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Noise-Induced Synchronization of Chaotic Oscillator by Natural Environmental Noise

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Abstract—Synchronization phenomenon of uncoupled chaotic oscillators with additive common noise input have been investigated in previous studies. In our research, we introduce natural environmental noise as the additive input noise to the chaotic oscillators. The natural environmental noises are correlated in neighboring area, and we have already shown that the limit-cycle oscillators can be synchronized by such correlated noise. Our proposed scheme enables to synchronize unconnected real devices without any interactions between them. In this paper, we investigate feasibility of the proposed natural synchronization scheme on the Rössler system as a chaotic oscillator. First, we evaluate synchronization performance by adding the correlated Gaussian noise, and we show the possibility of synchronization of the chaotic oscillators by the correlated noise. We also introduce a real environmental noise, the environmental sound data obtained at the neighboring microphone devices, and we show that the Rössler oscillators can be synchronized by such correlated environmental noise.

1. Introduction

The noise-induced phase synchronization [1, 2, 3] is a phenomenon that uncoupled nonlinear oscillators can be synchronized with each other by adding common identical noise to each of the oscillators. This synchronization phenomenon does not require any signal exchanges or interactions between the oscillators, for synchronization. The theory of this phenomenon has been already clarified for the limit-cycle oscillators [2, 3]. By applying the same identical noise sequences to the oscillators, the phase difference between the oscillators is gradually reduced, and they synchronize with each other. The chaotic oscillators can also be synchronized by adding common noises [1].

It has also be shown that the limit-cycle oscillators can be synchronized even when the input noises are not identical, but have high cross-correlation [4, 5]. Based on this noise-induced synchronization phenomenon with the correlated noise, we have proposed a natural synchronization scheme for uncoupled wireless devices [4]. We use environmental natural noises, such as the temperature and the humidity of the air, environmental sounds and so on [4, 6, 7]. Those environmental natural noises obtained at the neighboring devices have high similarity. By adding such similar noises to the nonlinear oscillators running in the neighboring devices, our proposed scheme realizes natural synchronization of those devices, without any interactions or exchanges of the signals. We have already shown the feasibility of the proposed synchronization on limit-cycle oscillators by using real natural environmental noises, such as humidity and temperature of the air, sound, electro magnetic wave, or so on [4, 6, 7].

In this paper, we introduce the chaotic oscillators and investigate the feasibility of the proposed natural synchronization on the chaotic systems. By using synchronization of the chaotic oscillators, unconnected devices will be synchronized with chaotic temporal behavior, which will be useful for various new applications.

In this paper, we investigate feasibility of the proposed scheme using chaotic oscillator. First, we evaluate the synchronization rate of the chaotic oscillators with the additive correlated Gaussian noises. Second, we introduce the pairs of real natural environmental data obtained in the neighboring area, and analyze feasibility of the proposed scheme using chaotic oscillators using the real environmental noise.

2. Noise-Induced Synchronization by Environmental Noise

Our proposed synchronization scheme is based on a noise-induced synchronization phenomenon, uncoupled oscillators can be synchronized by adding common noise to them. We have applied this phenomenon to synchronization among unconnected devices.

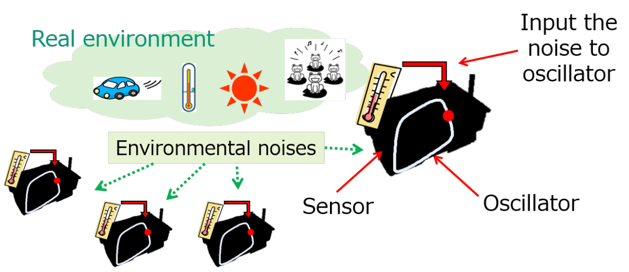

Figure 1: Schematic image of proposed system.

Fig. 1 shows schematic image of our proposed system. We assume that the devices are placed in neighboring area, and these devices have some sensors on each to obtain the natural environmental noises. Those environmental noises obtained in neighboring devices have high cross-correlation [4]. The devices also have their own nonlinear oscillator running independently. The obtained environmental noise is added to the oscillator at each. The oscillators of those devices synchronize in neighboring area based on noise-induced synchronization. Therefore, the unconnected devices running the oscillators can be synchronized by adding obtained environmental noise at each device [4].

We have already shown that the limit cycle oscillators can be synchronized by using real natural noises, such as the temperature, humidity, environmental sound and electro-magnetic wave [4, 6, 7]. We have implemented this proposed scheme on the wireless sensor network devices, and show the feasibility of the proposal on real unconnected devices [7]. We have also implemented the proposed synchronization scheme using several PCs equipped with microphones. Each PC in neighboring area obtains the environmental sound independently and inputs the sound to running nonlinear oscillator. The oscillators on the PC could be synchronized without any interactions between them [6].

3. Noise- Induced Synchronization of Chaotic Oscillators by Environmental Noise

In this paper, we introduce the chaotic oscillators to be synchronized by environmental noise in our proposed scheme. The chaotic oscillators can also be synchronized with each other by adding the identical noise [1]. There may be various applications in our proposed scheme to synchronize un-connected chaotic oscillators by natural environmental noises. For example, using the synchronized chaotic oscillators by our proposed natural synchronization scheme, it may be possible to realize chaotic intermittent communication.

4. Synchronization of Chaotic Oscillators by Natural Environmental Noises

In the previous research of noise induced synchronization for chaotic oscillators, identical white Gaussian noises have been used for the additive input noise [1]. In our proposal, we use the correlated noises obtained from the environment. Therefore, first, we investigate synchronization rate of the chaotic oscillators with adding correlated noise.

In this paper, we use the Rössler system as the chaotic oscillator. The Rössler oscillator is defined by the following equations,

$$\dot{x}(t) = -\omega y - z + \epsilon\xi, \quad (1)$$

$$\dot{y}(t) = \omega x + ay, \quad (2)$$

$$\dot{z}(t) = b + z(x - c), \quad (3)$$

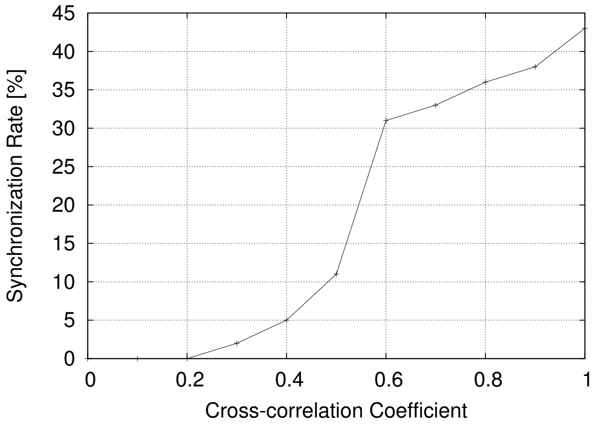

Figure 2: Relationship between synchronization rate and cross-correlation of input noises.

where $a = 0.15$, $b = 0.4$, $c = 8.5$ and $\omega = 0.97$.

We assume that there are two chaotic oscillators $\mathbf{X_1}$ and $\mathbf{X_2}$ with additive input noises ξ_1 and ξ_2.

In order to evaluate the temporal differences of the oscillators, we define the Poincaŕe section on $\{x > 0, y = 0\}$. The temporal differences is calculated at the times when oscillators pass through the Poincaŕe section.

Fig. 2 shows relationship between the synchronization rate and cross-correlation. In this case, we set the noise intensity ϵ at 0.3. From this figure, it is found that the synchronization rate is over 40 % when we use identical noise (cross-correlation coefficient 1), but the rate gets worse as the cross-correlation is reduced.

Subsequently, we use natural environmental fluctuations as the input noise. In this paper, we use environmental sound as the input noise of the chaotic noise-induced synchronization. The sound noise used in this paper have higher value of cross-correlation coefficient[6], and have normalized to have 0 mean and 1.0 of standard deviation. We input the sound noise to chaotic oscillator, and the intensity of noise is adjusted by the amplitude ϵ. We use Rössler oscillator as the chaotic oscillator.

Figs. 3a–3d show the time series of phase differences between two unconnected Rössler oscillators with additive natural environmental sounds for each. Different pairs of the environmental sound data collected in neighbors are used for each result. These results show that each pair of the environmental sound data collected in the neighbor can synchronize the unconnected chaotic oscillators. Fig. 4a and 4b show the time series of Rössler oscillators and the time that the oscillators have crossed the Poincaŕe section.. From Fig. 3b, we can find that the pairs of the chaotic oscillator could be synchronized in approximately 50 cycles for all results. These result show that our proposed natural synchronization scheme is feasible also for the chaotic oscillators.

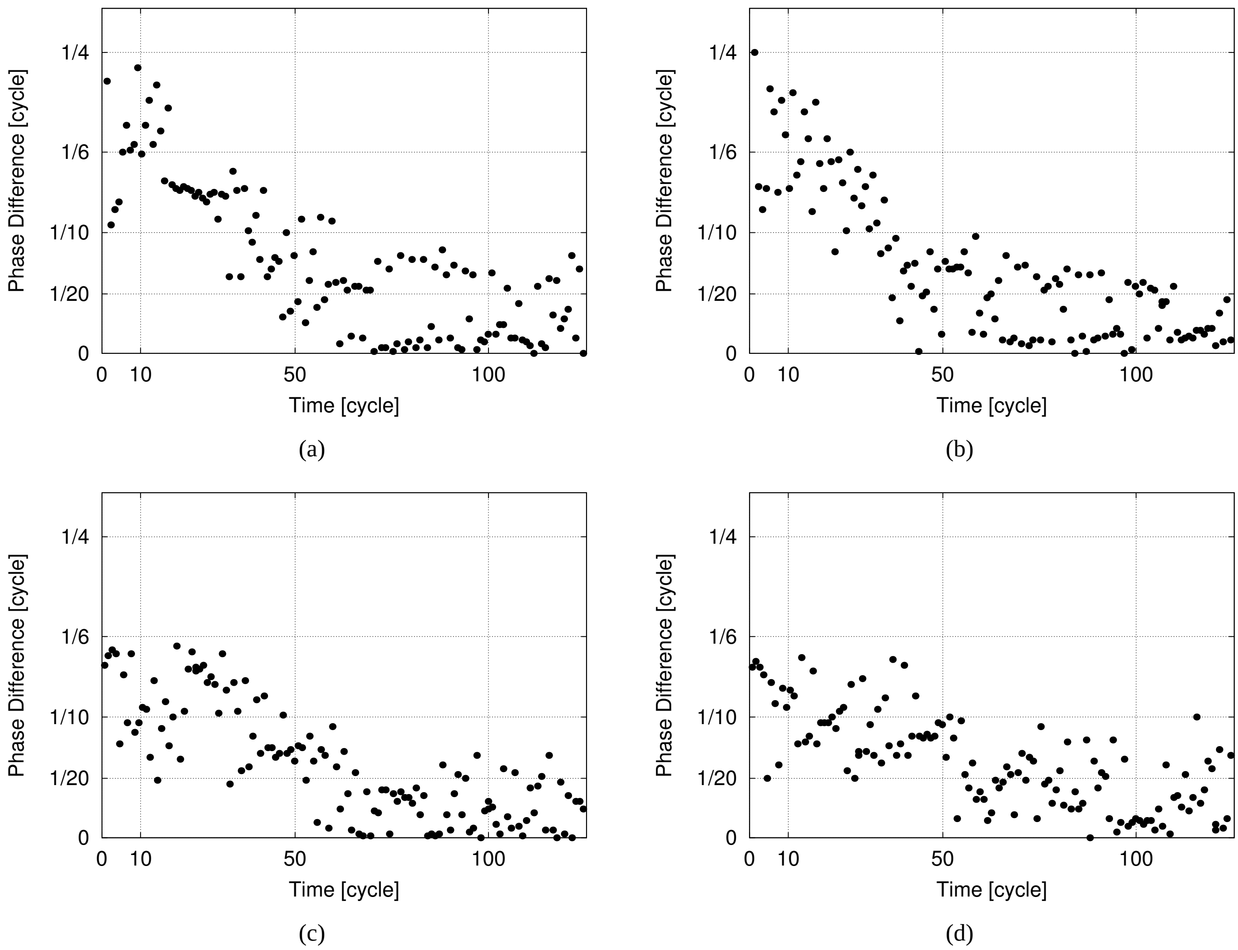

Figure 3: Time series of phase difference between Rössler oscillators adding natural environmental sound. The pair of (a) (b) and (c) (d) have same initial value of phase difference between the oscillators, and different sequences of environmental sound are used for every result.

5. Conclusion

In this paper, we have investigated the noise-induced synchronization between Rössler oscillators, by adding natural environmental noise obtained in neighboring area. The natural environmental noises are correlated in neighboring areas. We have evaluated occurrences of synchronization of the chaotic oscillators by adding such correlated white Gaussian noises.Our experimental results have clarified that the chaotic oscillators can be synchronized even by such correlated noises. Furthermore, we have also used natural environmental fluctuations, natural sound time series, as the input noise, to synchronize unconnected devices without any interactions. Our results have shown that the chaotic oscillators running on unconnected devices independently can be synchronized by applying natural signals, such as the natural sound time series.

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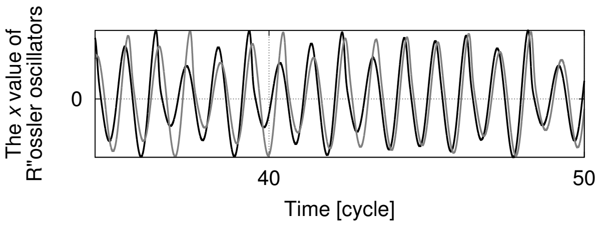
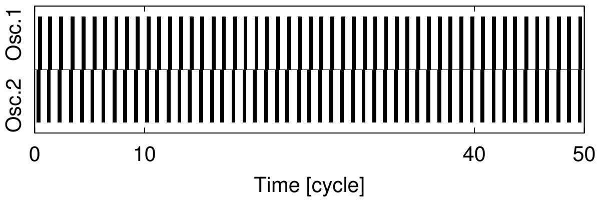


(a) Time series of Rössler oscillators.



(b) The timings that the oscillators have crossed the Poincaŕe section.

Figure 4: Results of synchronization experiments. (a) shows the time series of two Rössler oscillators with the natural environmental sound as additive noise. (b) shows the times that the oscillators cross at the Poincaŕe section.

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