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Long Seek Control of Hard Disk Drives Using Reference Governor

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Abstract—In this paper, we study the head position seek control of hard disk drives. We propose to combine the reference management method using reference governor with outer feedback and the model following control method. We apply the former method to the control model, which is a low-order nominal model, and the control input and the output for the control model are used the control input and the reference signal for the real hard disk systems, which has the high-order resonant term and with parametric uncertainly.

1. Introduction

Head position control of hard disk drives(HDDs) is a typical application in nano-scale servo control. Recently, many peoples extensively study this control issue[1]–[4].

When we consider the long seek control of HDDs, to achieve fast seek, we need to treat HDD system as a constrained system since armature current of voice-coil motor must be limited for the protection of the motor. Thus, we study the long seek control of HDDs in the framework of the control of constrained systems. Many control methods has been proposed[5]– [8]. One of the most popular approach is reference input shaping/management method.

In the HDD benchmark problem, the model of the swing arm consists of a double integrator, which is obtained by assuming it is a rigid body, and high-frequency resonant modes, and, hence, it is a $(3 + 12)$-th order system. Moreover, high-frequency resonant modes has some uncertainties. Therefore, the full-order model is too complicated, and it is not adequate to use it as a model for control.

In this paper, we propose to control HDD systems by combining the Reference input Governor (RG) with a outer feedback [8] and the model following control scheme [3]. Considering the double integrator as a nominal plant, we design a stable servo system (we call it as a nominal servosystem), and use the RG to governs the reference input so that the constraint condition is satisfied. To control the real HDD systems, we adopt the model following control scheme, that is, the control input for the nominal plant is also used as the control input of the real HDD system; the control output of the double integrator is used as the reference input for the real HDD system; the error between control output of the double integrator and the real HDD system is feedbacked so that the real HDD system follows the behavior of the nominal servosystem. Moreover, we apply the proposed method to the HDD benchmark problem [1, 2] and show that it works well and we can achieve 0.4848[ms] seek time, which is fastest seek time under the condition that we use a single rate control with sampling time 3.7879×10^{-2}[ms].

2. Problem Setting

2.1. Seek Control Problem of Hard Disk Drive

In the benchmark problem of HDD shown in [1, 2], the plant model $P_f(s)$ is given by

$$P_f(s) = K_p \left(\frac{1}{s^2} + \sum_{i=2}^{7} \frac{A_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \right) e^{-T_d \cdot s}, \quad (1)$$

where the first term (i.e., the double integrator $1/s^2$) is the model of the swing arm by considering it is a rigid body, the second term denotes high-frequency resonant modes, and T_d is the delay time to compute the control input and so on by computer (See [1, 2] for details).

We consider seek control of head position of HDD. We consider the following specifications.

1. Seek target is 10 [track], that is, the constant reference input is 10.

2. The disturbance (sensor noise, flutter disturbance, repeatable run out, force disturbance etc.) would be neglected. But uncertainty of the plant need to be considered. For this sake, the following specifications 3 and 4 must be satisfied for 18 full-order perturbed models P_f.

3. The input u_f, which is the armature current of voice-coil motor, has the constraint such that $|u_f(t)| \leq 0.1[A]$. [1]

4. Seeking time is defined as the time required for the system to settle within $\pm 0.1[\mathrm{track}]$ of the target track.

[1] In benchmark problem in [1, 2], there is no the input saturation.

2.2. Controller Design

To cancel or suppress the resonance modes included in $P_f(s)$, we use notch filter $N_f(s)$ given by

$$N_f(s) = \prod_{i=2}^{7} \frac{s^2 + d_i 2\zeta_i \omega_i s + \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{2}$$

where $\omega_i = 2\pi f_i$, ζ_i and d_i are frequency in center, width and depth of notch, respectively, and their values are shown in Table 1.

Table.1. Design parameters of notch filter.

i	f_i[Hz]	ζ_i	d_i	i	f_i[Hz]	ζ_i	d_i
2	3000	0.9	0.91	5	7000	0.4	0.05
3	4100	0.2	0.25	6	12300	0.5	0.17
4	5000	0.2	0.10	7	16400	0.5	0.17

The plant model $P_f(s)$ is $(3+12)$-th order system. and the total notch filter $N_f(s)$ is 12-th order, and, hence, Moreover, high-frequency resonant modes has some uncertainties. Therefore, $P_f(s)N_f(s)$ is too complicated, and it is not adequate to use it as a model for control. In this respect, we will use

$$P_m(s) = \frac{K_p}{s^2} e^{-T_d \cdot s}, \tag{3}$$

as a nominal model, and we adopt the model following control scheme [3].

Since we consider the digital control, we compute discrete time models $P_m(z)$ and $P_f(z)$ of $P_m(s)$ and $P_f(s)$, respectively. These discrete time models are computed using Matlab function c2d with the sampling time $T_s = 3.7879 \times 10^{-2}$[ms] and the option 'zoh'. We also compute discrete time notch filter $N_f(z)$ of the continuous time notch filter $N_f(s)$ by applying Matlab function c2d with the sampling time T_s and the option 'matched'.

In Figure 1, characteristic of $P_m(z)$, $P_f(z)$ and $P_f(z)N_f(z)$ are shown by the dotted line, the dashed line, and the solid line, respectively. From the figure, we can see that the characteristic of $P_m(z)$ gives a good approximation of that of $P_f(z)N_f(z)$ in low and middle frequencies.

To construct servo systems for the nominal system $P_m(s)$ and the full-order plant $P_f(s)N_f(s)$, we design controllers $C_m(s)$ and $C_f(s)$, respectively,

$$C_m(s) = K_m \frac{\alpha_m s + 2\pi F_m}{s + \alpha_m 2\pi F_m} \tag{4}$$

and

$$C_f(s) = K_f \frac{\alpha_f s + 2\pi F_{f1}}{s + \alpha_f 2\pi F_{f1}} \times \left(1 + \frac{2\pi F_{f2}}{s}\right), \tag{5}$$

where $K_m = 9.83 \times 10^{-4}$, $\alpha_m = 1.32 \times 10^2$, $F_m = 8.43$, $K_f = 3.78 \times 10^{-7}$, $\alpha_f = 2.65 \times 10^2$, $F_{f1} = 42.1$, $F_{f2} = 50.0$.

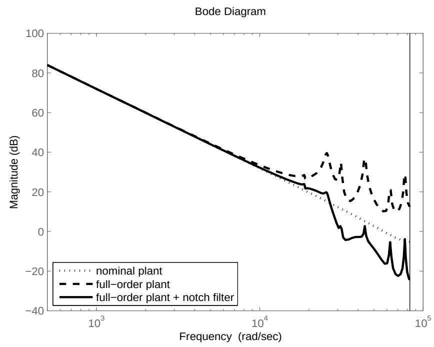

Figure 1. Frequency response of $P_m(z)$,$P_f(z)$, and $P_f(z)N_f(z)$.

Discrete time controllers $C_m(z)$ and $C_f(z)$ of continuous time controllers $C_m(s)$ and $C_f(s)$ are obtained, respectively, by applying Matlab function c2d with the sampling time T_s and the option 'matched'.

The nominal servo system is given by

$$\begin{cases} Y(z) = P_m(z)U(z), \\ U(z) = C_m(z)[R(z) - Y(z)]. \end{cases} \tag{6}$$

and the real servo system is given by

$$\begin{cases} Y_f(z) = P_f(z)N_f(z)[U(z) + U_{fl}(z)], \\ U_{fl}(z) = C_f(z)[Y(z) - Y_f(z)], \end{cases} \tag{7}$$

when we apply the model following control scheme.

2.3. Constrained Control Problem

Let the state space representation of the normal servo system be given by

$$\begin{cases} x[k+1] = Ax[k] + Br[k], \;\; x[0] = x_0 \\ y[k] = Cx[k], \quad u[k] = Lx[k] + Dr[k], \\ \qquad k \in \mathbf{Z}_+ = \{0, 1, 2, \cdots\}, \end{cases} \tag{8}$$

where $x, x_0 \in \mathbf{R}^n$, $r \in \mathbf{R}$, $y \in \mathbf{R}$ and $u \in \mathbf{R}$ are the state, the initial state, the reference input, the controlled output, and the control input of the servo system, respectively. Note that A is stable, (A,B) is reachable, and for any $\hat{r} \in \mathbf{R}^m$, there exists a unique $\hat{x}(\hat{r})$ such that

$$\hat{x}(\hat{r}) = A\hat{x}(\hat{r}) + B\hat{r}, \;\; C\hat{x}(\hat{r}) = \hat{r}. \tag{9}$$

We assume that the initial state x_0 locates in a neighborhood of $\hat{x}(r_0)$ for some r_0.

In our problem setting, $u[k]$ is the constrained variable and it is required that $|u[k]| \leq 0.1$, and we keep the constraint by managing $r[k]$.

In future, we may treat additional constraints. We use $z[k]$ instead of $u[k]$, that is, $z[k] = Lx[k] + Dr[k] \in \mathrm{R}^{N_z}$ is the vector of constrained variables. Let $x[k; x_0, r[\cdot]]$ is the state of (8) when both the initial condition $x[0; x_0, r[\cdot]] = x_0$ and the reference input $r[\cdot]$ are given. Then, the constraint is represented by

$$z[k; x_0, r[\cdot]] \in \mathcal{Z} \subseteq \mathbf{R}^{N_z} \;\; \forall k \in \mathbf{Z}_+, \tag{10}$$

where $\mathcal{Z} \subseteq \mathbf{R}^{N_z}$ is a polyhedral set defined by

$$\mathcal{Z} = \{z : \ h_i^\top z \leq 1, \ i \in \mathcal{N}_c\}, \ \ \mathcal{N}_c = \{1, 2, \cdots N_c\}. \quad (11)$$

3. Seek Control of HDD Benchmark Problem Using RG with an Outer Feedback and Model Following Control Scheme

3.1. Maximal Output Admissible Set

The concept of the maximal output admissible set [5] defined below plays crucial role to keep the constraint condition to be satisfied.

Definition 1 *For a constant reference $\hat{r} \in \mathbf{R}$, the Maximal output Admissible Set (MAS) corresponding to $\hat{r}$, denoted by $\Omega_\infty(\hat{r})$, is defined by*

$$\Omega_\infty(\hat{r}) = \{x_0 \in \mathbf{R}^n \mid z[k; x_0, \hat{r}] \in \mathcal{Z}, \ \forall k \in \mathbf{Z}_+\}.$$

Reference input Governor (RG) governs the reference input $r[\cdot]$ so that the constraint condition (10) is satisfied[5].

Note that MAS is a positively invariant set, that is,

$$x[k; x_0, \hat{r}] \in \Omega_\infty(\hat{r}) \quad \Rightarrow \quad x[k+1; x_0, \hat{r}] \in \Omega_\infty(\hat{r}).$$

Therefore, once the state $x[k_1]$ of system(8) enter the maximal output admissible set $\Omega_\infty(r^*)$, we set $r[k] = r^*$ and have $x[k; x[k_1], r^*] \in \Omega_\infty(r^*)$ for all $k \geq k_1$.

Remark 1 *Let $\hat{x}(\hat{r})$ be the equilibrium point of system* (8) *corresponding to $\hat{r}$ and let $\hat{z}(\hat{r}) = L\hat{x}(\hat{r}) + D\hat{r}$. Define $e[k] = x[k; x_0, \hat{r}] - \hat{x}(\hat{r})$. Then, we have $e[k+1] = Ae[k]$, and $e[k] = A^k e[0] = A^k(x_0 - \hat{x}(\hat{r}))$, $z[k; x_0, \hat{r}] = Le[k] + \hat{z}(\hat{r})$. Therefore, $\Omega_\infty(\hat{r})$ is also represented by*

$$\begin{aligned} \Omega_\infty(\hat{r}) &= \{e_0 + \hat{x}(\hat{r}) : LA^k e_0 + \hat{z}(\hat{r}) \in \mathcal{Z}, \ \ k \in \mathbf{Z}_+\} \\ &= \Omega'_\infty(\hat{r}) + \hat{x}(\hat{r}) \quad (12) \\ \Omega'_\infty(\hat{r}) &= \{e_0 : \ LA^k e_0 + \hat{z}(\hat{r}) \in \mathcal{Z}, \ \ k \in \mathbf{Z}_+\} \quad (13) \end{aligned}$$

Note that, in general, $\Omega'_\infty(\hat{r})$ depends on $\hat{r}$. However, from (13), we can see that $\Omega'_\infty(\hat{r}) = \Omega'_\infty(0)$ for all $\hat{r}$ such that $\hat{z}(\hat{r}) = 0$. For the HDD system we consider in Section 3, we have $\hat{z}(\hat{r}) = 0$.

3.2. Outer Feedback Controller

In this paper, we want to determine $\{r[k]\}_{k=0}^\infty$ so that not only the constraint is satisfied but also we have a good performance.

Suppose that an initial state x_0 of system (8) and a constant reference input r^* are given. Consider the following optimization problem[6, 7]:

$$\text{(QP1)} : \left\{ \begin{array}{ll} \min_{r[k]} & J(\{r[k]\}_{k=0}^N; x_0, r^*) \\ \text{subject to} & \text{(8) and (10)} \end{array} \right.$$

where $J(\{r[k]\}_{k=0}^N; x_0, r^*) = \sum_{k=0}^N (|y[k] - r^*|^2 + w_r^2 |r[k] - r^*|^2)$. The control method using the optimal solution $\{r[k]\}_{k=0}^N$ of (QP1) is called as Reference input Shaping(RS) method[6]. Note that (QP1) depends on x_0 and r^*, and, it takes a huge computing time when N is large, and, hence, we can not use RS method on line in such a case.

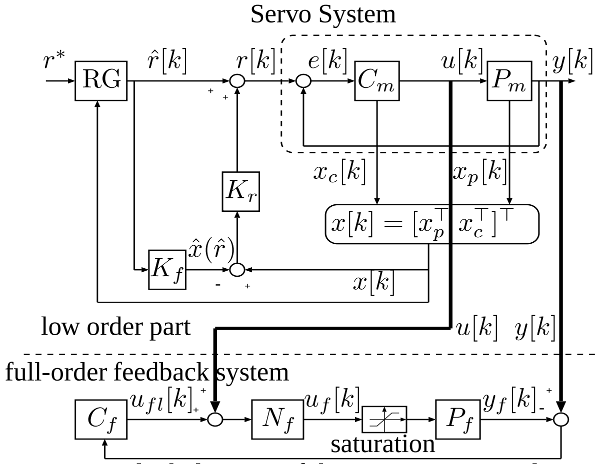

Figure 2. Block diagram of the proposing control system using a reference governor and the model following control scheme.

Lemma 1 *Let $(\hat{\varepsilon}, \hat{Q}, \hat{Y})$ be the optimal solution of (LMI 1).*

$$\textit{(LMI 1)} : \left\{ \begin{array}{ll} \min_{\varepsilon, Q, Y} & \varepsilon \\ \textit{subject to} & Q = Q^\top, Q - \mathtt{I} \succ 0, \\ \multicolumn{2}{l}{\begin{bmatrix} Q & (*_{21})^\top & (*_{31})^\top \\ (*_{21}) & Q & 0 \\ (*_{31}) & 0 & \varepsilon \mathtt{I} \end{bmatrix} \succ 0,} \end{array} \right.$$

where $*_{21} = -(AQ + BY)$, $\quad *_{31} = -(SQ + RY)$, $S = [C^\top \ 0]^\top$, and $R = [0 \ w_r]^\top$.

Set $\hat{P} = \hat{Q}^{-1}$, $K_r = \hat{Y}\hat{P}$, $r[k] = K_r[x[k] - \hat{x}(\hat{r}[k])] + \hat{r}[k]$ in (8)*. Then, closed system becomes*

$$x[k+1] \quad = \quad G[x[k] - \hat{x}(\hat{r}[k])] + \hat{x}(\hat{r}[k]), \quad (14)$$

where $G = A + BK_r$, $\hat{x}(\hat{r}[k]) = (\mathtt{I} - A)^{-1} B\hat{r}[k]$, and we have

$$G^\top PG - P + \frac{1}{\varepsilon}(S + RK_r)^\top (S + RK_r) \prec 0. \quad (15)$$

Moreover, if constraint (10) *is satisfied for a given initial state x_0 of system* (14) *and constant reference input $\hat{r}[k] \equiv r^*$, then we have*

$$\tilde{J}(\{r[k; r^*]\}_{k=0}^\infty; x_0, r^*) \leq \varepsilon |x_0 - \hat{x}(r^*)|^2. \quad (16)$$

For the closed loop system (14), the MAS $\tilde{\Omega}_\infty(0) = \{e_0 : \ LG^k e_0 \in \mathcal{Z}, \ k \in \mathbf{Z}_+\}$ is computed, and RG determines $\hat{r}[k]$ when $x[k]$ is given. More precisely, suppose that the constant reference input r^* is assigned. Let $r(\lambda) = (1 - \lambda)r_0 + \lambda r^*$ for $\lambda \in [0, 1]$, where r_0 is the reference input given in assumption (see blow (9)). When $x[k]$ is given, RG determines the maximum $\lambda_M \in [0, 1]$ such that $x[k] \in \Omega_\infty(r(\lambda_M))$ and set $r[k] = r(\lambda_M)$.

The structure of our proposing control systems is shown in Figure 2, where $K_f = (\mathtt{I} - A)^{-1} B = [0.00052 \ 0 \ 0 \ 0]^\top$ and $K_r = \hat{Y}\hat{Q}^{-1} = [-410.63 \ \ -183.34 \ \ -1.81 \ \ 1.01]$, where $(\hat{\varepsilon}, \hat{Q}, \hat{Y})$ is the optimal solution of (LMI 1).

4. Simulation Result

The simulation result of proposing control scheme is shown in Figures 3 and 4. In Figure 3, we show the reference input $r[k]$ and the output $y[k]$ of nominal servo system by solid lines when we use the proposing method (that is, the RG with an outer feedback). To show the validity of adopting the RG with outer feedback, we also show the reference input $r[k]$ and the output $y[k]$ by dotted lines when we apply the RS method. Note that the reference input $r[k]$ for the RS method is the optimal solution of (QP1).

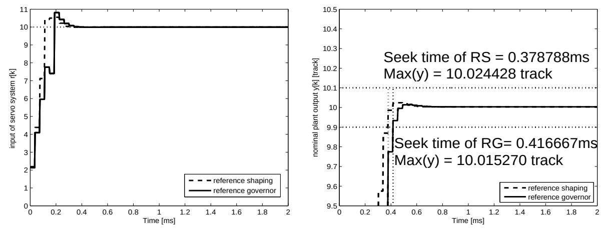

Figure 3. Reference input $r[k]$ of servo system and output $y[k]$ of nominal plant P_m.

In Figure 4, we show the control input $u[k]$ of the nominal servo system and the control input $u_f[k]$ of the real servo system by the solid line when the proposing method is applied. At Figure 4 left, the dashed line denotes $u[k]$ when the RS method is used. From Figures 3 and 4, we can see that the method using the RG with outer feedback gives a good approximation of the optimal solution of (QP1).

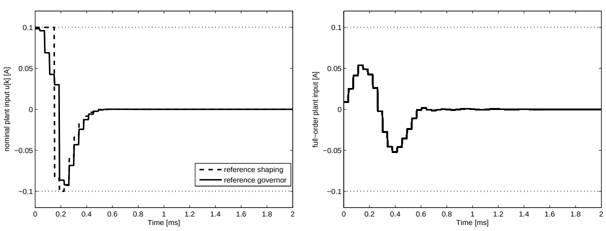

Figure 4. Control input $u[k]$ of the nominal servo system and and control input $u_f[k]$ of full-order plant P_f.

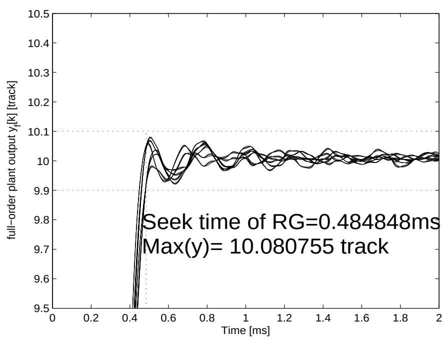

Figure 5. Output $y_f[k]$ of full-order plant P_f.

In Figure 5, we show the output $y[k]$ for 18 full-order perturbed models P_f when the proposing method is applied. Overshoot of output $y_f[k]$ of full-order plant P_f is about 0.08[track], and the seek time is about 0.48 [ms], and is shorter by 10% than seek time in paper [4].

5. Conclusion

In this paper, we propose to use combination of the reference governor with outer feedback and the model following control scheme for HDD benchmark problem. We achieved the shortest seek time for the benchmark problem using a single rate control. In this paper, we neglect the disturbance because of the benchmark problem setting. But the suppression of disturbance is needed for real applications.

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