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Bifurcation Phenomenon from Stick-slip Vibration to Slip Vibration in the Non-autonomous System with Dry Friction

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Abstract—In the non-autonomous system with dry friction, the bifurcation phenomena are observed upon varying parameters such as the coefficient of friction and the amplitude of the external force. In this paper, we discuss the bifurcation phenomenon from stick-slip vibration to slip vibration in the non-autonomous system with dry friction. First, we explain the physical model of the non-autonomous system with dry friction. Next, we define the Poincaré map by focusing on the local sections and the time. Finally, we investigate the bifurcation phenomenon.

1. Introduction

Non-autonomous system with dry friction is often observed in the mechanical fields. The typical examples of the system are seismic isolation systems [1], disk brakes [2], dynamic absorbers [3] and rail wheels [4]. It is known that the bifurcation phenomena are observed in these systems upon varying parameters such as the coefficient of friction and the amplitude of the external force [5]- [9]. Thus, it is important to analyze the bifurcation phenomena in a wide parameter area. However, the systems are difficult to analyze due to theirs complexity. Therefore, the physical model was proposed to analyze the system in detail [9]. There are many papers that have analyzed bifurcation phenomena occurring in the physical model. Some papers investigated generating region of chaos and periodic pull-in vibration of harmony vibration. We also proposed a simple stability analysis method for this model [10] and analyze bifurcation phenomena in a wide parameter area [11]. However, there are few papers discuss the bifurcation analysis from stick-slip vibration to slip vibration.

In this paper, we analyze the bifurcation phenomenon from stick-slip vibration to slip vibration in the non-autonomous system with dry friction. First, we explain the physical model of the non-autonomous system with dry friction. Next, we show the Poincaré map by focusing on the local sections and the time. Finally, we clarify the bifurcation phenomenon.

2. System and its behavior

Figure 1 shows the physical model of the non-autonomous system with dry friction, which has single degree of freedom. The following notation is used; mass m, spring constant k, displacement of mass $u(t)$ and velocity $\dot{u}$, gravity g, normal force P, external force $F_0\cos(\omega t)$, belt speed V, and relative velocity $V_{\rm r} = V - \dot{u}$. The motion of equation is described as follows:

$$m\ddot{u}(t) - f_0(V) + ku(t) = F_0\cos(\omega t) \tag{1}$$

During the slip mode ($V_{\rm r} \neq 0$), the friction force can be determined via the friction coefficient $\mu(V_{\rm r})$. The friction force is described as follows:

$$f_0(V_{\rm r}) = \mu(V_{\rm r})m\left(g + \frac{P}{m}\right) \tag{2}$$

where $\mu(V_{\rm r})$ is given by

$$\mu(V_{\rm r}) = \begin{cases} \gamma_0 - \gamma_1 V_{\rm r} + \gamma_3 V_{\rm r}^3, & V_{\rm r} > 0 \\ [-\gamma_0, \gamma_0], & V_{\rm r} = 0 \\ -\gamma_0 - \gamma_1 V_{\rm r} + \gamma_3 V_{\rm r}^3, & V_{\rm r} < 0 \end{cases} \tag{3}$$

where γ_0, γ_1 and γ_3 are decay constants. In particular, γ_0 is the maximum static fricttion force. We use the dimensionless value as follows:

$$\begin{aligned} &\omega_n^2 = \frac{k}{m}, \quad \tau = \omega_n t, \quad x = \frac{u\omega_n}{V}, \quad \nu = \frac{\omega}{\omega_n}, \\ &F = \frac{F_0\omega_n}{kv}, \quad A_0 = \frac{\gamma_0 c}{(V\omega_n)}, \quad A_1 = \frac{\gamma_1 c}{\omega_n}, \\ &A_3 = \frac{\gamma_3 V^2 c}{\omega_n}, \quad V_{\rm r} = 1 - y, \quad c = g + \frac{P}{m}. \end{aligned} \tag{4}$$

Thus, Eq. (1) can be described as follows:

$$
\begin{cases} \dfrac{dx}{d\tau} = y \\ \dfrac{dy}{d\tau} = -x + f_0(y) + F\cos(\nu\tau) \end{cases} \tag{5}
$$

where $f_0(y)$ is given by

$$
f_0(y) = \begin{cases} -A_0 + A_1(1-y) - A_3(1-y^3), & y > 1 \\ [-A_0, A_0], & y = 1 \\ A_0 + A_1(1-y) - A_3(1-y^3), & y < 1 \end{cases} \tag{6}
$$

Figure 2 shows stick-slip vibration of period-1 orbit. Figure 3 shows example of the waveforms of the displacement x and the velocity y. Black dots are the time when the slip mode (kinetic friction force) changes to the stick mode (static friction force). On the other hand, white dots are the time when the stick mode changes to the slip mode. The parameters are fixed as follows: $A_0 = 1.50$, $A_1 = 1.50$, $A_3 = 0.45$, $F = 0.35$.

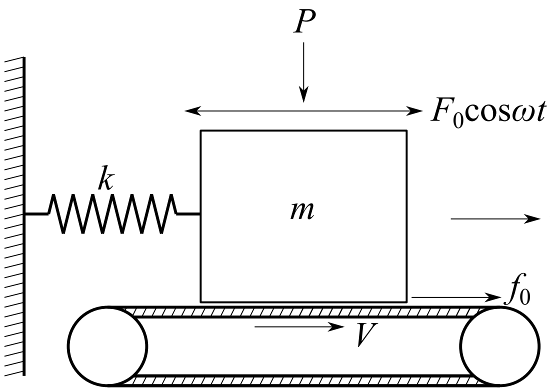


Figure 1: Physical model.

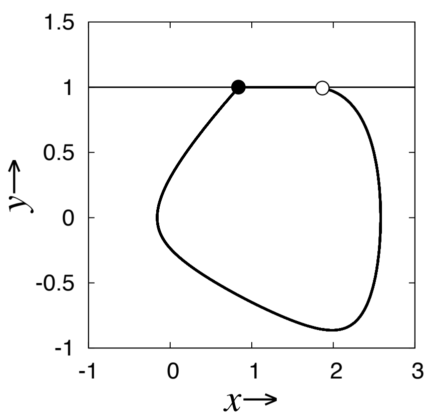


Figure 2: Example of the phase plane.

3. Analysis

3.1. Coexistence phenomenon

Figure 4 shows the phase planes for ν=0.44, 0.45 and 0.46. Figure 5 shows waveforms. Dotted line in Fig. 4(b) and 5(b) is slip vibration. Stick-slip vibration occurs at $\nu = 0.44$. Slip vibration occurs at $\nu = 0.46$. Here, we focus on the bifurcation parameter. Stick-slip and slip vibrations occur depending on the initial value at $\nu = 0.45$. Namely, coexistence phenomenon exists by changing the initial value.

3.2. Return map

We define the Poincaré map T from τ_k to τ_{k+1} for qualitative analysis in the system. Behavior of the system changes from stick mode to slip mode at a time $\tau = \tau_k$. Here, the deplacement x_k and velocity y_k are described as follows at the discrete time τ_k.

$$
x_k = A_0 + F\cos(\nu\tau_k), \tag{7}
$$

$$
y_k = 1. \tag{8}
$$

x_k is the function of the time τ_k and y_k is constant. Thus, the behavior of the system can be assumed as function of the time τ_k. The discretization makes the analysis of the behavior of the system easy. Moreover, we use modulus and function θ_k in order to calculate the fixed point [12]. Then, θ_k and $T(\theta_k)$ are given by:

$$
\theta_k = \frac{\nu}{2\pi}\tau_k \bmod 1, \tag{9}
$$

$$
\theta_{k+1} = T(\theta_k) = \frac{\nu}{2\pi}\tau_{k+1} \bmod 1. \tag{10}
$$

Figure 6 shows example of the return map of period-1 orbit. More detailed discussion is shown in Ref. [12].

3.3. Bifurcation phenomenon from stick-slip vibration to slip vibration

We focus on the return map in Fig. 7. The mapping point disappears at $0.88 < \theta_k < 0.92$. Here, this interval corresponds to slip mode. Therefore mapping point can not be defined.

In the following, we discuss bifurcation phenomenon from stick-slip vibration to slip vibration. When the parameter is $\nu = 0.455$, the slope of the return map for period-1 orbit is 1.0. We conclude that the saddle node bifurcation occurs at $\nu = 0.455$. After the saddle node bifurcation occurs, the fixed point disappears. As a result, the system settles into slip vibration by disappearing coexistence phenomenon of stick-slip and slip vibrations.

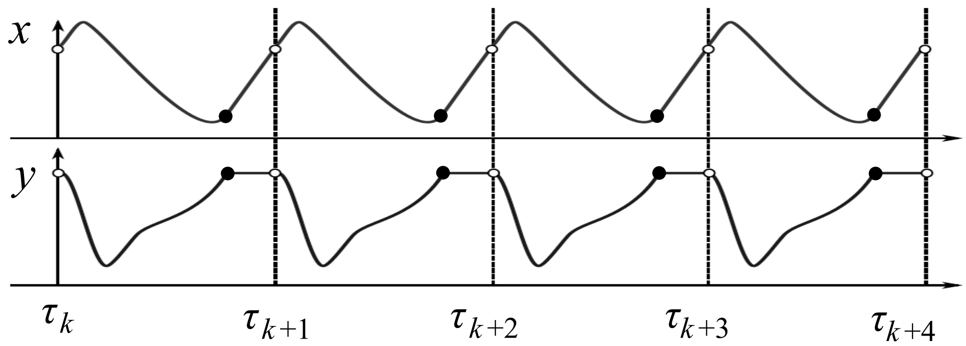
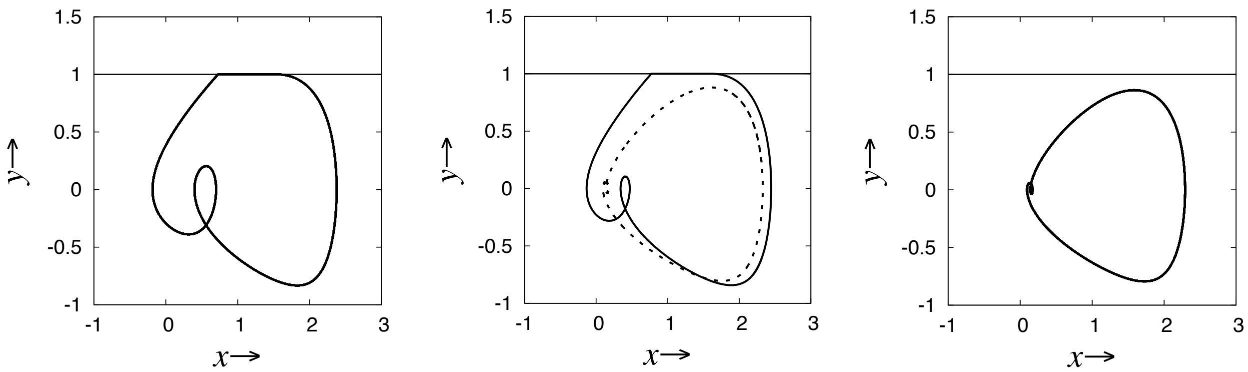
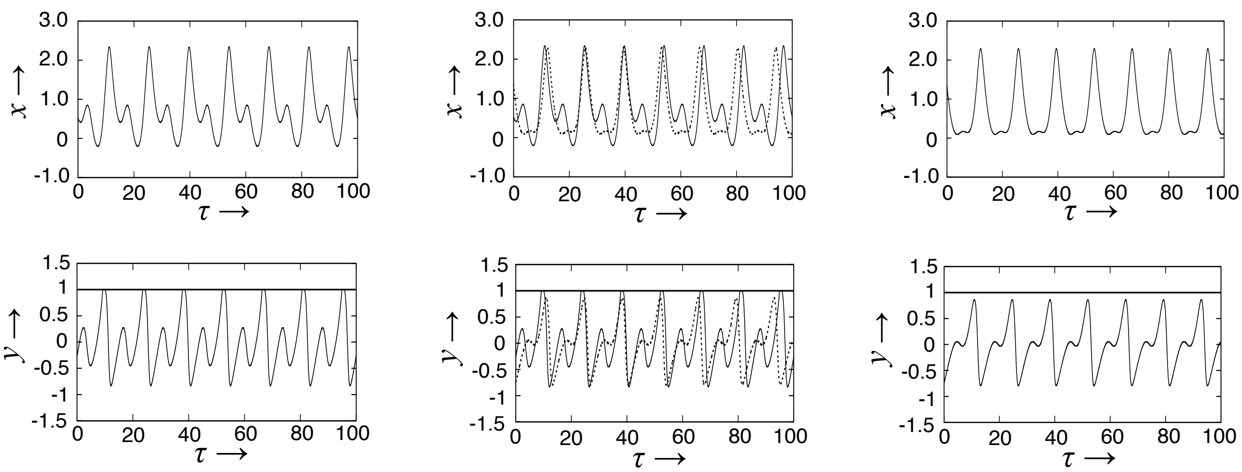


Figure 3: Example of the waveforms.



(a) $\nu = 0.44$ (b) $\nu = 0.45$ (c) $\nu = 0.46$

Figure 4: Phase planes.



(a) $\nu = 0.44$ (b) $\nu = 0.45$ (c) $\nu = 0.46$

Figure 5: Waveforms.

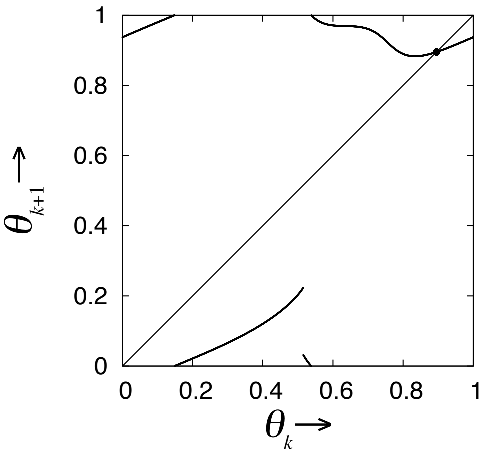


Figure 6: Example of return map (ν = 0.62).

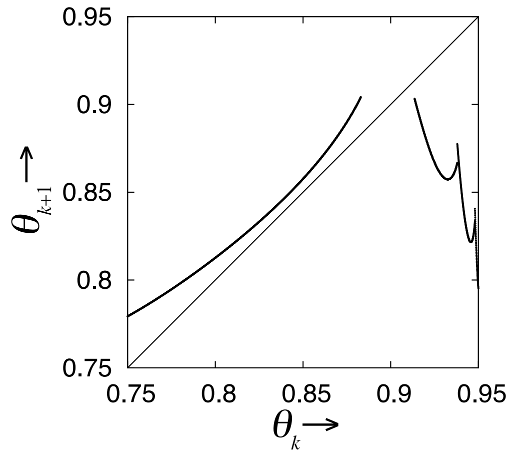


Figure 7: Return map with slip vibration (ν = 0.46).

4. Conclusion

In this paper, we reported the bifurcation phenomenon from stick-slip vibration to slip vibration in the non-autonomous system with dry friction. First, we explained the physical model of the non-autonomous system with dry friction. Next, we defined the Poincaré map by focusing on the local sections and the time. We showed the return map therefore the Poincaré map can be defined as one-dimensional. The mapping point disappears when slip mode is observed. As a result, the saddle node bifurcation caused slip vibration by disappearing coexistence phenomenon of stick-slip and slip vibrations.

5. Acknowledgment

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