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Modeling and Functional Estimation of Coupled Quantum Wave Guide Frequency Discriminator for Spread-Spectrum Frequency Shift Keying Communication

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Abstract—Coupled electron wave guide can be used as a frequency discriminator in the receiver part of a terahertz-band frequency shift keying communication system. However, failure of the discriminator arises because of quantum uncertainty. In this study, we examine the error reduction by direct sequence spread spectrum technique. We have created a mathematical model of the frequency discriminator based on stochastic quantization for quantum mechanical systems. By using the model, we confirmed that the data transmission error decreased with the spreading code length.

1. Introduction

Communication and sensing using terahertz (THz) frequency band has been researched. Recently, photo-electronic devices converting terahertz photons to single-electron streams have been built by using semiconductor quantum wells or carbon nano tubes [1, 2]. Such electromagnetic wave detectors will contribute to increasing performance of the communication and sensing systems.

Coupled electron wave guides (CEWG) can separate electrons by the difference in momenta [3]. If the kinetic energy of the outputted photoelectrons from the terahertz wave detectors is proportional to the frequency of the waves, the CEWG can be applied as frequency discriminators. Single-electron tunneling (SET) circuits processing streams of single-electrons have been developed. They include many static and dynamic functions useful for signal processing [4, 5]. Then, a very small and low power receiver for terahertz frequency shift keying (FSK) communication can be built of the detector, the CEWG, and the SET bit-stream processing circuit, as shown in Fig. 1.

For the devices and the circuits in the receiver, functional error which is due to quantum uncertainty or quantum noise is concerned. In this paper, we will present a procedure of quantitatively estimating the error of the CEWG frequency discriminator. We then will attempt to reduce the communication error caused by the discrimination error. Direct sequence spread spectrum (DSSS) technique [6] will be employed for the error reduction.

The behavior of electrons in the detector and the discriminator is described by the evolution of the wave function, that is, the solution of the Shrödinger equation. On the other hand, the electrons in the SET circuits are described as probabilistic particles in the circuit simulator [7]. It is necessary to obtain sample states of probabilistically behaving electrons from the wave function at the output terminals of the discriminator for the future simulation of whole the receiver. Stochastic quantization [8] is useful to obtain the samples. In this paper, we will employ it for the procedure of estimating the discrimination error. The error caused only by the quantization noise is evaluated. Errors caused by channel noise are not considered in this paper.

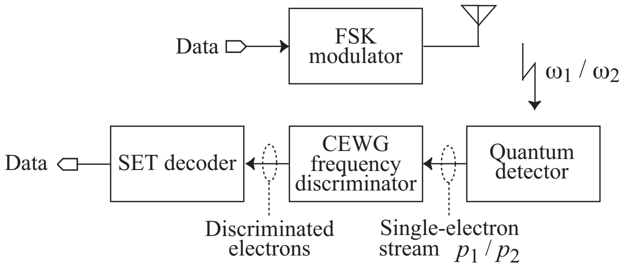


Figure 1: Terahertz communication system using quantum effect devices.

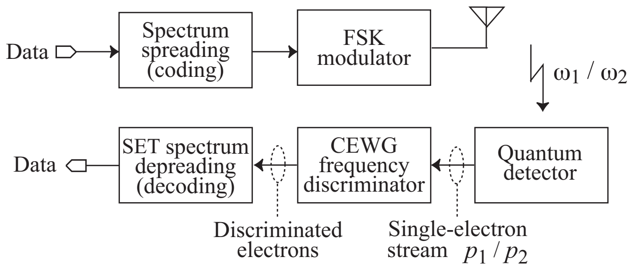


Figure 2: Spread-spectrum FSK communication system using CEWG discriminator.

2. Spread-spectrum FSK system using CEWG

Figure 2 shows a spread spectrum FSK communication system using the CEWG discriminator. Serial binary

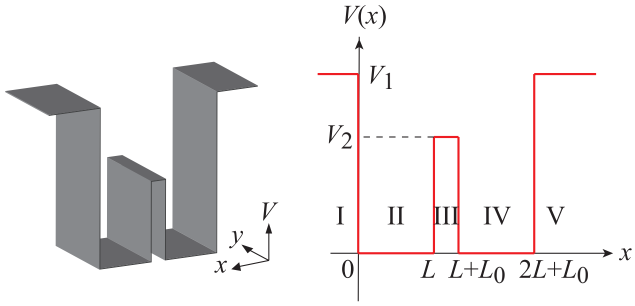


Figure 3: Potential structure of the CEWG discriminator.

data sequence is multiplied by a spreading sequence. The spread spectrum binary sequence is binary FSK modulated. The modulated terahertz signal switches its frequency between ω_1 and ω_2. The photoelectronic detector at the receiver outputs a photoelectron of momentum p_1 / p_2 if the frequency of the received terahertz signal is ω_1 / ω_2. The CEWG discriminator leads the photoelectron of momentum p_1 / p_2 to its left / right output. The SET circuit receiving the photoelectrons from the two outputs reconstructs a spread spectrum binary sequence and despreads it to obtain the transmitted data.

3. CEWG discriminator

3.1. Structure

The CEWG discriminator is a quantum potential system. Figure 3 shows the potential structure. Photo-electrons from the detector enter into the discriminator. The entrance is defined as an interval given by $0 < x \leq L + L_0/2$, $y = 0$. The left and the right outputs are defined as intervals given by $0 < x \leq L + L_0/2$, $y = l$ and $L + L_0/2 < x \leq 2L + L_0$, $y = l$, respectively. The potential height is given by

$$V(x,y) = \begin{cases} V_1 & \text{if } x \leq 0 \text{ or } 2L + L_0 < x \text{ (I, V)}, \\ V_2 & \text{if } L \leq x \leq L + L_0 \text{ (III)}, \\ 0 & \text{Otherwise, (II, IV)} \end{cases} \quad (1)$$

Potential $V(x,y)$ is independent of the y-coordinate.

3.2. Wave function expression

The state of a photoelectron in the CEWG discriminator is described by the Shrödinger equation,

$$i\hbar\frac{\partial}{\partial t}\Psi(x,y,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y)\right]\Psi(x,y,t) \quad (2)$$

where m is the electron mass and $\hbar$ is the Plank constant divided by 2π. Since $V(x,y) = V(x) + V(y)$ and $V(y) = 0$, the wave function can be the following product:

$$\Psi(x,y,t) = \psi_x(x,t)\psi_y(y,t) \quad (3)$$

Then, Eq. (2) separates into the following two equations:

$$i\hbar\frac{\partial}{\partial t}\psi_x(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi_x(x,t) \quad (4)$$

$$i\hbar\frac{\partial}{\partial t}\psi_y(y,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\psi_y(y,t) \quad (5)$$

We express the eigen wave function of Eq. (4) for discretized eigen energy E_n by $\psi_x(x,t) = \phi_n(x)T_{x,n}(t)$. In each region s ($\in \{\text{I, II}, \cdots, \text{V}\}$) of the discriminator shown in Fig. 3, $\phi_n(x)$ is given by

$$\phi_n^s(x) = A_n^s \exp(-i\alpha_n^s x) + B_n^s \exp(i\alpha_n^s x) \quad (6)$$

$$\alpha_n^s = \begin{cases} \dfrac{\sqrt{2m(E_n - V_1)}}{\hbar} & \text{if } s \in \{\text{I, V}\} \\ \dfrac{\sqrt{2m(E_n - V_2)}}{\hbar} & \text{if } s \in \{\text{III}\} \\ \dfrac{\sqrt{2mE_n}}{\hbar} & \text{if } s \in \{\text{II, IV}\} \end{cases} \quad (7)$$

$T_{x,n}(t)$ is a periodic function given by

$$T_{x,n}(t) = \exp(-\frac{i}{\hbar}E_n t) \quad (8)$$

Setting boundary conditions including the conditions that $\phi_n(x)$ and its derivative are continuous at the boundaries between regions s and $s+1$, we determine E_n, A_n^s, and, B_n^s. Then, $\phi_n(x)$ is computed [9, 10]. When initial distribution $\phi_0(x)$ of the wave function is given, $\psi_x(x,t)$ is obtained by

$$\psi_x(x,t) = \sum_{n=1}^{N_{max}} c_n\phi_n(x)T_{x,n}(t), \quad c_n = \int \phi_0(x)\phi_n(x)dx \quad (9)$$

where N_{max} is the number of the possible states that an electron in the CEWG can take.

We express the wave function of Eq. (5) by $\psi_y(y,p_y,t) = \phi(y,p_y)T_y(t)$ depending on the y-directional momentum p_y. Functions $\phi(y,p_y)$ and $T_y(t)$ are given by

$$\phi(y,p_y) = \exp(i\frac{p_y}{\hbar}y) \quad (10)$$

$$T_y(t) = \exp(-\frac{i}{\hbar}E(p_y)t), \quad E(p_y) = \frac{p_y^2}{2m} \quad (11)$$

When initial distribution $\phi_0(p_y)$ of the y-directional momentum is given, the wave function can be obtained by

$$\psi_y(y,t) = \int_{-\infty}^{+\infty} \phi_0(p_y)\psi_y(y,p_y,t)dp_y \quad (12)$$

3.3. Probabilistic particle expression

A stochastic quantization theory presented in [8] states that a classical probabilistic particle whose motion is governed by the Langevin equation,

$$\frac{dx}{dt} = b(x,t) + \frac{\hbar}{2m}\Gamma(t), \quad (13)$$
$$\Gamma(t): \text{Gaussian white noise},$$
$$\langle\Gamma(t_1)\Gamma(t_2)\rangle = \delta(t_1 - t_2)$$

has the same probability distribution $|\psi(x,t)|^2$ as a quantum system with wave function $\psi(x,t)$ if

$$b(x,t) = \Re[\chi(x,t)] + \Im[\chi(x,t)], \quad \chi(x,t) \equiv \frac{\hbar}{m}\frac{\partial}{\partial x}\ln\psi(x,t) \tag{14}$$

Sample two-dimensional trajectories of electrons in the discriminator are computed by applying the theory to system (4) for x-direction and to system (5) for y-direction.

4. Numerical experiments

This section presents a computation to estimate the rate of the discrimination error that CEWG causes because of the quantization noise. A possibility of DSSS to reduce the communication error caused by the discrimination error is also presented.

In this section, the following units are adopted: The Planck constant $\hbar$ is normalized to 1.0. The energy of a photon of frequency $\omega_p = 2\pi\times 10$THz is also normalized to 1.0. Thus, unit of energy (kinetic energy of electrons and potential energy) is $\hbar\omega_p$, which is approximately 10^{-20} Joule or 0.04 eV. Because of the energy normalization, unit time corresponds to 0.1psec. We also normalize electron mass m to 1.0 and the velocity of electron with kinetic energy $\hbar\omega_p$ to 1.0. Then, unit velocity and unit length correspond respectively to 10^5m/sec and 10nm.

4.1. Method and conditions

The parameters of the discriminator are as follows: $L = 3$, $L_0 = 0.1$, $l = 52$, $V_1 = 10$, $V_2 = 5$. Initial distribution $\phi_0(x)$ of x-directional location is given by the Gaussian distribution,

$$\phi_0(x) = \frac{1}{(2\pi\sigma_x^2)^{\frac{1}{4}}}\exp\left(-\frac{(x-x_0)^2}{4\sigma_x^2}\right) \tag{15}$$

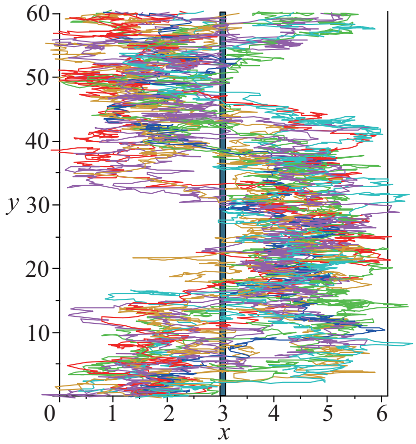
with expectation $x_0 = 2$, variance $\sigma_x^2 = 0.09$. Initial distribution $\phi_0(p_y)$ of y-directional momentum is given by the Gaussian distribution,

$$\phi_0(p_y) = \frac{1}{(2\pi\sigma_y^2)^{\frac{1}{4}}}\exp\left(-\frac{(p_y-p_0)^2}{4\sigma_y^2}\right) \tag{16}$$

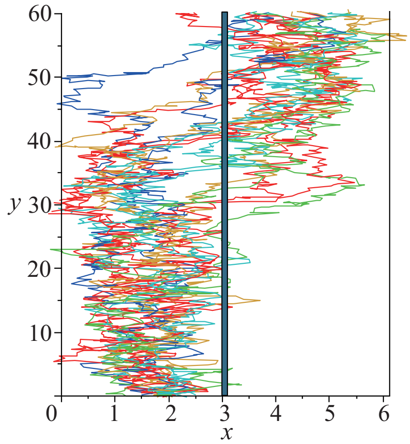
with expectation $p_0 = 2$ or 4, variance $\sigma_y^2 = 2.0$. The expectation is the average momentum of the photoelectrons outputted from the detector receiving terahertz waves of frequency ω_1 or ω_2.

We compute the trajectories of electrons in the CEWG discriminator by using the stochastic quantization in Section 3.3. Figure 4 shows 15 samples of the trajectories. We judge that the CEWG discriminator causes an error when a photoelectron with $p_0 = 2$ gets out of the right output terminal or a photoelectron with $p_0 = 4$ does from the left terminal of the discriminator .

The length of the spreading codes of chip sequence $\{c(n)\}$, $c(n) \in \{+1, -1\}$, $n = 0, \cdots, N-1$, which are expected



(a) Expectation of intial y-momentum : 2



(b) Expectation of intial y-momentum : 4

Figure 4: Sample trajectories of electrons in the CEWG discriminator.

to reduce communication error, is $N = 5$, 9, or 15. Let $\{d_m\}$, $d_m \in \{+1, -1\}$, denote a data symbol sequence generated randomly. The expectation of the initial momentum p_0 determined to be 2 / 4 when the level of the spread sequence is $d_m c(n) = +1 / -1$. Let sequence of left / right outputs from which the photoelectrons go out be represented by sequence $\{c_d(n)\}$, $c_d(n) \in \{+1, -1\}$. The data symbol is estimated by taking cross-correlation between $\{c_d(n)\}$ and $\{c(n)\}$.

4.2. Results

Table 1 shows discrimination error rate of the CEWG, the bit error rates when $N = 1$ and when spread spectrum is applied, i.e., $N = 5$, 9, and 15. The numerically estimated error rates are computed by the stochastic quantization. Since the structure of the CEWG discriminator is simple, the theoretical error rates are estimated by the fol-

Table 1: Bit error rate.

Code length (N)	Error rate	
	Theory	Numerical Expr.
1	1.1×10^{-1}	1.6×10^{-1}
5	1.2×10^{-2}	1.5×10^{-2}
9	1.6×10^{-3}	3.0×10^{-3}
13	8.0×10^{-5}	3.9×10^{-4}

lowing integration of the wave function of Eq. (2):

$$\int_0^\infty \int_{L_1}^{L_2} \int_0^\infty |\Psi(x,y,t)|^2 \delta(y-l) dy dx dt \qquad (17)$$

where $(L_1, L_2) = (0, L + L_0/2)$ if $p_0 = 4$ and $(L + L_0/2, 2L + L_0)$ if $p_0 = 2$. It seems that much large number of trajectories are necessary to get numerically estimated error rates close to the theoretically estimated error rates.

5. Conclusions

We have presented that CEWG have the capability to function as a discriminator for terahertz FSK communication and that DSSS technique is effective to reduce communication error caused by quantum noise. The stochastic quantization is useful when it is difficult to generate sample states of electrons according to the probability distribution, the square of wave function, at the outputs of the CEWG discriminator. Our future works include the optimization of the CEWG structure and building a model of whole the receiver for communication error estimation.

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