Amplification of Chaos

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Abstract–Using our previous results [1-5] on control and synchronization we design a coupling for obtaining amplification (attenuation) of nonlinear mismatched systems. Numerical results and experimental realization are shown.

1. Introduction

Amplifiers usually change the spectrum of the original signal if this is not small enough. If the linearity range is crossed then this frequently occurs. Using the methodology of the Open-Plus-Closed-Loop (OPCL) synchronization [1, 2] we obtained a method of amplification (attenuation) that keeps the spectrum of the original signal undistorted. In addition this methodology ensures stable synchronization (α=1) and antisynchronization (α=-1) as special cases. In other words we synchronize the dynamics of a system $x(t)$ with an amplified (attenuated) one, $\alpha y(t)$. For $\alpha > 0$ we have complete synchronization (CS) between x and $\alpha y(t)$ and for $\alpha < 0$ we have antisynchronization (AS) between x and $\alpha y(t)$. The OPCL synchronization works for master-slave synchronization [2, 3], mutual synchronization [4, 5] and synchronization of networks [6].

2. General Coupling for Amplification (Attenuation)

2.1. OPCL Method

The OPCL method offers a driving for a general system
$$\dot{x} = F(x), x \in R^n \tag{1}$$
in order to obtain a desired dynamics $g(t)$. This coupling is
$$D(x,g) = \dot{g} - F(g) + (H - \frac{\partial F(g)}{\partial g})(x - g) \tag{2}$$
where H is a Hurwitz matrix (a matrix with negative real part eigenvalues). The driven system
$$\dot{x} = F(x) + D(x,g) \tag{3}$$
will achieve the dynamics $g(t)$ [1,2] after a transient time that depends on H.
Let's consider a driver system,
$$\dot{y} = F(y) + \Delta F(y), \quad y \in R^n \tag{4}$$
where $\Delta F(y)$ contains the mismatch parameters. In this case, the driven system will be
$$\dot{x} = F(x) + D(x, \alpha y) \tag{5}$$

where
$$D(x, \alpha y) = \alpha[F(y) + \Delta F(y)] - F(\alpha y) + [H - \frac{\partial F(\alpha y)}{\partial(\alpha y)}](x - \alpha y) \tag{6}$$

2.2. Application to Lorenz Systems

We present the OPCL method for a Lorenz system
$$\dot{x}_1 = \sigma(x_2 - x_1); \; \dot{x}_2 = rx_1 - x_2 - x_1x_3; \; \dot{x}_3 = -bx_3 + x_1x_2 \tag{7}$$
We consider another Lorenz system with mismatch
$$\begin{aligned} \dot{y}_1 &= \sigma(y_2 - y_1) & + \Delta\sigma(y_2 - y_1) \\ \dot{y}_2 &= ry_1 - y_2 - y_1y_3 & + \Delta ry_1 \\ \dot{y}_3 &= -by_3 + y_1y_2 & - \Delta by_3 \end{aligned} \tag{8}$$
where $\Delta\sigma$, Δr and Δb are the mismatches in parameters. The coupling is derived using (6), and the driven Lorenz system (5) is obtained
$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) & + \alpha\Delta\sigma(y_2 - y_1) \\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3 + \alpha\Delta ry_1 + \alpha(\alpha-1)y_1y_3 \\ & \quad + (p_1 + \alpha y_3)(x_1 - \alpha y_1) \\ & \quad + (p_2 + \alpha y_1)(x_3 - \alpha y_3) \\ \dot{x}_3 &= -bx_3 + x_1x_2 & - \alpha\Delta by_3 + \alpha(1-\alpha)y_1y_2 \\ & \quad + (p_3 - \alpha y_2)(x_1 - \alpha y_1) \\ & \quad + (p_4 - \alpha y_1)(x_2 - \alpha y_2) \end{aligned} \tag{9}$$
The Hurwitz H (3x3 matrix) for the driven Lorenz system is
$$H = |-\sigma \quad \sigma \quad 0; \; r + p_1 \quad -1 \quad p_2; \; p_3 \quad p_4 \quad -b|^T \tag{10}$$
T denotes the transpose of a matrix. The coupling in (9) is simplified by an appropriate choice of the elements, p_k (k=1, 2, 3, 4), of the Hurwitz matrix H. A possible choice is $p_1 < 1-r$, p_2=0, p_3=0 and p_4=0 as elaborated earlier [2]. For further reduction in complexity of coupling, we choose the driver parameters as identical to the response except $r + \Delta r$=38. The number of mismatches is thereby limited to only one, Δr=10.

2.3. Numerical Results for Lorenz Systems

Results of numerical simulations of the coupled mismatch Lorenz system (8)-(9) are presented in Fig.1. The response variables $[x_1, x_2, x_3]$, and twice of the driver variables $[y_1, y_2, y_3]$ are plotted in the upper row where all the time series are identical in amplitude but opposite in phase. A plot of (x_1+2y_1) in the lower row left panel shows a

constant value at zero with time that confirms an amplification of the driver in the AS regime by a factor of two in the response as expected for α=-2. The amplification is also realizable in the CS regime by simply taking α=2 as shown in the lower row right panel of Fig.1. Similarly, attenuation is observed in both the AS and the CS regimes by taking $0 < |\alpha| < 1$. Stable synchronization is also achieved when all the parameters are different but figures are not presented.
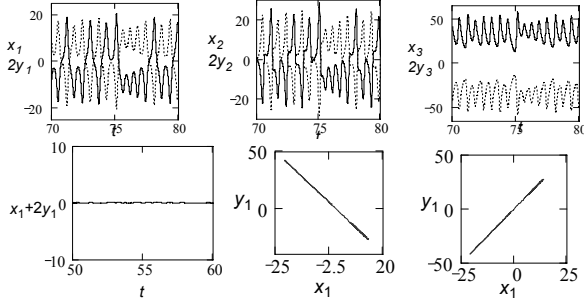

Fig.1. AS and CS and amplification in coupled Lorenz system. Driven system: r=28, σ=10, b=8/3, identical driver except Δr=10. Upper row AS (α=-2) : time series of x_1 and $2y_1$ in the left, x_2 and $2y_2$ in the middle, x_3 and $2y_3$ in the right panels. Driver in solid lines and response in dotted lines. Lower row: left panel confirms AS and amplification by the plot of $x_1+2y_1=0$ after the initial transients; plots of x_1 vs. y_1 confirm AS (α=-2) in the middle panel, CS (α=2) in the right panel.

2.4. Application to a Sprott System

We consider a Sprott system [7] with a single nonlinearity as given by

$$\dot{x}_1 = -ax_2; \dot{x}_2 = x_1 + x_3; \dot{x}_3 = x_1 + x_2^2 - x_3 \qquad (11)$$

Another mismatch Sprott system is chosen as a driver

$$\dot{y}_1 = -ay_2 - \Delta ay_2; \dot{y}_2 = y_1 + y_3; \dot{y}_3 = y_1 + y_2^2 - y_3 \qquad (12)$$

After the coupling, the driven system (11) becomes

$$\dot{x}_1 = -ax_2 - \alpha\Delta ay_2; \ \dot{x}_2 = x_1 + x_3 \qquad (13)$$

$$\dot{x}_3 = x_1 + x_2^2 - x_3 + \alpha(1-\alpha)y_2^2 + (p - 2\alpha y_2)(x_2 - \alpha y_2)$$

The driver and the response are chaotic before coupling for a=0.225, Δa=0.025. For an appropriate choice [3, 4] of the parameter p=-1, we find AS and amplification for α=-2 as shown in Fig.2. In the upper row, the middle attractor (2D projection) is the response, which is an amplified and inverted version of the driver attractor in the left. Plot of x_1 versus y_1 in upper row right confirms AS. The time series of the driver and the response in, in the lower row left, are clearly in opposite phase although their amplitudes are different. A measure of (x_1+2y_1) maintains a zero value after the initial transients as seen in the lower row right panel. This confirms amplification of the driver attractor in the response by a factor of two (for α=-2) in the AS regime.
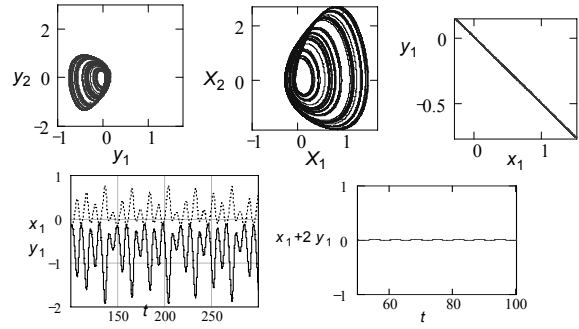

Fig.2. Amplification and AS in coupled Sprott system (12) and (13) . Upper row: phase portraits of driver in left, its enlarged version in the response at middle, axes are in same scale to compare attractor sizes; y_1 vs. x_1 plot in the right shows AS. Lower row: time series of y_1 in dotted line, x_1 in solid line in the left; plot of $x_1+2y_1=0$ in the right confirms AS and amplification by a factor of two (α=-2).
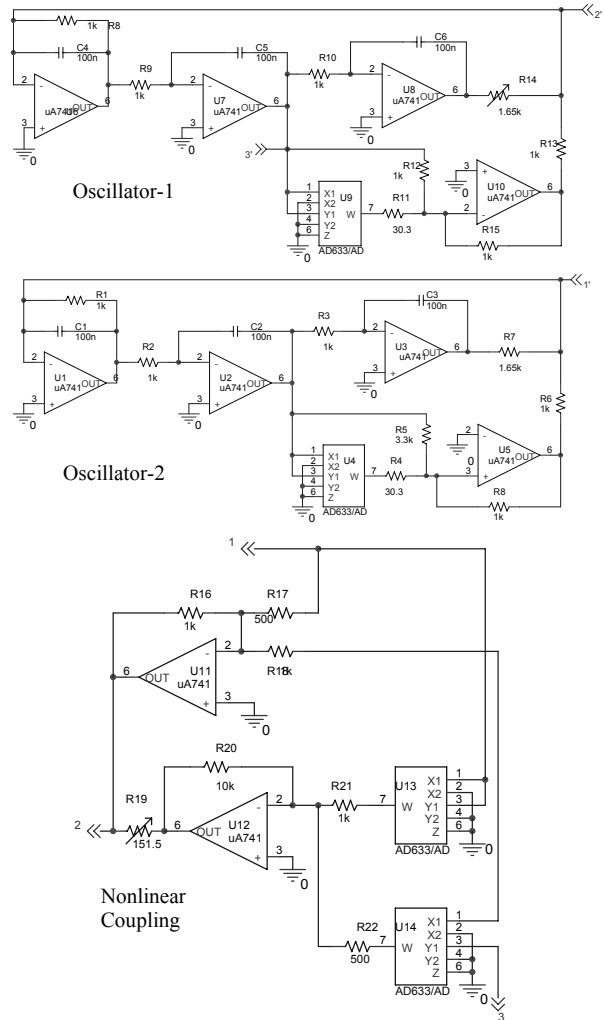

Fig.3. Two coupled Sprott circuits (12) and (13) under OPCL coupling. Oscillator-1 (12) is the driver and oscillator-2 (13) is the response. Nonlinear coupling from the driver to response is shown separately. Component values are noted in the circuit diagram.

3. Experimental Results

We present a physical realization of the coupled Sprott system in electronic circuit with experimental results. Figure 3 shows the circuit diagram of the coupled Sprott system. The Op-amp U1-U5 (U6-U10) with associated resistances R_1-R_8 (R_9-R_{16}) and capacitances C_1-C_3 (C_4-C_6) represent each Sprott circuit. The OPCL coupling is designed using U11-U15. The experimental results as shown in Fig.4 are in good agreement with numerical results in Fig.2.
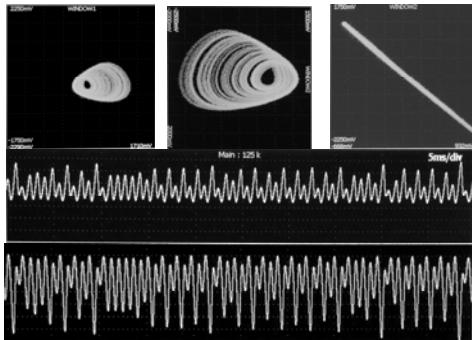

Fig.4. Oscilloscope pictures of AS and amplification in two Sprott circuits: Upper row: 2D projection of drive in left, of amplified response in middle, and synchronization manifold of AS in right. Middle row: time series of driver in yellow. Lower row: response time series in blue. All axes in the pictures are in same scale.

4. Conclusions

Here we presented a method of amplification (attenuation) of the signals of nonlinear systems keeping the spectrum. Numerical results and an experimental realization have been shown. More results are presented elsewhere [8].

We hope that present results will determine an increased interest in OPCL strategy and other engineering applications will be done in the near future.

Acknowledgments

References

[1] E.A.Jackson and I.Grosu, "An Open-Plus-Closed-Loop (OPCL) Control of Complex Dynamic Systems" P*hysica D*, vol.85, pp.1–9, 1995.

[2] I.Grosu, "Robust Synchronization" *Phys.Rev.E,* Vol.56, pp.3709-3712, 1997.

[3] A.I.Lerescu, N. Constandache, S.Oancea and I.Grosu, "Collection of Master-Slave Synchronized Chaotic Systems", *Chaos , Solitons and Fractals*, Vol.22, pp.599-604, 2004.

[4] A.I.Lerescu , S.Oancea and I.Grosu, "Collection of Mutually Synchronized Chaotic Systems" *Phys. Lett.A,* vol. 352, pp.222-226, 2006.

[5] I.Grosu, "General Coupling for Mutual Synchnization of Three Identical Oscillators" *Int. J. Bifur. chaos*, vol. 17(10), pp.3519-3522, 2007.

[6] Ch. Li , W. Sun and J. Kurths, " Synchronization between two Complex Networks", *Phys. Rev. E,* vol.76, 046204, 2007

[7] J.C. Sprott "Some Simple Chaotic Flows" *Phys. Rev. E ,*vol.55 , pp.5285-5290, 1994

[8] I.Grosu, E.Padmanaban, P.K.Roy and S.K.Dana, "Designing Coupling for Synchronization and Amplification of Chaos" (*submitted*).