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Phase Difference Propagation Phenomena on a Non-edge Lattice

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Abstract—In this study, we investigate and analysis some phenomena which are observed on a system that van der Pol oscillators are coupled by inductors as a non-edge lattice.

Furthermore, waveforms of double-scroll chaos are synthesized by using a circuit of coupled oscillators, and are predicted near future of the chaos waveform.

1. Introduction

A lot of things oscillate, and various phenomena are generated. For example, there are sea waves, motions of atoms, the sound of insects and so on. Specifically, a lot of things synchronize and regularities are observed in their motions. For example, neuronal cells, emissions of many fireflies, rumor synchronize, motion of stars, and so on. Furthermore, there are many things which continue to change while oscillating, for example, a change of temperature, stock prices, and exchange rate etc. Therefore, we can say that it is important that the coupled oscillator systems are researched.

Recently, we analyzed phenomena on a system which many oscillators are coupled as a lattice[1]. Furthermore, we made a system of coupled oscillators as a lattice, synthesized applied single-scroll chaos waveform and predicted a waveform of near future of the chaos waveform.

In this study, many oscillators are coupled as a lattice, and the one edge is coupled to a counter side edge. If phase difference is given to an oscillator, the phase difference is propagated when all oscillators are synchronized in-phase. The propagation phenomena are observed and classfied. We call the propagation phenomena a phase-wave. Furthermore, we show that complex chaos waveform with the double-scroll is synthesized by using a circuit of coupled oscillators, which we suggested before.

2. Circuit model

In this study, many van der Pol oscillators are coupled by inductors L_0 as a lattice(see Fig.1). The one edge is coupled to a counter side edge. Therefore, in the Fig. 1, the leftside "A" and "B" are coupled to the rightside "A" and "B" respectively, and the topside "C" and "D" are coupled to the bottomside "C" and "D" respectively. The number of row of the system is assumed as M. The number of column of the system is assumed as N. Each oscillator is named as OSC(k,l), a voltage of each oscillator is named as $v_{(k,l)}$, a current that flows to inductor of each oscillator is named as $i_{(k,l)}$ (see Fig.1).

The circuit equations of the circuit model of Fig.1 are normalized by Eq.(1). The normalized circuit equations are shown as Eq.(2)~Eq.(10).

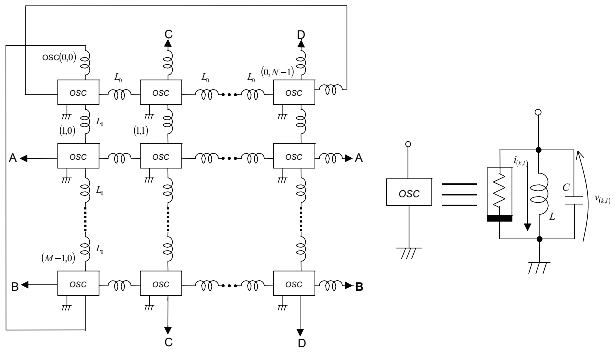


Figure 1: Circuit Model.

$$i_{(k,l)} = \sqrt{\frac{Cg_1}{3Lg_3}}x_{(k,l)}, \quad v_{(k,l)} = \sqrt{\frac{g_1}{3g_3}}y_{(k,l)},$$
$$t = \sqrt{LC}\tau, \quad \frac{d}{d\tau} = \text{“ } \cdot \text{ ”}, \alpha = \frac{L}{L_0}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}}. \tag{1}$$

[Corner–top left]

$$\frac{dx_{(0,0)}}{d\tau} = y_{(0,0)} \tag{2}$$

$$\frac{dy_{(0,0)}}{d\tau} = -x_{(0,0)} + \alpha(x_{(0,N-1)} + x_{(M-1,0)} + x_{(0,1)} + x_{(1,0)} - 4x_{(0,0)}) + \varepsilon(y_{(0,0)} - \frac{1}{3}y^3_{(0,0)})$$

[Corner–bottom left]

$$\frac{dx_{(M-1,0)}}{d\tau} = y_{(M-1,0)} \tag{3}$$

$$\frac{dy_{(M-1,0)}}{d\tau} = -x_{(M-1,0)} + \alpha(x_{(M-1,1)} + x_{(0,0)} + x_{(M-2,0)} + x_{(M-1,N-1)} - 4x_{(M-1,0)}) + \varepsilon(y_{(M-1,0)} - \frac{1}{3}y^3_{(M-1,0)})$$

[Corner–top right]

$$\frac{dx_{(0,N-1)}}{d\tau} = y_{(0,N-1)} \tag{4}$$

$$\frac{dy_{(0,N-1)}}{d\tau} = -x_{(0,N-1)} + \alpha(x_{(0,N-2)} + x_{(0,0)} + x_{(1,N-1)}$$

$$+ x_{(M-1,N-1)} - 4x_{(0,N-1)}) + \varepsilon(y_{(0,N-1)} - \frac{1}{3}y^3_{(0,N-1)})$$

[Corner–bottom right]

$$\frac{dx_{(M-1,N-1)}}{d\tau} = y_{(M-1,N-1)} \quad (5)$$

$$\frac{dy_{(M-1,N-1)}}{d\tau} = -x_{(M-1,N-1)} + \alpha(x_{(M-1,N-2)} + x_{(M-2,N-1)} + x_{(M-1,0)} + x_{(0,N-1)} - 4x_{(M-1,N-1)}) + \varepsilon(y_{(M-1,N-1)} - \frac{1}{3}y^3_{(M-1,N-1)})$$

[Center($0 < k < M-1$ *and* $0 < l < N-1$)]

$$\frac{dx_{(k,l)}}{d\tau} = y_{(k,l)} \quad (6)$$

$$\frac{dy_{(k,l)}}{d\tau} = -x_{(k,l)} + \alpha(x_{(k-1,l)} + x_{(k+1,l)} + x_{(k,l-1)} + x_{(k,l+1)} - 4x_{(k,l)}) + \varepsilon(y_{(k,l)} - \frac{1}{3}y^3_{(k,l)})$$

[Edge–top]

$$\frac{dx_{(0,l)}}{d\tau} = y_{(0,l)} \quad (7)$$

$$\frac{dy_{(0,l)}}{d\tau} = -x_{(0,l)} + \alpha(x_{(0,l+1)} + x_{(1,l)} + x_{(0,l-1)} + x_{(M-1,l)} - 4x_{(0,l)}) + \varepsilon(y_{(0,l)} - \frac{1}{3}y^3_{(0,l)})$$

[Edge–bottom]

$$\frac{dx_{(M-1,l)}}{d\tau} = y_{(M-1,l)} \quad (8)$$

$$\frac{dy_{(M-1,l)}}{d\tau} = -x_{(M-1,l)} + \alpha(x_{(M-1,l-1)} + x_{(M-2,l)} + x_{(M-1,l+1)} + x_{(0,l)} - 4x_{(M-1,l)}) + \varepsilon(y_{(M-1,l)} - \frac{1}{3}y^3_{(M-1,l)})$$

[Edge–left]

$$\frac{dx_{(k,0)}}{d\tau} = y_{(k,0)} \quad (9)$$

$$\frac{dy_{(k,0)}}{d\tau} = -x_{(k,0)} + \alpha(x_{(k-1,0)} + x_{(k,1)} + x_{(k+1,0)} + x_{(k,N-1)} - 4x_{(k,0)}) + \varepsilon(y_{(k,0)} - \frac{1}{3}y^3_{(k,0)})$$

[Edge–right]

$$\frac{dx_{(k,N-1)}}{d\tau} = y_{(k,N-1)} \quad (10)$$

$$\frac{dy_{(k,N-1)}}{d\tau} = -x_{(k,N-1)} + \alpha(x_{(k,N-2)} + x_{(k-1,N-1)} + x_{(k+1,N-1)} + x_{(k,0)} - 4x_{(k,N-1)}) + \varepsilon(y_{(k,N-1)} - \frac{1}{3}y^3_{(k,N-1)})$$

The α corresponds to the coupling of the oscillators. The ε corresponds to nonlinearity of each oscillator. These equations are calculated by the fourth Runge-Kutta methods.

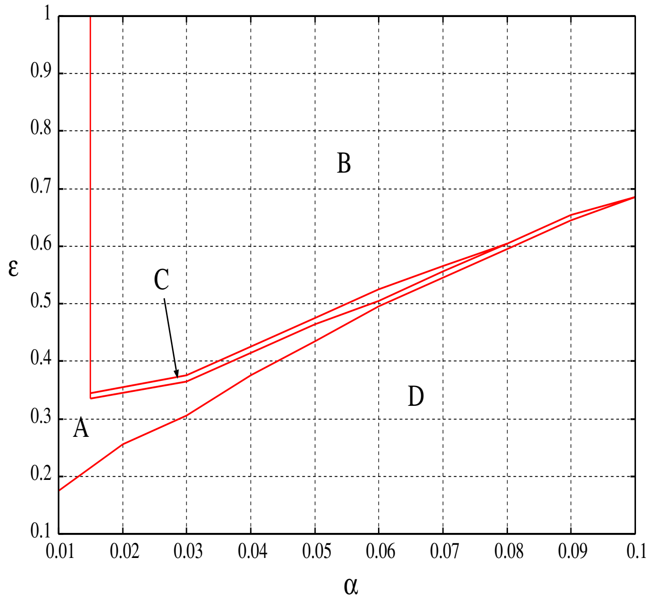


Figure 2: Domains of observation phenomena(α vs ε).

3. The propagation phenomena of phase-waves

In-phase synchronization of all oscillators can be observed in this circuit. We can observe phenomena that phase differences between adjacent oscillators propagate on this system. We call this phenomena phase-waves. The phase-waves are observed.

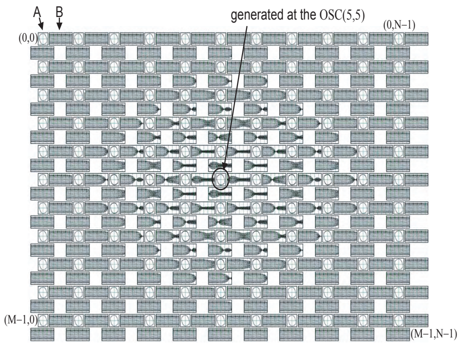
3.1. observation conditions

In this section, observation conditions are fixed as follows.

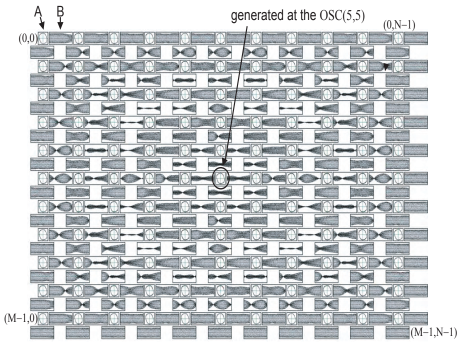
1. M and N are fixed as 11.
2. The initial values of all oscillators are entered same value.
3. The phase-wave is generated from OSC(5,5).
4. The coupling parameter α is changed from 0.01 to 0.1 every 0.01, and nonlinearity ε is changed from 0.1 to 1 every 0.01.

3.2. Observation results

Observed phenomena are classified into four patterns. The observation results are shown from Figs. 2–5. Figure 2 shows the domain of the observed phenomena. The vertical axis expresses nonlinearity, and the horizontal axis expresses coupling. Figure 3–"A" shows the attractor of each circuit. The horizontal axis expresses voltage $y_{(n,m)}$, and the vertical axis expresses current $x_{(n,m)}$. Figure 3–"B" shows a transition of a phase difference between adjacent oscillators. The horizontal axis expresses time, and the vertical axis expresses a voltage which is sum of two voltages of adjacent oscillators. The vertical axis is $y_{(n,m)} + y_{(n+1,m)}$ or $y_{(n,m)} + y_{(n,m+1)}$. The amplitude doubles if in-phase synchronization is observed between adjacent oscillators. On the other hand, the amplitude becomes zero if anti-phase synchronization can be observed between adjacent oscillators. We show our results of each class.



(a) τ = 0 ~ 10000.



(b) τ = 10000 ~ 20000.

Figure 3: Observation phase-waves in domain A (α=0.01 ε=1.00).

3.2.1. domain A (see Fig.3)

Figures 3 shows observed phase-waves. Figure 3(b) is a figure of continuation of Fig. 3(a). The phase-wave is generated at OSC(5, 5). The phase-wave is propagated like the ripple of water from the OSC(5,5). The phase-wave arrives at OSC(5,1), OSC(5,9) and OSC(3,8) on the same time in Fig. 3(a). The distances between OSC(5,1) or OSC(5,9) and OSC(5,5) are 4, if distance from OSC(5,5) to OSC(5,4) is counted as 1. However,the distance between OSC(5,5) and OSC(3,8) is 5. Therefore, the propagating phase-wave diagonal of this system is easy than propagating phase-waves on the column and the row. Further, the phase-wave seems not to reach the edge of the figure. However, the phase-wave arrives to the edge of figure, collides a phase-wave from other side, and reflects. Finally, complex phenomena can be observed.

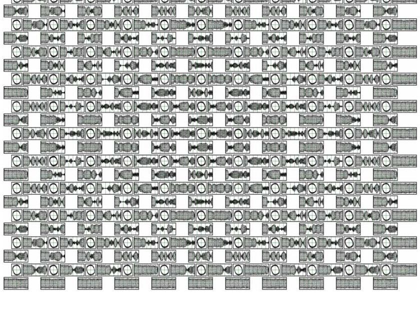
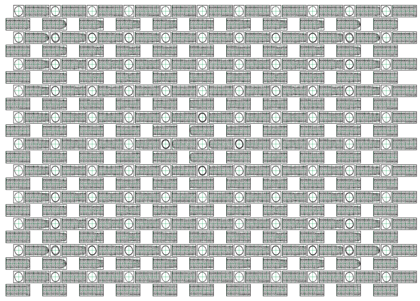
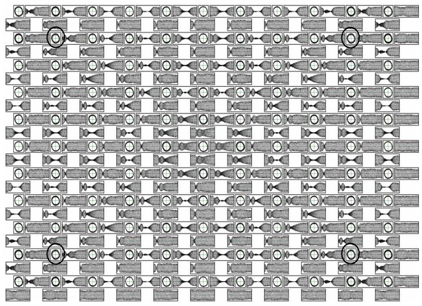


Figure 4: Observation phase-waves in domain B (α=0.05 ε=0.93).



(a) τ = 0 ~ 10000.



(b) τ = 10000 ~ 20000.

Figure 5: Observation phase-waves in domain C (α=0.05 ε=0.47).

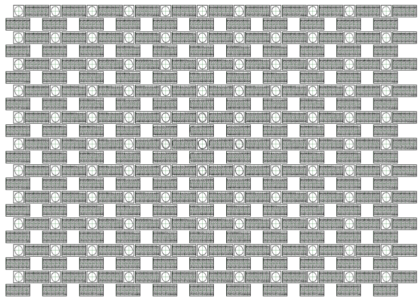


Figure 6: Observation phase-waves in domain D (α=0.05 ε=0.47).

3.2.2. domain B (see Fig.4)

Figures 4 shows observed a phenomenon. A phase difference is given to OSC(5, 5). The phase difference does not become the clear phase-wave. Only complex phenomenon can be observed at once.

3.2.3. domain C (see Fig. 5)

Figure 5 shows observed phase-waves. Figure 5(b) is a figure of continuation of Fig. 5(a). A phase difference is given to OSC(5, 5). Only phase-waves which propagate diagonally can be observed, but the propagation phenomenon is hard to see. The phase-waves arrives at OSC(2,1), OSC(2,10), OSC(10,1) and OSC(10,10) (see markings of a circle in Fig. 5(b)). The three phase-waves coming from three directions are collided around at these oscillator, and the phase-waves with large phase difference are generated. Finally, these phase-waves are propagated to all oscillators.

When the number of oscillators is an odd number, the phenomenon in domain C is ovserved. Therefore, obeserving phenomena in domainC depend on the numberof oscillators.

3.2.4. domain D (see Fig. 6)

Figure 6 shows observed phase-waves. A phase difference is given to OSC$(5,5)$. Generated phase-waves become to disappear while propagating, or collide phase-waves from other side and become to disappear.

4. the generating arbitrary waveform

We developed a technique, which uses coupled oscillators system, for prediction of an arbitrary time-series data(see Fig.7)[1]. In this section, we use 9 oscillator for this system. In this paper, a time-series data of Chua's chaos circuit is used as the arbitrary original time-series data. Chua's chaos circuit have double-scroll attractor. Therefore, the time-series data of the chaos circuit is complex more than the original data used in our previous study. The original time-series data is entered through the inductors $L_{out(k,l)}$(see Fig.7). The arbitrary time-series data is assumed $f(t_1)$. A prediction of the arbitrary time-series data is assumed $g(t_2)$.

This circuit equations include coupling parameter $\alpha_{out(k,l)}$ which is used for connecting with a signal generator generating the arbitrary time-series data. The coupling parameters and nonlinearity are individually changed for each oscillator. Therefore, a nonlinearity for an oscillator OSC(x,y) is assumed $\varepsilon(x,y)$. Four coupling parameters of between adjacent oscillators are expressed as $\alpha_{h(x,y)-(x,y+1)}$, $\alpha_{h(x,y)-(x,y-1)}$, $\alpha_{v(x,y)-(x+1,y)}$, and $\alpha_{v(x,y)-(x-1,y)}$. The equations of OSC(0,0) is shown as Eq.(11).

[Corner–top left]

$$
\begin{aligned}
\frac{dx_{(0,0)}}{d\tau} &= y_{(0,0)} \qquad (11)\\
\frac{dy_{(0,0)}}{d\tau} &= -x_{(0,0)} + \alpha_{h(0,0)-(0,1)}(x_{(0,1)} - x_{(0,0)}) \\
&\quad + \alpha_{v(0,0)-(1,0)}(x_{(1,0)} - x_{(0,0)}) \\
&\quad + \alpha_{out(0,0)}(x_s - x_{(0,0)}) + \varepsilon(y_{(0,0)} - \frac{1}{3}y^3_{(0,0)})
\end{aligned}
$$

$k = 0$, and $l = 0$.

The center oscillator OSC(2,2) is not used $\alpha_{out(k,l)}$. These equations are calculated by the fourth Runge-Kutta methods in this study.

The prediction data of $f(t_1)$ is made by method as follows:

1. The arbitrary time-series data from $f(0)$ to $f(2T)$ is added. Generating data $g(t_2)$:$(T < t_2 \leq 2T)$ are adjusted to values close to $f(t_1)$:$(3T < t_1 \leq 4T)$ by individually changing α, ε, α_{out}, and eighteen initial values of each oscillator. The "T" expresses arbitrary time.
2. The arbitrary time-series data from $f(2T)$ to $f(3T)$ is added. We can think that generating data $g(t_2)$:$(2T < t_2 \leq 3T)$ are data close to $f(t_1)$:$(4T < t_1 \leq 5T)$. In other word, we can think that $f(t_1)$ can be predicted.

In this section, the initial value shifts positive and negative as follows.

$$g_{(0)} = \pm f_{(2T)} \qquad (12)$$

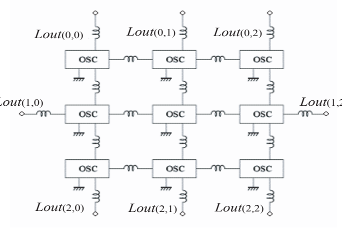


Figure 7: System for synthsisis and prediction.

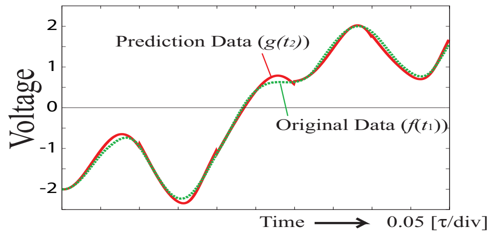


Figure 8: Prediction data and data waveform.

The results of prediction are shown in Fig.8 while t_1 and t_2 are changed from $4T$ to $6T$. The T equals 2000[τ].

The plain-line shows the prediction data $g(t_2)$. The dot-line shows the original data $f(t_1)$. We can observe similar waveform between the prediction waveform and the original waveform.

5. Conclusion

In this study, we investigated and analyzed some phenomena which are observed on a system where oscillators are coupled by inductors as a lattice coupling opposite edges. The observed phenomena were able to be classified into four domains. In the domain A, the phase-waves are propagated like the ripple of water from OSC(5,5). Finally, only the complex phenomena can be observed. In the domain B, the phase difference, which is applied into OSC(5,5), does not become the clear phase-waves. Only complex phenomena can be observed at once. In the domain C, only phase-waves which propagate diagonally can be observed, but the propagation phenomenon is hard to see. The three phase-waves coming from three directions collide, and the phase-waves, which have a large phase difference, were generated and become a complex phenomenon. In the domain D, generated phase-waves become to disappear while propagating, or collide phase-waves from other side and become to disappear.

We were able to synthesize and predict a waveform of Chua's circuit generating double-scroll attractor which is more complex than waveform synthesized in our previous study. The synthesis waveform included errors, but, synthesis waveform closed to an original waveform. Therefore, we can think that our method have possibilities to be able to predict more complex waveform of natural world.

6. Acknowledgements

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References

[1] S. Yamane, T. Imoto, and M. Yamauchi, "Investigation of Phase-Waves and Prediction of Waveform using System of Coupled Oscillators," *Proc. of NCSP'08*, pp. 148–151, Mar. 2008.