mapped into one of sequence set $S_c \equiv \{c_1, \cdots, c_{2^N}\}$ of integer symbols belonging to set $S_s \equiv \{s_1, \cdots, s_M\}$. The numbers of the elements of S_c and S_s are 2^N and M respectively. That is, $(b_1, \cdots, b_N) \rightarrow c_i = (s_1, \cdots, s_L) \in S_c$, $s_i \in S_s$. The length of the sequences is L. Receiving the symbol sequence, the m-FSK modulator generates terahertz wave of frequency $\omega_i \in \{\omega_1, \cdots, \omega_M\}$ changing correspondingly to the symbol s_i in the sequence. The terahertz wave detector at the receiver outputs a photoelectron of momentum $p_1, \cdots, p_M$ corresponding to the frequencies. The EWL discriminator having M outputs leads the photoelectron of momentum p_i to the i-th output, one of its M outputs. The SET circuit receiving the photoelectrons from the M outputs reconstructs the symbol sequence and inversely transforms it into the transmitted N-bit data. This communication system is a kind of FHSS system.

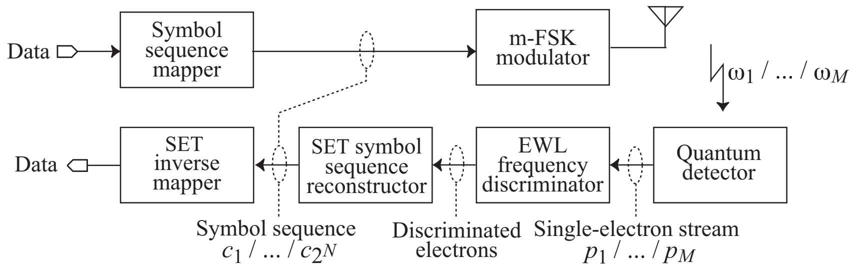
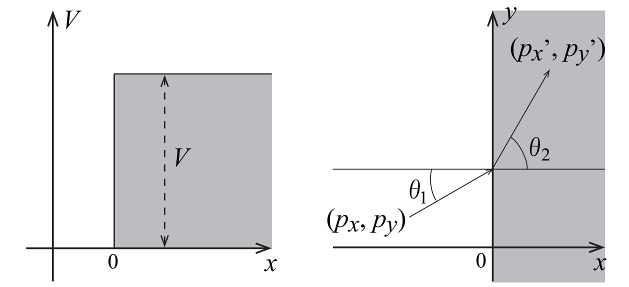


Figure 2: M-FSK spread spectrum communication system using EWL discriminator.



(a) x-directional step potential (b) Incident and refraction angles

Figure 3: Potential structure of the EWL discriminator and the refraction of electron waves.

3. EWL discriminator

3.1. Structure

Figure 3 shows a quantum step potential which changes its height in x-direction but is constant in y-direction. The potential is given by

$$V(x) = \begin{cases} V & \text{if } x \geq 0, \\ 0 & \text{if } x < 0 \end{cases} \quad (1)$$

The potential system functions as an electron wave lens. When electron wave in area $x < 0$ propagates into area $x \geq 0$, its x-directional momentum decreases. Then, refraction is observed. Let the x-directional momentum of the electron be p_x and p'_x in areas $x < 0$ and $x \geq 0$, respectively. Since y-directional momentum does not change, we have from the energy conservation low the following equation:

$$E_x = \frac{p_x^2}{2m} = \frac{p_x'^2}{2m} + V \quad (2)$$

where m is the mass of an electron. Then, the relation between the expectations θ_1, θ_2 of the incident and refraction angles is given by

$$\frac{\sin\theta_2}{\sin\theta_1} = \sqrt{\frac{E_x}{E_x - V}} \quad (3)$$

3.2. Wave function expression

The state of a photoelectron in the EWL discriminator is described by the Shrödinger equation,

$$i\hbar\frac{\partial}{\partial t}\psi(x,y,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y)\right]\psi(x,y,t) \quad (4)$$

Since potential $V(x,y) = V(x) + V(y)$, $V(y) = 0$, the wave function can be the following product:

$$\psi(x,y,t) = \psi_x(x,t)\psi_y(y,t) \quad (5)$$

Then, Eq. (4) separates into the following two equations:

$$i\hbar\frac{\partial}{\partial t}\psi_x(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi_x(x,t) \quad (6)$$

$$i\hbar\frac{\partial}{\partial t}\psi_y(y,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\psi_y(y,t) \quad (7)$$

We express the wave function of Eq. (6) by

$$\psi(x,p_x,t) = \phi(x,p_x)\exp(-\frac{i}{\hbar}E_x(p_x)t), \quad E_x(p_x) = \frac{p_x^2}{2m} \quad (8)$$

with x-directional momentum p_x as a parameter. Function $\phi(x,p_x)$ is the superposition of incident and reflected electron waves in area $x < 0$ and the transmitted wave in area $x \geq 0$ [5], that is,

$$\phi_x(p_x,x) = \begin{cases} B\exp(i\frac{p'_x}{\hbar}x) & \text{if } x \geq 0 \\ \exp(i\frac{p_x}{\hbar}x) + A\exp(-i\frac{p_x}{\hbar}x) & \text{if } x < 0 \end{cases} \quad (9)$$

As we have Eq. (2), p'_x does not need to be a parameter of $\psi(x,p_x,t)$. Setting boundary conditions that $\phi(x,p_x)$ and its derivative are continuous at the boundary $x = 0$, we obtain coefficients A and B. Let $X(p_x)$ be the initial x-directional momentum distribution. Wave function of Eq. (6) with initial state $X(p_x)$ is given by

$$\psi_x(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} X(p_x)\psi(x,p_x,t)dp_x \quad (10)$$

We may let the wave function of Eq. (7) be given by

$$\psi(y,p_y,t) = \exp(i\frac{p_y}{\hbar}y)\exp(-\frac{i}{\hbar}E_y(p_y)t), \quad E_y(p_y) = \frac{p_y^2}{2m} \quad (11)$$

Table 1: The symbols, frequencies, momenta, refraction angles, and output locations for the m-FSK communication system.

Symbol s_i	Carrier freq. ω_i [THz]	Momentum p_i [(Kg.eV)$^{1/2}$]	Refraction angle θ_2	Outputs x $5.29\cdot10^{-11}$[m]
+2	104.5	14.46	50°	$x <$ 62.1
+1	120.0	15.49	45°	$62.1 \leq x <$ 86.0
-1	151.9	17.43	40°	$86.0 \leq x < 102.8$
-2	249.9	22.36	35°	$102.8 \leq x$

When initial y-directional momentum distribution $Y(p_y)$ is given, $\psi_y(y,t)$ is obtained similarly as we get $\psi_x(x,t)$ from Eq. (10). That is, $\psi_y(y,t)$ is given by

$$\psi_y(y,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Y(p_y)\psi(y,p_y,t)dp_y \qquad (12)$$

3.3. Probabilistic particle expression

A stochastic quantization theory presented in [4] states that a classical probabilistic particle whose motion is governed by the Langevin equation,

$$\frac{dx}{dt} = b(x,t) + \frac{\hbar}{2m}\Gamma(t), \qquad (13)$$
$$\Gamma(t) : \text{Gaussian white noise},$$
$$\langle \Gamma(t_1)\Gamma(t_2)\rangle = \delta(t_1 - t_2)$$

has the same probability distribution $|\psi(x,t)|^2$ as a quantum system with wave function $\psi(x,t)$ if

$$b(x,t) = \Re[\chi(x,t)] + \Im[\chi(x,t)], \quad \chi(x,t) \equiv \frac{\hbar}{m}\frac{\partial}{\partial x}\ln\psi(x,t) \qquad (14)$$

Sample two-dimensional trajectories of electrons in the discriminator are computed by applying the theory to system (6) for x-direction and to system (7) for y-direction.

4. Numerical Experiments

This section presents a computation to estimate the rate of the discrimination error that EWL causes because of the quantization noise. A possibility of FHSS to reduce the communication error caused by the discrimination error is also presented.

4.1. Method and conditions

The communication system in this section uses (M =) 4 integer symbols and correspondingly 4 carrier frequencies. They are shown in Tab. 1. The detector receiving electromagnetic waves of one of the four frequencies emits electrons with one of four moment expectations. They are also

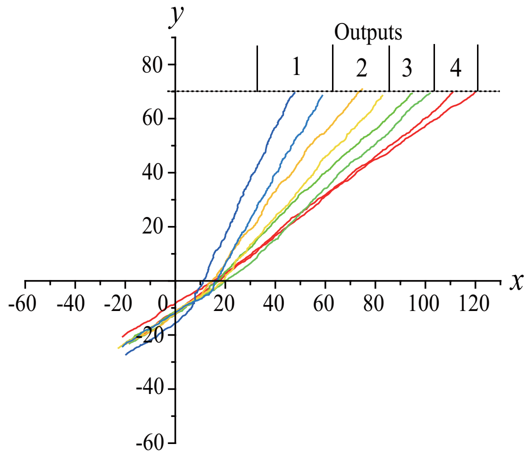


Figure 4: Sample trajectories of electrons in the EWL discriminator.

shown in the table. The EWL discriminator has the step potential given by Eq. (1). Its height is V = 60 [eV]. The electron waves of the momenta are refracted at the angle expectation shown in the table when incident angle expectation is $\theta_1 = 30°$. The four outputs of the discriminator are then set as given in the table. The difference between symbols $s_i - s_j$ is set large / small when the distance between the i-th and the j-th outputs is long / short.

We compute the trajectories of electrons in the EWL discriminator by using the stochastic quantization in Section 3.3. Figure 4 shows examples of the computed trajectories. Initial momentum distributions in x and y-directions are as follows:

$$X(p_x) = \frac{1}{(2\pi\sigma^2\hbar^2)^{\frac{1}{4}}} \exp\left(-i\frac{p_x}{\hbar}x_0 - \frac{1}{4}\frac{(p_x - p_{0,x})^2}{\sigma^2\hbar^2}\right) \quad (15)$$

$$Y(p_y) = \frac{1}{(2\pi\sigma^2\hbar^2)^{\frac{1}{4}}} \exp\left(-i\frac{p_y}{\hbar}y_0 - \frac{1}{4}\frac{(p_y - p_{0,y})^2}{\sigma^2\hbar^2}\right) \quad (16)$$

where x_0, y_0 and $p_{0,x}$, $p_{0,y}$ are the expectations of initial position of an electron and the expectations of its initial momenta in x and y-directions and σ^2 is the initial variance of the momenta. Momenta $p_{0,x}$, $p_{0,y}$ are set so that the absolute momenta and the incident angle expectation are given as mentioned above. Variances of $p_{0,x}$, $p_{0,y}$ are $\sigma^2\hbar^2$ = 0.08. We judge that the EWL discriminator causes a symbol identification error when a photoelectron with momentum expectation of p_i gets out of the j-th output, $j \neq i$, of the discriminator. That is, the error rate is the conditional probability Prob(output j | moment p_i), $j \neq i$.

The transmitter maps (N =) 2 or 3 bit data into 4 or 8 symbol sequences of length L = 1, 4, or 8. We examine 4 mapping, SM1, $\cdots$, SM4 shown in Tab. 2(a)-(d). Mapping SM1 with L = 1 does not use FHSS technique. When L = 4 or 8, the cross-correlations between the sequences are shown in Tabs. 2. Since they are equal or smaller than 0.5,

Table 2: Symbol sequences.

(a) SM1

Data	Symbol Sequence $c_i = (s_j, s_k, s_l, ...)$
00	c_1 = (-2)
01	c_2 = (-1)
10	c_3 = (+1)
11	c_4 = (+2)

(d) SM4

Data	Symbol Sequence $c_i = (s_j, s_k, s_l, ...)$	Correlation c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
000	c_1 = (+1,-1,+2,-2)	1	3/10	0	-1/10	1/10	-9/10	5/10	-2/10
001	c_2 = (-2,+1,+2,-1)	--	1	-9/10	0	0	1/10	-2/10	5/10
010	c_3 = (+2,-2,-1,+1)	--	--	1	3/10	-3/10	-3/10	5/10	-4/10
011	c_4 = (-1,-2,+1,+2)	--	--	--	1	-1/10	3/10	4/10	5/10
100	c_5 = (+1,+2,-1,-2)	--	--	--	--	1	-3/10	4/10	-5/10
101	c_6 = (-2,+1,-1,+2)	--	--	--	--	--	1	-5/10	5/10
110	c_7 = (+2,-1,+2,+1)	--	--	--	--	--	--	1	3/10
111	c_8 = (-1,+1,+2,+2)	--	--	--	--	--	--	--	1

(b) SM2

Data	Symbol Sequence $c_i = (s_j, s_k, s_l, ...)$	Correlation c_1	c_2	c_3	c_4
00	c_1 = (+1,-1,+2,-2)	1	3/10	0	-1/10
01	c_2 = (-2,+1,+2,-1)	--	1	-9/10	0
10	c_3 = (+2,-2,-1,+1)	--	--	1	3/10
11	c_4 = (-1,-2,+1,+2)	--	--	--	1

(c) SM3

Data	Symbol Sequence $c_i = (s_j, s_k, s_l, ...)$	Correlation c_1	c_2	c_3	c_4
00	c_1 = (+1,-1,+2,-2,-1,-2,+1,+2)	1	6/20	-1/20	-1/20
01	c_2 = (-2,+1,+2,-1,+2,-2,-1,+1)	--	1	-9/20	-9/20
10	c_3 = (+2,-2,-1,+1,+1,-1,+2,-2)	--	--	1	6/20
11	c_4 = (-1,-2,+1,+2,-2,+1,+2,-1)	--	--	--	1

Table 3: Sequence identification error rates for 4000 symbols.

Mapping	Number of data bits (N)	Number of sequences (2^N)	Sequence length (L)	Error rate
SM1	2	4	1	0.205
SM2	2	4	4	0.001
SM3	2	4	8	0.000
SM4	3	8	4	0.051

the receiver can identify the sequences by computing the cross-correlations.

4.2. Results

We computed 4000 trajectories of photoelectrons in the EWL discriminator by the stochastic quantization. We obtained the rates of symbol identification error that the discriminator caused and of sequence identification error, as shown in Tab. 3. The sequence identification error can be small, naturally, as we use mapping such that cross-correlation is smaller in average.

5. Conclusions

We have presented that EWLs have the capability to function as a discriminator for terahertz m-FSK communication and that FHSS technique is effective to reduce communication error caused by quantum noise. The stochastic quantization is useful when it is difficult to generate sample states of electrons according to the probability distribution, the square of wave function, at the outputs of the EWL discriminator. Our future works include applying high mobility graphene material to EWL, the optimization of the EWL structure, and building a model of whole the receiver for communication error estimation.

References

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