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A Study on Perfect Synchronization Method using Environmental Fluctuations

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Abstract—Noise-induced synchronization is a phenomenon that uncoupled nonlinear oscillators synchronize by adding common noise. We have proposed a new synchronization method using the cross-correlated natural environment data instead of a common input noise. Wireless devices obtain environmental data by their own sensors, and input these data independently. Based on noise-induced synchronization theory, nonlinear oscillators running in wireless devices are synchronized without any interaction or communications. In our implementation, devices use time average values of the environmental data as an input noise. There is interval of noise input because devices calculate time average of obtained data independently. Therefore, the timing of inputting noise is different among devices. In this paper, we propose a novel data input method, that adjusts data input interval to achieve synchronization of timing of input between devices. We use time series of Rössler system as input data in our numerical simulation. Our proposed scheme minimize different of input timing among two FitzHugh-Nagumo oscillators by adding time series of Rössler system at some specific phases.

1. Introduction

There are many researches about a phenomenon of synchronization between nonlinear oscillators with external force, such as periodic injection, noise injection and so on. As one of those phenomena, there is the phenomenon that uncoupled nonlinear oscillators are synchronized by common noise, which is called noise-induced synchronization[1, 2]. In the previous studies, some types of noises, such as white Gaussian noise and so on, are used for the input of noise-induced synchronization. On this phenomenon, it is not necessary that the inputs of noise-induced synchronization are perfectly common. We apply this phenomenon to synchronization of wireless devices.

We have already investigated the feasibility of our proposed synchronization scheme that uses natural environmental fluctuation, such as temperature, environmental sound, electromagnetic wave and so on, and propose novel natural synchronization scheme[3, 4, 5]. Environmental fluctuations are similar to each other in neighboring area[3, 4, 5], and we use these fluctuations as cross-correlated input noise to achieve synchronization between oscillators running on neighboring wireless devices. Based on noise-induced synchronization theory, if each devices actuate according to some timing of phases of the oscillator running on their own, the actuation of these devices will be synchronized.

Previously, we have investigated feasibility of our proposed natural synchronization scheme using various types of natural environmental fluctuations. And, we have already shown feasibility of the natural synchronization scheme by real implementation using temperature data, natural environmental sound and so on[3, 4, 5].

In this paper, we study a perfect synchronization method of our proposed natural synchronization scheme. In our proposed natural synchronization scheme, every wireless device have their own sensors to obtain the natural environmental data. To avoid the effect of measurement errors, time average of sampling data is used for the input noise of noise-induced synchronization, which are intermittently input to the oscillators. However, the noise input timing are not synchronized among the devices.

In this paper, we propose a method that is able to synchronize noise input timing. Time average value of natural environmental data is used as input noise in our synchronization scheme. Therefore, if the timing of input is different among devices, cross-correlation coefficients among input noise of each device differ from other devices. We propose new method to adjust the input timing. Time averaged environmental data is input at the timing of specific phases of oscillator in the devices. If the synchronization of phase of oscillators is achieved, the input timing also synchronize. To investigate the feasibility of this new method, we introduce the input sequence of deterministic process assuming natural environmental fluctuation. We evaluate the phase difference of oscillators adding these correlated noise sequences by the our new approach.

2. Synchronization of uncoupled nonlinear oscillators by natural environmental noises

Our proposed synchronization scheme can make wireless devices synchronized by using environmental fluctuation. As the environmental fluctuation, we obtain the circumference environmental data, such as temperature data, environmental sound data, electromagnetic wave and so on, by sensors, such as heat indicator, microphone, and so on. Our scheme is based on the phenomenon that uncoupled nonlinear oscillators are synchronized by adding common noise sequence or periodic sequence. Oscillators can achieve synchronization autonomously even if there is not any interaction or communication between them[1, 2]. These oscillators also synchronize even though environmental data is used as input noise to them[3, 4, 5]. In our proposed natural synchronization scheme, nonlinear oscillators synchronize by natural environmental data, we achieve perfect synchronization between wireless devices according to the phase of their own phase synchronized oscillators.

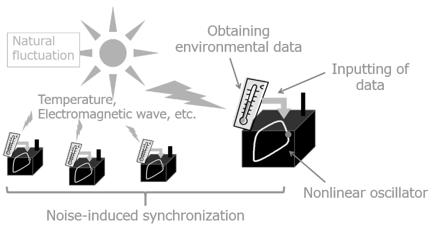

Figure 1: Schematic image of our proposed natural synchronization scheme.

3. Our proposed synchronization scheme using natural environmental noises

Figure 1 shows the schematic image of our proposed scheme. In our proposed synchronization scheme, we use wireless devices which have their own sensors. They obtain environmental data by their own sensor and calculate time average of obtained data, and input these data to their own oscillator running independently. Based on noise-induced synchronization, these oscillators are synchronized independently. Therefore, if each device actuate according to the phase of their own oscillator, the timing of actuation is synchronized, that means wireless devices which use our proposed synchronization scheme can achieve time synchronization. In this case, these devices use only circumference environmental fluctuation without any interactions or communications. For the instance, if this time synchronization apply to the communication, it is able to achieve intermittent communication. In the intermittent communication, it is necessary to boot the communication function of wireless devices simultaneously to communicate each other. By applying our proposed synchronization scheme to such communication method, we achieve most efficient power saving.

We have already shown that the limit cycle oscillators can be synchronized by using real natural noises, such as the temperature, humidity, environmental sound and electro-magnetic wave [3, 4, 5]. We have implemented this proposed scheme on the wireless sensor network devices, and show the feasibility of the proposal on real unconnected devices [5]. We have also implemented the proposed synchronization scheme using several PCs equipped with microphones. Each PC in neighboring area obtains the environmental sound independently and inputs the sound to running nonlinear oscillator. The oscillators on the PC could be synchronized without any interactions between them [4].

In our proposed method, devices calculate time average to avoid the effect of measurement error, and the timing of calculation and input is different among the devices. Obtained natural environmental fluctuation is similar to each other in neighboring area at the same time. However, if the time of calculating time average of environmental fluctuation is different among devices, cross-correlation of these time averaged value of environmental fluctuation should be low. Therefore, it is necessary to synchronize not only the phase of oscillators but also the input timing of noise.

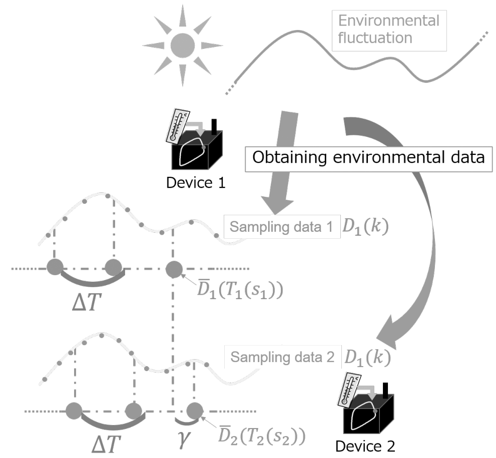

Figure 2: The image of time average and gap of calculation interval.

4. Interval of input timing in proposed time synchronization method

Nonlinear oscillator $\mathbf{X}_i$ with adding noises is expressed by following equation.

$$\dot{\mathbf{X}}_i(t) = \mathbf{F}(\mathbf{X}_i(t)) + \alpha Y_i(t) + \epsilon \xi_i(t). \quad (1)$$

Where, α means amplitude of input, $Y_i(t)$ means environmental data. ϵ, ξ_i are amplitude of error noise and measurement error.

$D_i(t)$ is the discrete sensing data obtaining by device i. Time average $\bar{D}_i(t)$ is expressed like,

$$\bar{D}_1(T_1(s_1)) = \frac{1}{n(T_1(s_1+1)) - n(T_1(s_1))} \sum_{k=n(T_1(s_1)+1)}^{n(T_1(s_1+1))-n(T_1(s_1))} D_1(k) \quad (2)$$

$$\bar{D}_2(T_2(s_2)) = \frac{1}{n(T_2(s_2+1)) - n(T_2(s_2))} \sum_{k=n(T_2(s_2)+1)}^{n(T_2(s_2+1))-n(T_2(s_2))} D_2(k) \quad (3)$$

where $T_i(s_i)$ means discrete time $n(T_i(s_i))$ means number of data obtained at device i by the time $T_i(s_i)$. Difference of input timing of device 1 and device 2 is expressed as $\gamma = |T_1(s_1) - T_2(s_2)|$, input interval is expressed as $\Delta T = T_i(s_i + 1) - T_i(s_i)$.

The average of all sensing data $\sum_{k=n(T_i(s_i)+1)}^{n(T_i(s_i+1))-n(T_i(s_i))} D_i(k)$ obtained within the time interval ΔT is time average value $\bar{D}_i(T_i(s_i))$. Time difference of input timing between devices is γ.

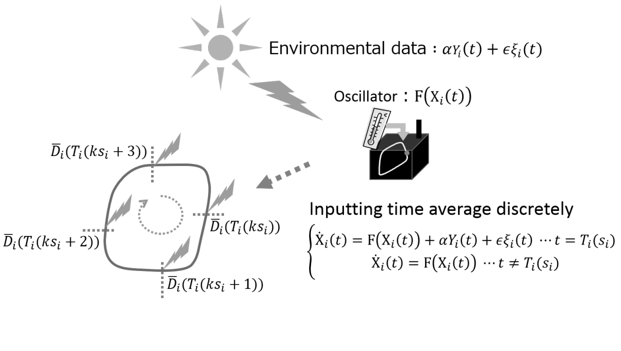

Figure 3: the image of proposed method that input noise to oscillator at specific phase (example of inputting 4 times per cycle).

5. Proposed method to adjust the gap of input timing

In this paper, we propose new method that adjust the initial gap of input timing to achieve perfect synchronization. Figure 3 shows the image of proposed input method. Sensing data $D_i(k)$ is saved on the memory of the device by the timing $T_i(s_i)$. Devices calculate time average $\bar{D}_i(T_i(s_i))$ by the equation (2) at input timing $T_i(s_i)$. Therefore, they input time average value to their own oscillator as $Y(t) = \bar{D}_i(T_i(s_i))$, and sample the sensing data by the time $T_i(s_i + 1)$. Devices run this loop.

In the new input method, we input time average at some specific phases. After achieving synchronization of phase of oscillator, the input timing of devices should be synchronized. We investigate feasibility of new input method by numerical simulation.

In this paper, we use two FitzHugh-Nagumo oscillators $\mathbf{X}_1$ and $\mathbf{X}_2$ as nonlinear oscillator. FitzHugh-Nagumo oscillator is expressed by following equation.

$$\dot{v}(t) = v - v^3/3 - u + I \quad (4)$$

$$\dot{u}(t) = \varepsilon(v + a - bu) \quad (5)$$

The parameters of the oscillator is set to $\varepsilon = 0.08,\ a = 0.7,\ b = 0.8,\ I = 0.4$. And we also use Rössler system which have deterministic chaos fluctuation as input noise. Rössler system is written like following equation.

$$\dot{x}(t) = -wy - z \quad (6)$$

$$\dot{y}(t) = wz + ay \quad (7)$$

$$\dot{z}(t) = b + z(x - c) \quad (8)$$

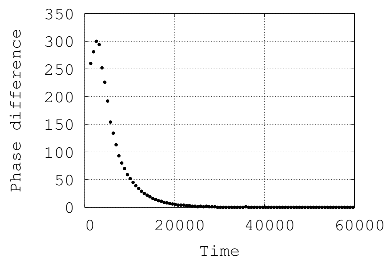

Figure 4: Time series of phase difference between oscillators without noise.

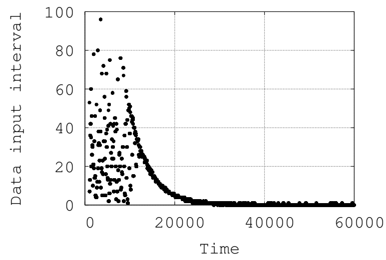

Figure 5: Time series of difference between the input timing of oscillators without noise input.

We set parameters as $w = 0.97,\ a = 0.15,\ b = 0.4,\ c = 8.5$. In this paper, we use x value of Rössler system as the input $Y_i(t)$ of equation (1). We input $Y_i(t)$ to the u value of FitzHugh-Nagumo oscillator $\mathbf{X}_1$ and $\mathbf{X}_2$. We investigate the phase difference and time difference of input timing.

Figure 4 shows the phase difference between two FitzHugh-Nagumo oscillators with inputting sequences of Rössler system. In this case, we input the sequences at the phases of $\{0, 0.13\pi, 0.25\pi, 0.51\pi, 0.75\pi, \pi, 1.254\pi, 1.63\pi\}$. Input amplitude is set $\alpha = 0.4$. From the time series of Figure 4, we can find that phase difference is gradually reduced, and achieve synchronization finally. Figure 5 shows the time series of time difference γ of input timing. We can find that time difference between the input timing of devices is gradually reduced.

We also investigate about the situation that there are measurement errors. In this paper, we use Rössler system as the natural environmental fluctuation $Y_i(t)$ and white Gaussian noise as the measurement error $\xi_i(t)$. The inputting phases are set to $\{0, 0.13\pi, 0.25\pi, 0.51\pi, 0.75\pi, \pi, 1.254\pi, 1.63\pi\}$. Figure 6 shows the time series of phase difference between oscillators. We set the amplitude of Rössler system to $\alpha = 0.4$ and the amplitude of white Gaussian noise to $\epsilon = 2.0$. From the Figure 6, we can find that phase difference is gradually reduced, and finally achieve synchronization. It shows that oscillators can synchronize by inputting of Rössler system even if there is white Gaussian noise. Figure 7 shows time series of time difference of input timing between oscilla-

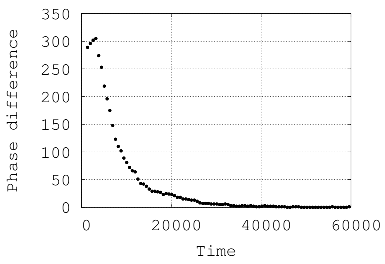

Figure 6: Time series of phase difference between oscillators with noise (with applying proposed method of this paper).

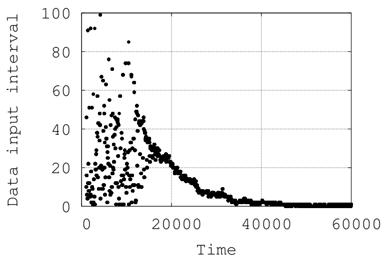

Figure 7: Time series of difference between the input timing of oscillators with noise input(with applying proposed method of this paper).

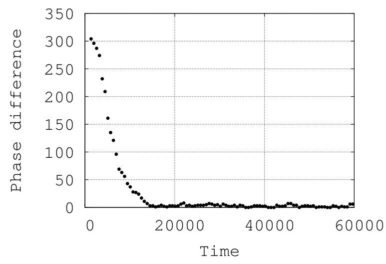

Figure 8: Time series of phase difference between oscillators with noise (without applying proposed method of this paper).

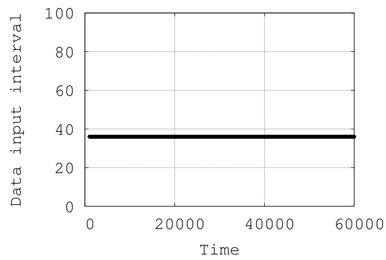

Figure 9: Time series of difference between the input timing of oscillators with noise input(without applying proposed method of this paper).

tors. From the Figure 7, we can find that time difference between the input timing of devices is gradually reduced. Figures 8 and 9 shows the result without our proposed input method. The parameters and initial values are same as the case of using our proposed method. From the Figure 8, oscillators can achieve phase synchronization even if we don't use our proposed input method. However, time series of Figure 9 have constant value, that means the timing of input is not synchronized. From these results, FitzHugh-Nagumo oscillators can achieve perfect synchronization even when there are some errors in input sequences.

6. Conclusion

In our proposed natural synchronization scheme, we use natural environmental data as the input sequence, and oscillators synchronize based on noise-induced synchronization theory. To avoid the effect of measurement errors, time average of sampling data is used for the input sequences of our method. In this paper, we have proposed the method that is able to synchronize calculation timing of time average.

We have used two FitzHugh-Nagumo oscillators as the nonlinear oscillator with adding the sequences of Rössler system, and the input timing of these oscillator is different. These two oscillators have been run in the situation with errors and without errors by the simulation. We have shown that the synchronization of phase and input timing are able to achieve even when there are some errors in input sequences.

As the future work, we study about the situation when other deterministic system is used as input sequences, and we will show the most efficient input sequence to achieve synchronization between wireless digital devices.

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