2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Switching Control between Stable Periodic Vibrations in a Nonlinear MEMS Resonator

Atsushi YAO† and Takashi HIKIHARA‡

†‡Department of Electrical Engineering, Kyoto University
Katsura, Nishikyo, Kyoto, 615-8510 Japan
Email: †yao@dove.kuee.kyoto-u.ac.jp, ‡hikihara.takashi.2n@kyoto-u.ac.jp

Abstract—Doubly clamped mechanical resonator substantially exhibits bistable, jump, or hysteretic response when the nonlinear characteristics appear. Recently, switching between stable coexisting periodic vibrations has been studied in a micro electromechanical resonator with hysteresis at a single excitation frequency. In this paper, we focus on switching control between stable periodic vibrations. The availability of delayed feedback control is also shown by numerical simulations.

1. Introduction

Nonlinear characteristics of a mechanical resonator with doubly clamped end have been known to produce bistable, jump, or hysteretic responses [1]. The doubly clamped beam has the fixed supports inducing a membrane tension at finite deflection. It causes a hardening nonlinear stiffness modeled by cubic term [1].

Micro Electro Mechanical Systems (MEMS) resonators also exhibit nonlinear response at large amplitude excitation [2]. It is reported that these nonlinear MEMS resonators can be applied for a mechanical memory elements [3]. The doubly clamped beam structure also has a potential to be a memory switching between two stable vibrations [3]. The operation of memory requires deterministic control methods to read and write logic states between coexisting vibrations.

Recently, Unterreithmeier *et al.* suggested a switching control between two stable periodic vibrations in a doubly clamped beam resonator at a single excitation frequency [4]. They applied a radio-frequency (RF) pulse to control two stable states. They showed clear possibilities of switching states.

Their results motivated us to find other appropriate continuous control method for the practical use in MEMS resonators in order to achieve the deterministic control by feedback. From this standpoint, we apply a delayed feedback control method (DFC) [5] for switching the coexisting vibrations. In the following sections, the idea of control and the possibilities will be confirmed by numerical simulations.

2. MEMS resonator and its steady state

Figure 1 shows a schematic diagram of MEMS resonator with a doubly clamped beam [6]. The motion of the first mode of the doubly clamped beam is given by the following non-dimensionalized equation:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{1}{Q}\frac{\mathrm{d}x}{\mathrm{d}t} + x + \alpha_3 x^3 \quad = \quad k_{\mathrm{e}} \cos \omega t, \qquad (1)$$

where x denotes the resonator displacement, ω the driving frequency, Q $(= 1.2 \times 10^5)$ the quality factor, α_3 $(= 1.273 \times 10^{-5})$ the coefficient of cubic correction to the linear restoring force, and k_{e} $(= 9.818 \times 10^{-5})$ the amplitude of the driving force. The parameter settings are due to Ref. [4]. Fig. 2 shows the frequency response curve of the resonator, which has two stable periodic vibrations and an unstable periodic vibration. In this figure, the solid (red) and broken (aqua) lines are the stable solutions and the unstable solution, respectively. When we consider the discrete initial condition plane sampled by excitation period, these two stable solutions possess their basins completely separated by a stable manifold of unstable solution. We aim for smooth switching between two stable periodic vibrations at a single excitation frequency. In the following calculations, the driving frequency ω_1 $(= 1.0001245210)$, owing to Ref. [4], is used as shown in Fig. 2. Hereafter the symbol A denotes non-resonance solution and the symbol B resonance solution.

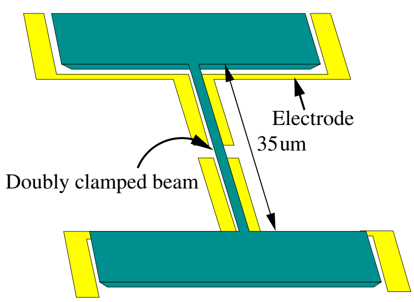


Figure 1: Schematic diagram of MEMS resonator with doubly clamped beam.

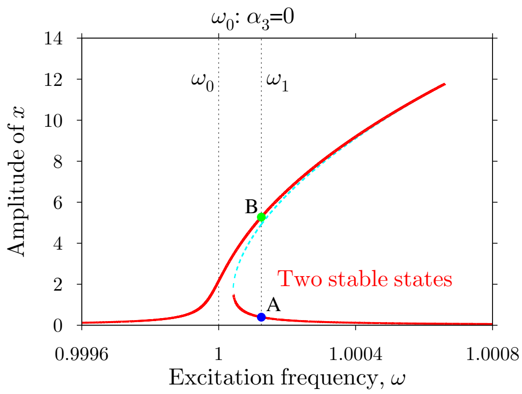


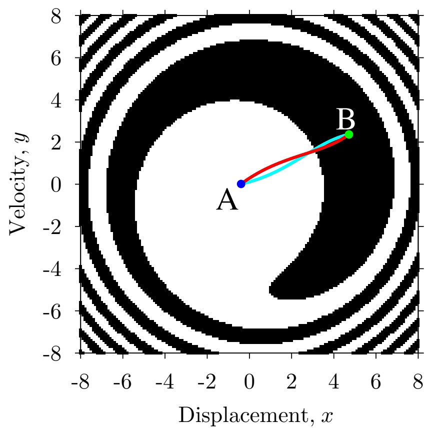
Figure 2: Frequency response curve: The solid (red) and broken (aqua) lines correspond to two stable solutions and an unstable solution, respectively.

The system (1) with control signal $u(t)$ is given by

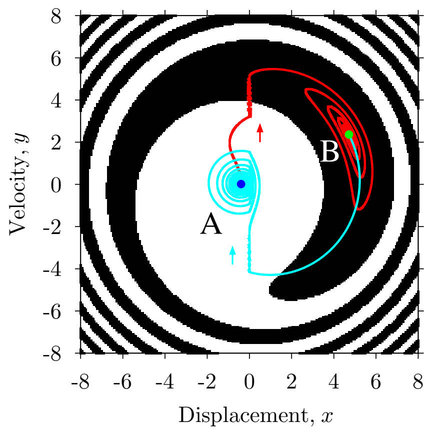
$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = y, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{y}{Q} - x - \alpha_3 x^3 + k_{\mathrm{e}} \cos \omega t + u(t), \end{cases} \tag{2}$$

where y denotes the resonator velocity. Fig. 3 depicts the initial value phase plane at $\omega = \omega_1$ without control signal in Eq. (2). In the figure, the white and black indicate the basins of two stable solutions [7]. Every initial state obviously corresponds to the convergence to A (white) and B (black).

While the system is controlled by $u(t)$, the basin, which is obtained by estimating the convergence without $u(t)$, has no practical meaning for understanding their response. Therefore, we consider the trajectories of the operating point under control for the estimation of the ability of control in the sections 3 and 4.



(a) Switching by RF pulse control



(b) Switching by DFC

Figure 3: Switching between stable periodic vibrations: The basins of attraction corresponding to the two states are separated by the manifold of the unstable solutions. The red and aqua lines show the switching from A to B and from B to A, respectively.

3. Pulsive control at radio frequency

In this section, we confirm the previously achieved results by Unterreithmeier *et al.* [4]. They applied a radio-frequency (RF) pulse to control signal $u(t)$ [4] :

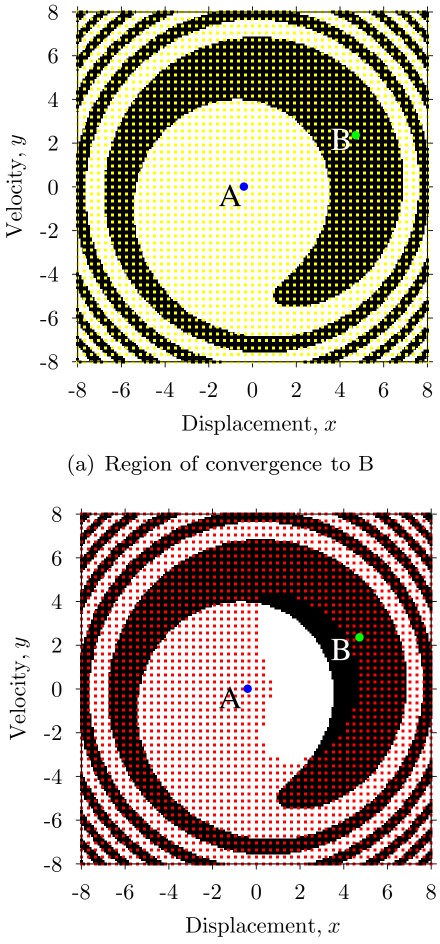
$$u(t) = k_{\mathrm{p}} \cos(\omega t + \varphi), \tag{3}$$

where k_{p} $(= 1.767 \times 10^{-3})$ denotes the amplitude, and φ the phase of the RF pulse. The phase of the RF pulse φ is set at 77° (or 233°) for the control to A (or B). The RF pulse with adjustable duration $t_{\mathrm{p}} (= 6470)$ is used. The RF pulse control possibly switches the state between the stable vibrations when the amplitude, phase, and duration of the RF pulse are adjusted. Fig. 3(a) shows the results controlled by the RF pulse. The red and aqua lines show the loci of map with the RF pulse from A to B and from B to A, respectively. Unterreithmeier *et al.* employ their extended knowledge to perform fast switching between the stable states no longer bound by relaxation times. The duration of the RF pulse was about 1030 periods.

The amplitude of the RF pulse seems too intense for the beam to avoid the stress. From the engineering point of view, the control is an open loop without any guarantee to the result. Therefore, we should seek another possible closed loop way of control.

4. Delayed feedback control

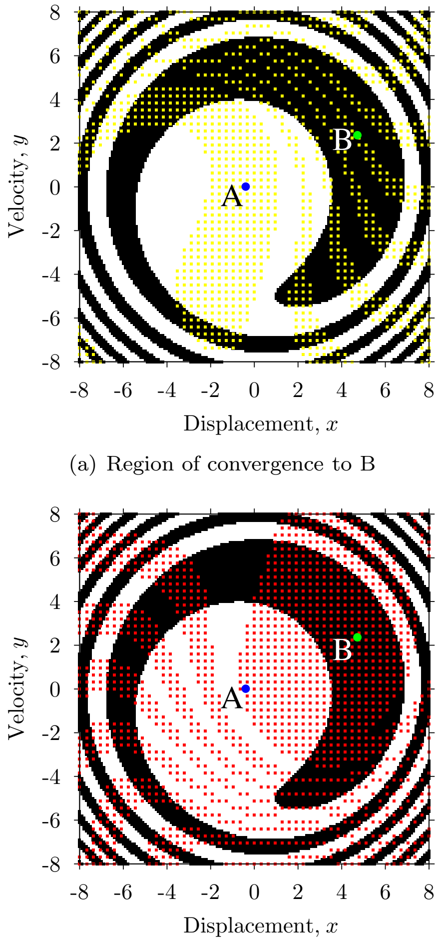
Pyragas proposed a delayed feedback control (DFC) that can be used for controlling chaos [8] in continuous-time system. It was experimentally [9, 10], numerically [11, 12], and theoretically [12] confirmed that DFC can stabilize unstable periodic orbits embedded



(a) Region of convergence to B

(b) Region of convergence to A

Figure 4: Initial value dependence of convergence by DFC: Yellow and Red grid points show the region of convergence to B and A, respectively.



(a) Region of convergence to B

(b) Region of convergence to A

Figure 5: Initial value dependence of convergence by RF pulse control: Yellow and Red grid points show the region of convergence to B and A, respectively.

in the chaotic attractor. We apply DFC to switching between stable periodic vibrations because DFC does not need any exact model except the periodic of the target solution. DFC employs the control signal as

$$u(t) = K\{x(t) - \mathrm{sgn}(x(nT)) \cdot x(t - T)\}, \qquad (4)$$

where T $(= 2\pi/\omega_1)$ denotes the period, K $(= -0.0001)$ the feedback gain, n natural number, at $t \in [nT, (n+1)T)$. $\mathrm{sgn}(x(nT))$ is defined by

$$\mathrm{sgn}(x(nT)) \equiv \begin{cases} +1 & (x(nT) \geq 0), \\ -1 & (x(nT) < 0), \end{cases} \qquad (5)$$

when the states are requested to come towards B. The following function

$$\mathrm{sgn}(x(nT)) \equiv \begin{cases} +1 & (x(nT) \leq 0) \\ -1 & (x(nT) > 0) \end{cases} \qquad (6)$$

is set bound for A. Fig. 3(b) shows the controlled results in the phase space by DFC. The red and aqua lines show the loci of map with DFC from A to B and from B to A, respectively. It should be noted that the states slowly transition from A to B or from B to A. It took around 150,000 periods before converging to another stated. The switching speed of RF pulse method was about 145 times that of DFC.

5. Initial value dependence and stress of beam

DFC is a continuous closed loop control method. Here, we investigate the initial value dependence of the convergence. The results are shown in Fig. 4. In Fig. 4(a), yellow grid points indicate the initial states that converge to B. In the initial state plane, DFC can control the vibration to B without exception. In Fig. 4(b), red grid points indicate the initial states of convergence to A. There still remain the uncontrollable initial states.

Figure 5 shows the initial value dependence by RF

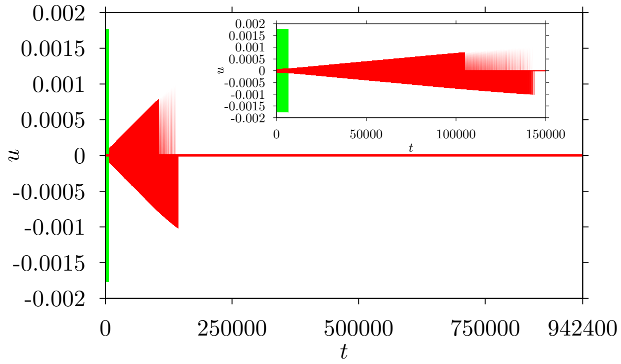


Figure 6: Time evolution of control signal from A to B: Red and green lines display the control signal of DFC and the RF pulse control, respectively. (Inset) Extension of the initial part.

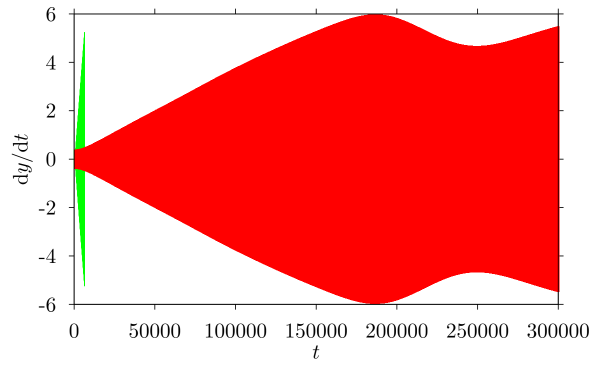


Figure 7: Time evolution of stress applied to beam from A to B: Red and green lines correspond to stress by DFC and the RF pulse control, respectively.

pulse control. In Fig. 5(a), yellow grid points indicate the initial states of convergence to A and red grid points in Fig. 5(b) to B. These results show that the RF pulse control can not insure the convergence exactly in the initial state plane for both directions. The complicated convergence area appears in the simple basin. The background mechanism is not clear at this moment. Therefore, the RF pulse control is weak for application from the standpoint of control engineering.

Figure 6 depicts the time evolution of the control signal from A to B. The red and green plots correspond to the control signal of DFC and the RF pulse control, respectively. The result shows that the control signal of DFC disappears after the establishment of switching. It implies the converged state is exactly one of the stable periodic vibrations in the resonator.

On the other hand, the amplitude of the RF pulse is extremely strong so that the stress on the beam becomes large. It implies that the beam may be broken if the control is applied repeatedly. We can roughly estimate the stress F applied to the beam by $\mathrm{d}y/\mathrm{d}t$. That is, the stress is estimated at the value in proportion to $\mathrm{d}y/\mathrm{d}t$. Fig. 7 shows one of the estimated results in the case for switching from A to B. The red and green lines are the stress by DFC and RF pulse control, respectively. The peak value of the stress seems almost the same in two control methods. We are currently investigating the effects to stress in more details.

6. Summary

In this paper, we have analyzed the switching control between two periodic vibrations in a nonlinear MEMS resonator. The availability of the delayed feedback control is numerically confirmed. We believe this result can provide a vital clue to development of MEMS active memory.

Acknowledgments

One of the authors (AY) would like to express his gratitude to Mr. Unterreithmeier for fruitful comments through wire communication. The authors acknowledge the partially supports by Global COE of Kyoto University and Regional Innovation Cluster Program "Kyoto Environmental Nanotechnology Cluster".

References

[1] J. M. T. Thompson and H. B. Stewart, "Nonlinear Dynamics and Chaos," John Wiley & Sons, 1986.

[2] V. Kaajakari, "Practical MEMS," Small Gear Publishing, 2009.

[3] R. L. Badzey *et al.*, *Appl. Phys. Lett.*, vol.85, pp.3587–3589, 2004.

[4] Q. P. Unterreithmeier *et al.*, *Phys. Rev. B*, vol.81, 241405, 2010.

[5] K. Pyragas, *Phys. Lett. A*, vol.170, pp.421–428, 1992.

[6] Q. P. Unterreithmeier *et al.*, Nature (London), vol.458, pp.1001–1004, 2009.

[7] J. Guckenheimer and P. Holmes, "Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields," Springer-Verlag, p. 34, 1983.

[8] E. Ott *et al.*, *Phys. Rev. Lett.*, vol.64, pp.1196–1199, 1990.

[9] T. Hikihara and T. Kawagoshi, *Phys. Lett. A*, vol.211, pp.29–36, 1996.

[10] K. Yamasue *et al.*, *Phys. Lett. A*, vol.373, pp.3140–3144, 2009.

[11] K. Yamasue and T. Hikihara, *Rev. Sci. Instrum.*, vol.77, 053703, 2006.

[12] T. Hikihara and Y. Ueda, *Chaos*, vol.9, pp.887–892, 1999.