2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Amplitude death in a pair of time-delayed chaotic oscillators coupled by a static connection

Luan Ba Le, Keiji Konishi, Hideki Kokame and Naoyuki Hara

Department of Electrical and Information Systems
Graduate School of Engineering, Osaka Prefecture University
1-1 Gakuen-cho, Nakaku, Sakai, Osaka, 599-8531 Japan
Email: konishi@eis.osakafu-u.ac.jp

Abstract—Amplitude death, the coupling induced stabilization of unstable fixed points, is observed in a pair of time-delayed chaotic oscillators coupled by the static connection. The cluster treatment of characteristic roots methodology allows us to estimate the boundary curves of the death region in a parametric space. This region can be utilized to streamline the design of the long delay times appropriately for inducing amplitude death. The analytical results are verified by electronic circuit experiments.

1. Introduction

Amplitude death, a diffusive connection induced stabilization of unstable fixed points in coupled oscillators, has been a subject of extensive investigation in the last 15 years [1, 2]. Aronson *et al.* analytically investigated death phenomenon for two coupled nonlinear oscillators [2]. They reported that death never occurs in coupled identical oscillators. Reddy, Sen, and Johnston showed that a time-delayed coupling effect, which exists owing to the finite speed of data propagation, is able to induce the amplitude death even in coupled identical oscillators [3]. Their report has considerably intrigued in the field of nonlinear physics [4]. Atay showed that the distributed time-delay connections facilitate amplitude death [5]. Konishi *et al.* reported that the multiple delay [6] and time-varying delay [7] connections can also facilitate amplitude death in coupled oscillators.

In order to use death in practical situations, one has to design a mutual connection that induces death. However, one inevitably confronts two problems for such a design. The first problem is how to select the type of mutual interactions and how to determine the coupling parameters. The second one is how to deal with high-dimensional oscillators. The previous paper provided a solution for the above problems [8] and, in particular, focused on the amplitude death in a pair of time-delayed chaotic oscillators [9] coupled by three types of mutual interactions: static connections, dynamic connections, and delayed connections.

Since time delay systems have been widely employed to model undesirable nonlinear phenomena such as chatter and chaos in metal cutting tools [10, 11], the stabilization of time-delay systems has been an important issue for engineering applications. The previous paper concluded that the amplitude death can occur with the dynamics and delayed connections, but cannot with the static one [8]. Although the dynamic and delayed connections are not easy to implement in experimental situations due to their complicated structure, the static connection has a simple structure. Therefore, the static one is the best solution from a cost standpoint.

This work was partially supported by NEC C&C Foundation.

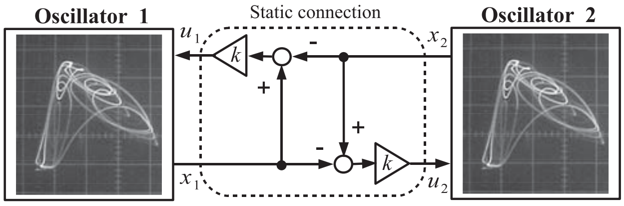


Figure 1: Block diagram of a pair of oscillators (1) coupled by connection (2).

The present paper theoretically and analytically considers the stability of a pair of non-identical time-delayed oscillators coupled by the static connection. The traditional procedure for the stability analysis is difficult to employ for deriving the boundary curves of death, since its characteristic equation includes a cross-talk term between the non-identical delays. To overcome this difficulty, the present paper applies the methodology proposed by Sipahi and Olgac [12] to the characteristic equation. This methodology allows us to derive the boundary curves of death which are useful to design the delay times without trial-and-error testing. Furthermore, our theoretical results are verified by electronic circuit experiments.

2. Time-delayed chaotic coupled oscillators

Let us consider the pair of non-identical scalar time-delayed chaotic oscillators (see Fig. 1) [8]:

$$\begin{cases} \dot{x}_1 &= f(x_{1\tau_1}) - \alpha x_1 + u_1 \\ \dot{x}_2 &= f(x_{2\tau_2}) - \alpha x_2 + u_2 \end{cases}, \tag{1}$$

where $x_{1,2} \in \mathbb{R}$ and $u_{1,2} \in \mathbb{R}$ are the system states and coupling signals, respectively. $x_{1\tau_1,2\tau_2} := x_{1,2}(t - \tau_{1,2})$ are the delayed states, $\tau_{1,2} \geq 0$ are the delay times and $\alpha > 0$ is a parameter, $f : \mathbb{R} \to \mathbb{R}$ represents a nonlinear function. The symbol $\mathbb{R}$ denotes the set of real numbers. These oscillators are coupled by the static connection described as

$$u_{1,2} = k(x_{1,2} - x_{2,1}), \tag{2}$$

where $k \in \mathbb{R}$ is the coupling strength.

Each individual oscillator without coupling (i.e., $u_{1,2} \equiv 0$) has the fixed points

$$x^* : 0 = f(x^*) - \alpha x^*. \tag{3}$$

Throughout this paper, it is assumed that there is one unstable fixed point. The location of the fixed point x^* never changes even by coupling; in other words, the static connection changes only the stability of the point. The characteristic equation of linearized system at the fixed point x^* can be rewritten as

$$a_0(\lambda) + a_1(\lambda)e^{-\lambda\tau_1} + a_2(\lambda)e^{-\lambda\tau_2} + a_3 e^{-\lambda(\tau_1+\tau_2)} = 0, \tag{4}$$

where

$$\begin{aligned} a_0(\lambda) &:= s^2 + 2(\alpha - k)\lambda + (\alpha - k)^2 - k^2, & (5)\\ a_1(\lambda) &= a_2(\lambda) := -\beta\lambda - \beta(\alpha - k), & (6)\\ a_3 &:= \beta^2, & (7)\\ \beta &:= \{\mathrm{d}f(x)/\mathrm{d}x\}_{x=x^*}. & (8) \end{aligned}$$

The nonlinear function f is given by

$$f(x) = \begin{cases} 0 & \text{if } x \leq -4/3 \\ -1.8x - 2.4 & \text{if } -4/3 < x \leq -0.8 \\ 1.2x & \text{if } -0.8 < x \leq 0.8 \\ -1.8x + 2.4 & \text{if } 0.8 < x \leq 4/3 \\ 0 & \text{if } x > 4/3 \end{cases}, \tag{9}$$

as a typical case study [13]. The three fixed points of individual oscillator located at the intersection of $f(x)$ and αx: $x^* = \pm 6/7$ and 0. The slopes of $f(x)$ at x^* can be estimated as $\beta(\pm 6/7) = -1.8$ and $\beta(0) = 1.2$. The parameters are fixed at $\alpha = 1$ and $k = -2$. Additionally, characteristic equation (4) in the absence of delays (i.e., $\tau_1 = \tau_2 = 0$) has two roots -6.79 and -2.80, thus the number of unstable roots is zero.

3. Death region and boundary curves

The cluster treatment of characteristic roots (CTCR) paradigm is capable of deriving the boundary curves of a stability region in (τ_1, τ_2) space [12]. To simplify the stability analysis, we only identify the roots lied on the imaginary axis: $\lambda = j\lambda_{\mathrm{Im}}$, where the symbol j is denoted as $j = \sqrt{-1}$. The stability posture in (τ_1, τ_2) space is shown in Fig. 2, where the amplitude death region is symbolized by Ω. Every point on the boundary curves correspond to the purely imaginary root. The numbers of unstable roots are stated in several regions.

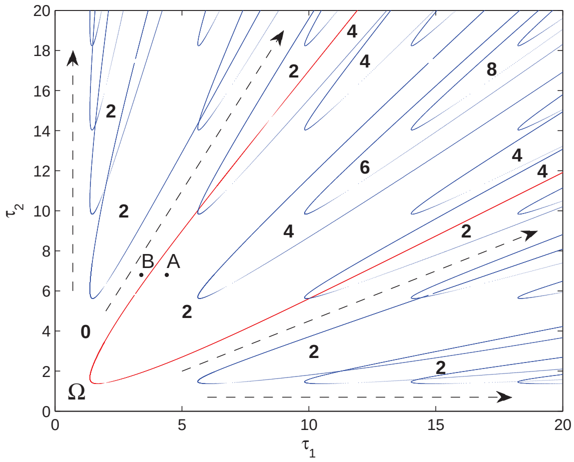


Figure 2: Death region Ω and the boundary curves of stability.

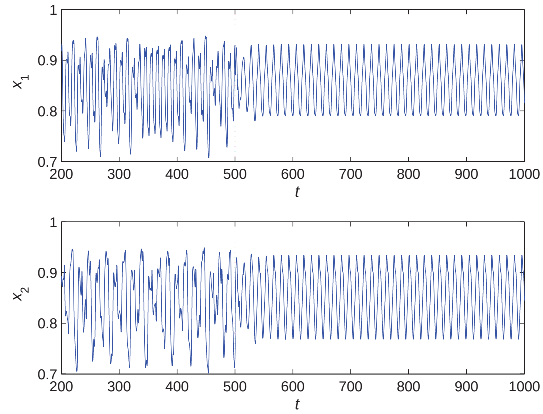


Figure 3: Time series data $x_{1,2}(t)$ of coupled oscillators (numerical simulation): (A) $\tau_1 = 4.4$, $\tau_2 = 6.8$.

Figures 3 and 4 illustrate the time series data of the two non-identical oscillators for the two parameter sets: (A) $\tau_1 = 4.4$, $\tau_2 = 6.8$; (B) $\tau_1 = 3.4$, $\tau_2 = 6.8$. These sets are indicated in Fig. 2. For the parameter set (A), the two identical oscillators without coupling behave chaotically until $t = 500$, and become periodical after coupling. On the other hand for the parameter set (B), the two individual oscillators before coupling has the chaotic motion. After coupling the states $x_{1,2}(t)$ converge on $x^*_{1,2}$.

From the death region on a wide scale as shown in Fig. 5, it seems that one may choose (τ_1, τ_2) arbitrarily large to obtain death, if they are within the strips (see the dotted lines in Fig. 5), $\tau_{1,2} = 2\tau_{2,1} + \xi$, $0 \leq \xi \leq 2$. The static connection can induce death over a long delay-time region Ω. However, we do not still have the proof for this fact.

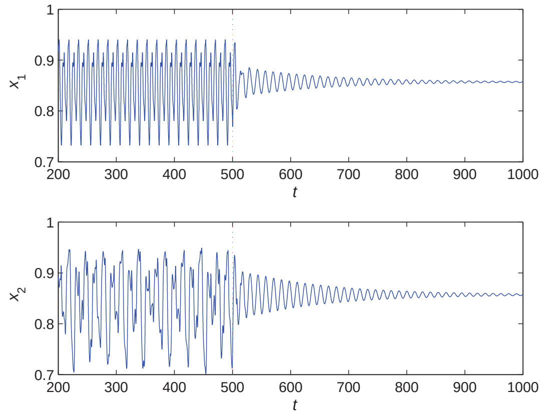


Figure 4: Time series data $x_{1,2}(t)$ of coupled oscillators (numerical simulation): (B) $\tau_1 = 3.4$, $\tau_2 = 6.8$.

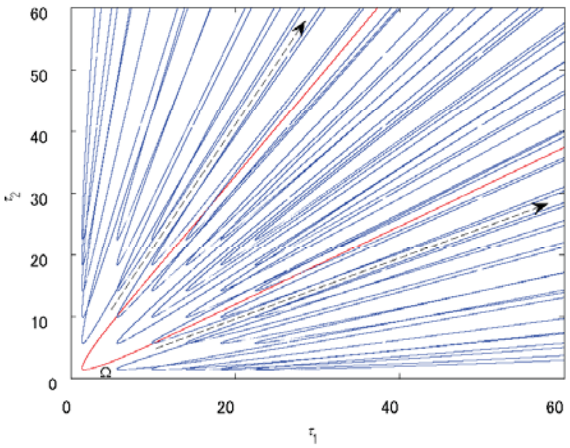


Figure 5: Death region and the boundary curves of stability on a wide scale.

4. Electronic circuit experiments

The two chaotic oscillators coupled by the static connection are sketched in Fig. 6. The delay units employ the bucket brigade delay line MN3011 (Panasonic) to generate the delayed signal [8] and the nonlinear function is implemented by the three opamps and the two diodes.

Here, x_i denotes the voltage of the i-th oscillator. The boxes labeled $-\tau_i$ and f are the time delay unit and the nonlinear function unit, respectively. These oscillators are governed by

$$\begin{cases} C_n\dot{x}_1(t) &= \dfrac{1}{R_n}\left\{f(x_1(t-\tau_1)) - x_1(t)\right\} + u_1(t) \\ C_n\dot{x}_2(t) &= \dfrac{1}{R_n}\left\{f(x_2(t-\tau_2)) - x_2(t)\right\} + u_2(t) \end{cases}, \qquad (10)$$

where R_n, C_n are a resistor and capacitor, respectively. The coupling terms $u_{1,2}(t)$ are the currents from the connection circuit to the oscillators. The static connection is described by

$$u_{1,2}(t) = -\frac{1}{R}\left\{x_{1,2}(t) - x_{2,1}(t)\right\}. \qquad (11)$$

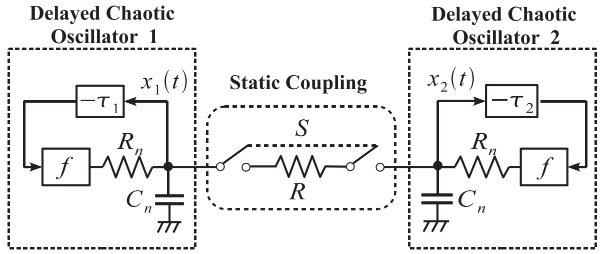


Figure 6: Two time-delay chaotic oscillators coupled by a static connection.

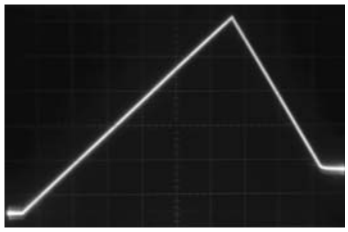


Figure 7: Nonlinear function $f(x)$. Horizontal axis: x (1V/div); vertical axis: $f(x)$ (1V/div).

It is sufficient to treat the above circuits as the dimensionless oscillators (1) on the following relations:

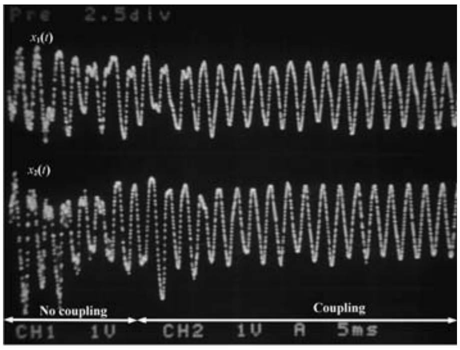
$$\tilde{t} := \frac{t}{R_nC_n},\ \tilde{\tau}_{1,2} := \frac{\tau_{1,2}}{R_nC_n},$$
$$\dot{x}_{1,2} := \frac{dx_{1,2}(\tilde{t})}{d\tilde{t}},\ x_{1,2} := x_{1,2}(\tilde{t}),\ x_{1,2\tau} := x_{1,2}(\tilde{t}-\tilde{\tau}_{1,2}),$$
$$u_{1,2} := u_{1,2}(\tilde{t}),\ k := -\frac{R_n}{R}.$$

These relations show that circuit equation (10) is identical to equation (1) with $\alpha = 1$. The input-output characteristic of the nonlinear function unit is shown in Fig. 7. From the characteristic, the fixed point $x^* = 2.2$ V and the slope $\beta(x^*) = -1.8$ are approximately estimated. The circuit parameters and the coupling resistor are fixed at $R_n = 1.0$ kΩ, $C_n = 1.0$ μF, and $R = 0.5$ kΩ. We consider two parameter sets: (A) $\tau_1 = 4.4$ ms, $\tau_2 = 6.8$ ms; (B) $\tau_1 = 3.4$ ms, $\tau_2 = 6.8$ ms. In dimensionless oscillators (1), $\tilde{\tau}_{1,2}$ for the two parameter sets correspond to (A) $\tilde{\tau}_1 = 4.4$, $\tilde{\tau}_2 = 6.8$; (B) $\tilde{\tau}_1 = 3.4$, $\tilde{\tau}_2 = 6.8$, which indicated as the points A and B on Fig. 2.

Figures 8(a) and 8(b) show that the individual oscillator at the parameter set (A) exhibits chaotic behavior. When the switch S shown in Fig. 6 is closed, the two individual circuits are connected by the coupling resistor. After S is closed, the chaotic behavior changes to a periodic motion, but death is not observed as shown in Fig. 8(c). For the parameter set (B), the individual oscillator exhibits chaotic behavior (see Figs. 9(a) and 9(b)). The oscillators behave chaotically until S is closed. As shown in Fig. 9(c), the states $x_{1,2}(t)$ gradually converge on $x^*_{1,2}$: the am-

(a)

(b)

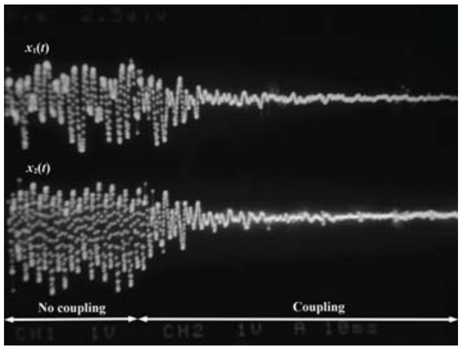


(c)

Figure 8: Experimental verification at parameter set (A): (a) chaotic behavior of oscillator 1; (b) chaotic behavior of oscillator 2 (horizontal axis: $x(t-\tau)$ (0.5V/div), vertical axis: $x(t)$ (0.5V/div)); (c) time series data $x_{1,2}(t)$ (V) just before and after coupling.

(a)

(b)



(c)

Figure 9: Experimental verification at parameter set (B): (a) chaotic behavior of oscillator 1; (b) chaotic behavior of oscillator 2 (horizontal axis: $x(t-\tau)$ (0.5V/div), vertical axis: $x(t)$ (0.5V/div)); (c) time series data $x_{1,2}(t)$ (V) just before and after coupling.

plitude death occurs, after S is closed. This experimental study confirms that the estimated death region by analytical insight in the preceding section agrees well with the electronic circuit experiments.

5. Conclusion

The present paper investigated the static connection induced amplitude death in the two non-identical time-delayed chaotic oscillators. The precise shape of the death region derived by the CTCR paradigm leads to the conclusion that even the relatively long delay times can induce the amplitude death. This perspective seems to be important, because systems with large delay are frequently met in various fields of applications. Moreover, our analytical results are experimentally confirmed by the real electronic oscillators.

References

[1] Y. Yamaguchi and H. Shimizu, Physica D, vol. 11, pp. 212–226, 1984.

[2] D. Aronson, G. Ermentout, and N. Kopell, Physica D, vol. 41 pp. 403–449, 1990.

[3] D. Reddy, A. Sen, and G. Johnston, Phys. Rev. Lett., vol. 80, pp. 5109–5112, 1998.

[4] S. Strogatz, Nature, vol. 394, pp. 316–317, 1998.

[5] F. M. Atay, Phys. Rev. Lett., vol. 91, pp. 94101, 2003.

[6] K. Konishi, H. Kokame, and N. Hara, Phys. Lett. A, vol. 374, pp. 733–738, 2010.

[7] K. Konishi, H. Kokame, and N. Hara, Phys. Rev. E, vol. 81, pp. 016201, 2010.

[8] K. Konishi, K. Senda, and H. Kokame, Phys. Rev. E, vol. 78, pp. 056216, 2008.

[9] J.D. Farmer, Physica D, vol. 4, pp. 366–393, 1982.

[10] F. Moon, Dynamics and Manufacturing Processes (J Wiley, New York), 1998.

[11] G. Radons and R. Neugebauer, Nonlinear Dynamics of Production Systems (Wiley, New York), 2004.

[12] R. Sipahi and N. Olgat, Systems & Control Letters, vol. 55, pp. 819–825, 2006.

[13] D.V. Senthilkumar and M. Lakshamanan, Phys. Rev. E, vol. 71, pp. 016211, 2005.