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Measuring two-dimensional translations with nanoscale-resolution by using quasiperiodic dynamics in a laser diode

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Abstract—We describe a novel approach for position sensing using the complex dynamics of a diode laser. Specifically, we show that the quasi-periodic dynamics of the optical field generated by a diode laser emitting in the telecom band ($\lambda \sim 1.55\,\mu$m) subjected to a dual coherent optical feedback changes in response to independent nanoscale translation of the mirrors in each feedback path. We obtain sub-wavelength precision of $\sim\lambda/160$ (~9.8 nm) on the distance measurements. Such sensing capabilities rely on mirror translations mapping uniquely and independently mapped to a change in the value of two incommensurate frequencies present in the quasi-periodic radio-frequency spectrum of the light emitted by the laser diode. This approach opens new avenues towards realizing an all-optical, nanoscale-resolved two-dimensional position sensor.

1. Introduction

It is well known that a semiconductor laser subjected to weak optical back reflection causes dramatic changes in the dynamics of the slow-varying envelope of the optical field, such as periodic, quasi-periodic, or even chaotic evolution [1, 2]. An important parameter that control the dynamics is the roundtrip time delay (proportional to the cavity length) of the back-reflected light. For example, it was shown that a sub-wavelength change in the delay time induces visible changes in physical observables, such as the relaxation-oscillation frequency of the laser [3].

Such a system belongs to a class of laser-feedback interferometry techniques [4] and is well suited for detecting one-dimensional sub-wavelength translations because the change of the distance between the laser cavity and the reflecting object can be uniquely mapped onto a single measurable quantity (namely an observable): A frequency component in the radio-frequency (RF) spectrum of the optical field. However, to detect simultaneously multiple sub-wavelength translations in different spatial dimensions requires access to several observables that will provide independent evolutions so that the translations can mapped onto them uniquely.

Cohen *et al.* have already shown how to exploit the RF spectrum of quasi-periodic (QP) dynamics generated by a system with time-delay feedback provided by a wave-chaos cavity and track the position of a sub-wavelength scatterer. The tracking was performed by monitoring independent frequency shifts associated with two incommensurate frequencies [5]. The study was later adapted in the optical domain using a semiconductor laser with dual optical feedback [6].

In this paper, we review how the RF spectrum generated by a laser diode with dual optical feedback can be harnessed to recover sub-wavelength nano-scale translations as in [6]. In a two-dimensional space spanned by the position of two independently shifted mirrors, we show that we can reconstruct an arbitrary sub-wavelenght trajectory with a precision of $\sim\lambda/160$.

2. Description of the Experimental Setup

The experimental setup is illustrated in Fig.1. It consists of a semiconductor diode laser emitting in the telecom band ($\lambda \sim 1550$ nm) subjected to a dual coherent optical feedback with two arms of different length. Each arm is bound by a mirror ($M_{x,y}$) mounted on a piezoelectric transducer ($PZT_{x,y}$), which can realize nano-scale translations $\Delta_{x,y}$ in the horizontal (x) and vertical (y) directions with a resolution of ±10 nm. This induces change of time-delays $\Delta\tau_{x,y} = 2\Delta_{x,y}/c$, where c is the speed of the light in vacuum.

3. Quasi-periodic Spectrum for Detecting Nano-scale Translations

The laser diode with dual optical feedback is in a quasi-periodic (QP) regime comprising multiple incommensurate frequency components when the proper experimental conditions are met (see caption of Fig.2 for the experimental details). In our experimental system, the spectrum of such a QP regime under these conditions is shown in Fig. 2(a). One can notice a complex optical spectrum with six clusters of frequency components distributed on the entire observation bandwidth of the oscilloscope. Each cluster is

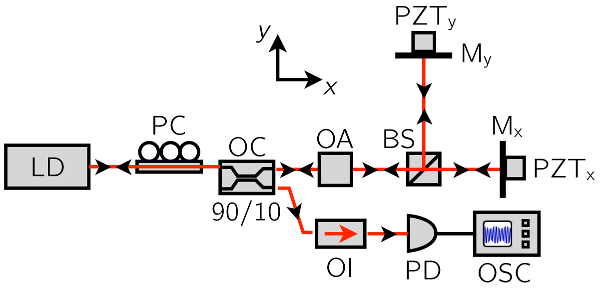

Figure 1: Experimental setup of a diode laser (LD) with a dual optical feedback with mirrors ($M_{x,y}$) mounted on piezoelectric transducers Burleigh PZO-015 ($PZT_{x,y}$) realizing translation in the horizontal (x) and vertical (y) directions, respectively. The dual optical feedback comprises a 50/50 beam splitter (BS) and an optical attenuator (OA) to control the strength of the total optical feedback. The setup also uses a polarization controller (PC) to ensure that optical feedback is coherent. A 90/10 optical coupler (OC) is used to send a fraction of the light power to a photodiode (PD) after passing an optical isolator (OI), which prevents back reflection. The optical signal is converted into an electrical signal so that the RF spectra and temporal evolution can be monitored on a broadband oscilloscope Agilent Infinuum DSO90804 (OSC) with 8 GHz bandwidth and sampling rate of 40 GSa/s.

centered on frequency $f_{\mathrm{QP},i}$ with $i = 1, \ldots, 6$ and the frequency components are distributed symmetrically on both sides of the central frequency $f_{\mathrm{QP},i}$. When each arm of the cavity is translated by $\Delta_{x,y}$, the various components of RF spectrum changes by small amount $\Delta f_{\mathrm{QP},i}$. Because the translations are significantly below the wavelength of the light emitted $\Delta_{x,y} \ll \lambda$, the laser remains in a QP state that is similar to that observed prior to the mirror translation. Among the various clusters, we follow the change in frequency $\Delta f_{\mathrm{QP},2}$ and $\Delta f_{\mathrm{QP},6}$ of the central frequency of the second and sixth cluster, respectively, because they lead to the best nano-scale sub-wavelength resolution in our experiment (see [6]).

We observe dependence of these two frequency shifts on $\Delta_{x,y}$. Towards this end, we create a two-dimensional spatial grid of approximately 100 nm $\times$100 nm using incremental steps of 10 nm in both the horizontal (x) and vertical (y) directions. In that range of variation of the dual external cavity, the change in time-delays $\tau_{x,y}$ is so small that the system stays in its QP regime and the frequency variation can be monitored continuously. The experimental method to track these small frequency variations is detailed in [6].

The frequency variations of $\Delta f_{\mathrm{QP},2}$ ($\Delta f_{\mathrm{QP},6}$) for each couple of discrete translations are represented by blue circles (red squares) in Fig. 2(b) [Fig. 2(c)]. They unveil a nonlinear dependence of the frequency variations with respect to the translation of each mirror. These discrete points associated with frequency shifts belong to smooth surfaces that can be modeled mathematically with simple second-degree, multivariate, polynomial functions given by

$$\Delta\hat{f}_{\mathrm{QP},2} = \sum_{0\le i+j\le 2} a_{ij}\Delta_x^i\Delta_y^j, \tag{1}$$

$$\Delta\hat{f}_{\mathrm{QP},6} = \sum_{0\le i+j\le 2} b_{2|ij}\Delta_x^i\Delta_y^j, \tag{2}$$

where $\Delta\hat{f}_{QP,2|6}$ are the continuous mathematical estimate of the frequency shifts of the central frequency of cluster #2 and #6, respectively, and a_{ij}, $b_{ij} \in \mathbb{R}$ are coefficients estimated by linear regression using the experimental data. These two polynomial functions define a continuous approximation of the surface on which the experimental points lie. They are displayed on the top of the experimental data on Fig. 2(b)-(c). We observe that these surfaces have different geometrical features : (i) ranges of variation over the grid and (ii) local curvature, which suggests that the two observables have independent evolution when each mirror is translated. Hence, they are suitable candidates as independent observables for recovering the nano-scale translations information.

Equations (1)-(2) realize a simple mapping between the frequency and the position of the mirror $(\Delta_x, \Delta_y) \to (\Delta f_{\mathrm{QP},2}, \Delta f_{\mathrm{QP},2})$ on the two-dimensional grid considered in the experiment. This mapping is invertible, and hence it is possible to trace-back the frequency shifts to the estimated position $(\hat{\Delta}_x, \hat{\Delta}_y)$ of the mirrors allowing for the discrimination between translation of each mirror. This is in contrast with typical interferometric methods that are only sensitive to the relative difference in the two optical paths, which could come from either of the two mirrors being translated in our setup.

The second degree multivariate polynomial functions $(\Delta\hat{f}_{QP,2}, \Delta\hat{f}_{QP,6})$ can be numerically inverted to retrieve estimation of the translation $(\hat{\Delta}_x, \hat{\Delta}_y)$. The numerical inversion consists of solving the optimization problem

$$\begin{bmatrix} \hat{\Delta}_x \\ \hat{\Delta}_y \end{bmatrix} = \underset{[\Delta_x,\Delta_y]\in grid}{\operatorname{argmin}} \left\| \begin{matrix} \Delta\hat{f}_{\mathrm{QP},2}\left(\Delta_x, \Delta_y\right) - \Delta f_{\mathrm{QP},2} \\ \Delta\hat{f}_{\mathrm{QP},6}\left(\Delta_x, \Delta_y\right) - \Delta f_{\mathrm{QP},6} \end{matrix} \right\|_2, \tag{3}$$

where $||\cdot||_2$ is the squared Euclidean norm, $(\Delta f_{\mathrm{QP},2}, \Delta f_{\mathrm{QP},6})$ are the couple of experimental frequency shifts measured in the second and sixth frequency clusters. The inversion via numerical optimization leads to root-mean square (RMS) differences between with actual translations with an average sub-wavelength resolution of $\sim\lambda/160$ (approximately 9.8 nm) over the two-dimensional grid defined in our experiment.

4. Reconstruction of an Arbitrary Sub-wavelength Trajectory

The previous experiment is used as a calibration grid to create the mapping between frequency shift of the QP fre-

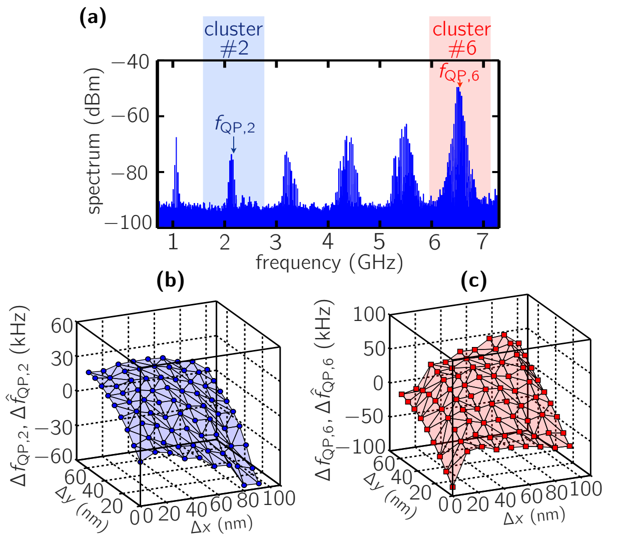

Figure 2: (a) Experimental power spectral density (PSD) of a QP regime corresponding to feedback strength of ∼2%, pumping current of $I = 23.6$ mA (threshold current is $I_{th}{\approx}8$ mA) and external cavity with time delay $\tau_x{\approx}55.5$ ns and $\tau_y{\approx}55.6$ ns. The two frequency clusters of interest are shadowed and their central frequency $f_{QP,2|6}$ are indicated by arrows. (b)-(c) Experimental frequency shifts of cluster central frequencies $\Delta f_{QP,2|6}$ as a function of translation steps of each mirror $\Delta_{x,y}$. Experimental points are represented by blue circles (●) and red squares (■). Multivariate polynomial functions $\Delta\hat{f}_{QP,2|6}(\Delta_x, \Delta_y)$, which realize the invertible mapping, are represented by the light colored surfaces displayed on the top of the experimental points.

quencies and position of the mirrors. If we create an unknown sequence of translation for the two mirrors, it is possible to recover an estimation of the trajectory by solving sequentially the optimization problem of Eq. (3).

Specifically, each time we apply the couple of translations $(\Delta_{x,i}, \Delta_{y,i})$, we record the associated frequency shifts $(\Delta f_{QP,2,i}, \Delta f_{QP,6,i})$. We recover the estimated translation from the frequency measurement by repeating the optimization problem given by Eq. 3 each time. We apply this protocol to an arbitrary trajectory defined within the 2D grid (not necessarily coinciding with the grid points). It comprises 32 points that are recorded immediately after those of the calibration grid. We choose this recording protocol because the sensitivity of our experiment to thermal fluctuations is such that the calibration grid is valid only for a few minutes.

The experimental results are shown in Fig. 3. We observe that the reconstructed path has similar geometric features to those of original path. However, because of the inaccuracy of inversion process, the limited resolution of the piezoelectric actuators, and inherent thermal fluctuations, the calibration grid happens to be too coarse to allow more precise reconstruction of the actual trajectory. Nevertheless, if we measure the average RMS error between the original and reconstructed curves by computing $\epsilon = 1/N \sum_{i=1}^{N} \left\|(\hat{\Delta}_{x,i}, \hat{\Delta}_{y,i}) - (\Delta_{x,i}, \Delta_{y,i})\right\|_2$, we find that $\epsilon \approx \lambda/160$, thus being sub-wavelength with nano-scale precision. This is consistent with the average resolution found with the inversion of the frequency-shift mapping obtained from the calibration grid [7].

5. Conclusion

In this paper, we show that it is possible to harness the RF spectrum of quasi-periodic dynamics in a diode laser emitting in the telecom band (λ∼1550 nm) subjected to dual optical feedback. We find that independent frequency shifts of two incommensurate frequencies in the RF spectrum are associated with independent translations of the two mirrors comprising the dual external cavity. The existence of an invertible mathematical map between the frequency shifts and translations allows us to estimate the position of the mirrors with nano-scale precision of ∼$\lambda/160$. These experimental results are the first step towards realizing multi-dimensional, nano-scale precision sensors based on the complex dynamics occuring in nonlinear photonics systems.

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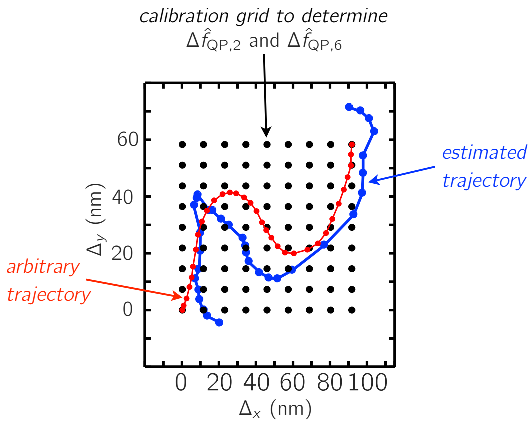

Figure 3: Reconstruction of an arbitrary path. The path is defined within the calibration grid represented by black dots, which is used to create the multivariate continuous approximation of the frequency shifts evolution. The actual arbitrary path is given by individual translation pairs $(\Delta_{x,i}, \Delta_{y,i})$ with $i = 1, \ldots, 32$ positions (non-overlapping with the grid points) and is represented by the red curve. The reconstructed path of estimated position $(\hat{\Delta}_{x,i}, \hat{\Delta}_{y,i})$ with $i = 1, \ldots, 32$ is represented by the blue curve.

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