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Experimental verification of delayed feedback control based on act-and-wait concept

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Abstract—The act-and-wait concept, which reduces continuous-time systems with delays (i.e., infinite-dimensional systems) to discrete-time systems (i.e., finite-dimensional systems), was applied to the delayed feedback control (DFC) in our previous study [Konishi, Kokame, Hara, 2011]. The present paper provides an experimental verification of the DFC based on the act-and-wait concept by using the well-known double-scroll circuit. The act-and-wait DFC is mainly implemented by a peripheral interface controller (PIC). We confirm that the experimental results roughly agree with the stability analysis.

1. Introduction

Controlling chaos has been a focus of intensive research for more than two decades in nonlinear science [1]. Delayed feedback control (DFC) [2], which is one of the most popular methods for controlling chaos, is now widely used to control various systems [3]. The two limit sets, unstable periodic orbits (UPOs) and unstable fixed points (UFPs), can be stabilized by this method. Although the stability analysis of DFC systems is needed to design controller, it is difficult to analyze the stability because the stability is identical to that of a time-periodic time-delay linear system. There have been some theoretical interest in the stability analysis [4, 5, 6].

The stabilization of UFPs has created a growing attention [7, 8, 9]. Kokame *et al.* showed a necessary and sufficient condition for the existence of controller, and provided a systematic procedure for designing it [10]. Unfortunately, this procedure can be used only for a sufficiently short delay time. On the other hand, in experimental situations, delayed feedback signals are often realized by a bucket brigade delay line device or a digital computer with an analog-digital-analog converter [11, 12, 13]. As these electronic devices have the finite operating speed, it is difficult to realize short delay times. This fact implies that the UFPs in fast dynamical systems cannot be experimentally stabilized by these devices. Although the multiple DFC [14, 15] and time-varying DFC [16, 17] methods can solve this problem, they require the implementation cost of multiple delay times or a high-speed time-varying delay.

In recent years, the periodic switching on-and-off (act-and-wait) feedback, called the act-and-wait control concept, was proposed [18, 19]. This concept reduces the dynamics of the control systems with delay to that of discrete-time systems without delay when the waiting time is longer than the feedback delay [20, 21, 22]. Very recently, we proposed the DFC method based on the act-and wait concept, which can solve the above problem. Furthermore, we provided a systematic procedure to design a dead-beat delayed feedback controller for the prototype two-dimensional limit-cycle oscillator [23][1].

The present paper provides an experimental verification of the act-and-wait DFC: UFPs in a well-known double-scroll circuit are experimentally stabilized by switching on-and-off delayed feedback. The controller is implemented by a peripheral interface controller (PIC). We show that the experimental results roughly agree with the stability analysis.

2. DFC based on the act-and-wait concept [23]

Consider a chaotic oscillator,

$$\left\{ \begin{array}{lcl} \dot{\boldsymbol{x}} & = & \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}u \\ y & = & \boldsymbol{c}\boldsymbol{x} \end{array} \right. , \tag{1}$$

where $\boldsymbol{x} \in \boldsymbol{R}^m$, $y \in \boldsymbol{R}$, and $u \in \boldsymbol{R}$ are the state variable, the output signal, and the control signal, respectively. $\boldsymbol{b} \in \boldsymbol{R}^m$ and $\boldsymbol{c} \in \boldsymbol{R}^{1\times m}$ are the input and output vectors. There exits an unstable fixed point $\boldsymbol{x}^*$ satisfying $\boldsymbol{0} = \boldsymbol{f}(\boldsymbol{x}^*)$. The controller with a time-varying gain $k(t) \in \boldsymbol{R}$ is described by

$$u = k(t)(y_\tau - y), \tag{2}$$

where $y_\tau := y(t - \tau)$ is the delayed output signal. Figure 1 illustrates the block diagram of the control systems. The time-varying gain,

$$k(t) = \begin{cases} 0 & t \in [nT, nT + t_w) \\ k_0 & t \in [nT + t_w, (n+1)T) \end{cases} , \tag{3}$$

[1]Aside from our previous study, Ueta *et al.* proposed a partial DFC method for stabilizing UPOs in a hybrid chaotic circuit [24].

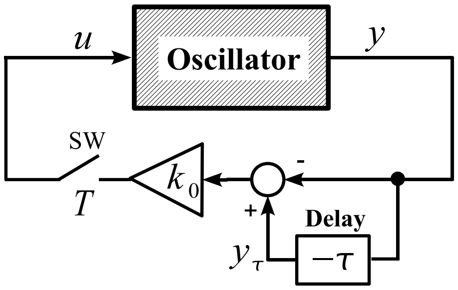


Figure 1: Block diagram of control system consisting of oscillator (1) and act-and-wait DFC (2).

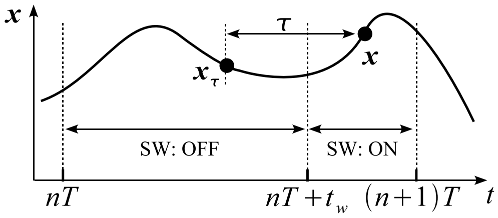


Figure 2: Sketch of system state $\boldsymbol{x}$ and switch timing.

for $n = 0, 1, \ldots$, is realized by turning switch SW on and off with period $T > 0$. The system state $\boldsymbol{x}$ and switch timing are sketched in Fig. 2. Here $n \in \boldsymbol{Z}_+$ is the number of switching operations. For $t \in [nT, nT + t_w)$, SW is turned off, where t_w is the waiting period. For $t \in [nT + t_w, (n+1)T)$, SW is turned on, where $T - t_w$ is the active period. The assumption, $T - t_w \leq \tau \leq t_w \leq T$, is always used: this assumption reduces the dynamics of control systems [18, 19].

Oscillator (1) with controller (2) is linearized around $\boldsymbol{x}^*$,

$$\begin{cases} \delta\dot{\boldsymbol{x}} &= \boldsymbol{A}\delta\boldsymbol{x} + \boldsymbol{b}\delta u, \\ \delta u &= k(t)\boldsymbol{c}(\delta\boldsymbol{x}_\tau - \delta\boldsymbol{x}), \end{cases} \tag{4}$$

where $\delta\boldsymbol{x} := \boldsymbol{x} - \boldsymbol{x}^*$, $\delta\boldsymbol{x}_\tau := \delta\boldsymbol{x}(t-\tau)$, and $\boldsymbol{A} := \partial\boldsymbol{f}(\boldsymbol{x}^*)/\partial\boldsymbol{x}$.

The mapping from the state at the start of switching period to that at the end of it is given by a m-dimensional discrete-time system,

$$\delta\boldsymbol{x}\left[(n+1)T\right] = \boldsymbol{\Phi}\delta\boldsymbol{x}\left[nT\right],$$

where the transition matrix $\boldsymbol{\Phi}$ is obtained by

$$\boldsymbol{\Phi} := \exp\left\{(\boldsymbol{A} - k_0\boldsymbol{bc})\,T + t_w k_0 \boldsymbol{bc}\right\} + \int_{t_w}^{T} \exp\left\{(\boldsymbol{A} - k_0\boldsymbol{bc})\,(T-s)\right\} k_0\boldsymbol{bc}\exp\left\{\boldsymbol{A}(s-\tau)\right\}\mathrm{d}s.$$

Note that $\boldsymbol{x}^*$ is stable if and only if $\boldsymbol{\Phi}$ is a stable matrix (i.e., Schur matrix).

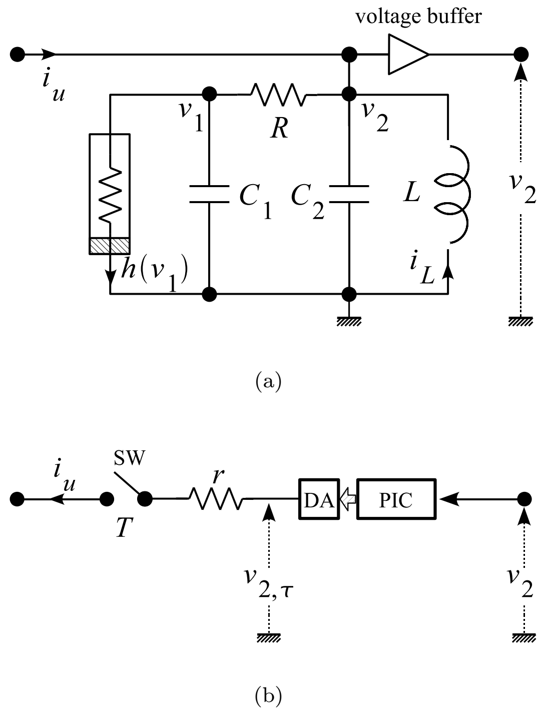


Figure 3: Circuit diagrams: (a) double scroll circuit; (b) act-and-wait DFC circuit.

3. Control of double-scroll oscillator

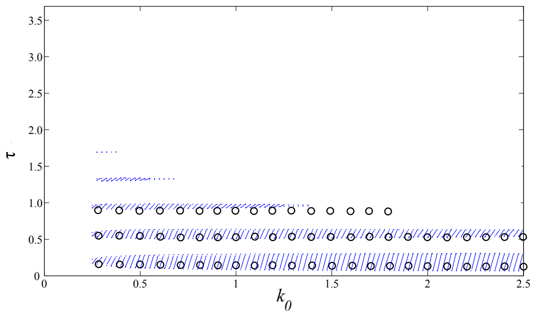
The double scroll oscillator [25], as illustrated in Fig. 3(a), is governed by

$$\begin{cases} C_1\dfrac{\mathrm{d}v_1}{\mathrm{d}t} &= \dfrac{1}{R}(v_2 - v_1) - h(v_1) \\ C_2\dfrac{\mathrm{d}v_2}{\mathrm{d}t} &= \dfrac{1}{R}(v_1 - v_2) + i_{\mathrm{L}} + i_u \\ L\dfrac{\mathrm{d}i_{\mathrm{L}}}{\mathrm{d}t} &= -v_2 \end{cases} . \tag{5}$$

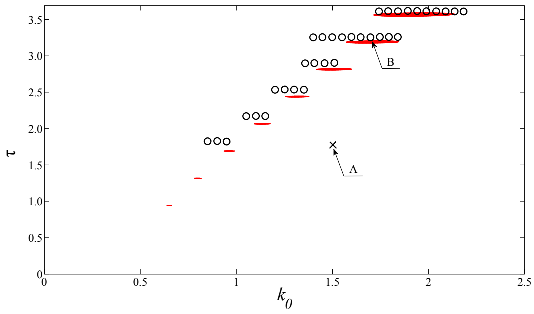
v_1 [V], v_2 [V], and i_{L} [A] denote the voltage across C_1 [F], C_2 [F], and the current through L [H], respectively. Current $h(v_1)$ [A] flows through the nonlinear resistor:

$$h(v) := m_0 v + \frac{1}{2}(m_1 - m_0)\left|v + B_{\mathrm{p}}\right| + \frac{1}{2}(m_0 - m_1)\left|v - B_{\mathrm{p}}\right|.$$

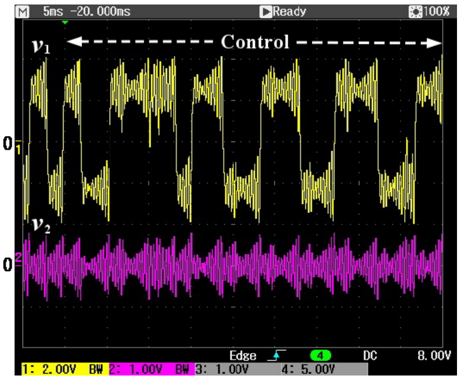
i_u [A] denotes the control current from the controller to the oscillator. Figure 3(b) sketches the circuit diagram of act-and-wait DFC. The voltage v_2 is applied to PIC (PIC18F2550) device; this device generates the delay digitized signal. The digital to analog converter (DA) using R/2R resistor network transforms the digitized signal into the delayed voltage $v_{2,\tau} := v_2(t-\tau)$. SW is implemented by the analog switch (4066): the switching period T [s] and the waiting period t_w [s].



(a)



(b)



(a)

(b)

Figure 4: Stability regions in k_0-τ plane: red and blue regions are numerically estimated by $|\lambda_{\max}(\mathbf{\Phi})| < 1$; The symbol $\circ$ denotes the parameter set (k_0, τ) where the stabilization is experimentally observed: (a) original DFC method, (b) act-and-wait DFC method. Vertical axis: τ [ms].

Figure 5: Time series data of the circuit voltages $v_{1,2}$: (a) point A ($k_0 = 1.5$ and $\tau = 1.80$ [ms]) and (b) point B ($k_0 = 1.7$ and $\tau = 3.24$ [ms]) in Fig. 4(b). Horizontal axis: t (5 ms/div); vertical axis: v_1 (2 V/div) and v_2 (1 V/div).

The current i_u during the active period $T - t_w$ is described by

$$i_u = \frac{1}{r}(v_{2,\tau} - v_2).$$

A dimensionless form of oscillator (5) is described by Eq. (1) with

$$\boldsymbol{f}(\boldsymbol{x}) := \begin{bmatrix} \alpha\{x_2 - x_1 - g(x_1)\} \\ x_1 - x_2 + x_3 \\ -\beta x_2 \end{bmatrix}, \ \boldsymbol{b} := \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \boldsymbol{c} := \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T,$$

where the dimensionless time $t/(RC_2)$ is used instead of the real time t. The state variables, the parameters, and the nonlinear function are rewritten as

$$x_1 := v_1/B_{\mathrm{p}},\ x_2 := v_2/B_{\mathrm{p}}, x_3 := i_L R/B_{\mathrm{p}},$$
$$a := m_1 R,\ b := m_0 R,\ \alpha := C_2/C_1,\ \beta := R^2 C_2/L,$$
$$g(x) := bx + (b-a)\{|x-1| - |x+1|\}/2.$$

The oscillator without control (i.e., $u \equiv 0$) has three fixed points: $\boldsymbol{x}^*_\pm := \begin{bmatrix} \pm p & 0 & \mp p \end{bmatrix}^T$ and $\boldsymbol{x}^*_0 := \boldsymbol{0}$, where $p := (b-a)/(b+1)$. The present paper focuses on the stabilization of $\boldsymbol{x}^*_+$ below[2].

The dynamics of oscillator (1) with controller (2) around $\boldsymbol{x}^*_+$ is described by Eq. (4), where

$$\boldsymbol{A} = \begin{bmatrix} -\alpha(b+1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix},\ k_0 = \frac{R}{r}.$$

The dimensionless periods, $T/(RC_2)$ and $t_w/(RC_2)$, are used in controller (2) instead of the real time periods, T and t_w.

[2]It should be noted that the same results are obtained for $\boldsymbol{x}^*_-$.

Here, the parameters are fixed at

$$C_1 = 0.01 \times 10^{-6}\text{F},\ C_2 = 0.1 \times 10^{-6}\text{F},$$
$$L = 18 \times 10^{-3}\text{H}, B_p = 1.0\text{V},\ R = 1800\Omega,$$
$$T = 4.48 \times 10^{-3}\text{s},\ t_w = 3.58 \times 10^{-3}\text{s},$$
$$m_0 = -0.4 \times 10^{-3},\ m_1 = -0.8 \times 10^{-3},$$

where the double scroll attractor exists in oscillator (1) without control. The stability regions in $k_0 - \tau$ plane on our numerical estimation and circuit experiments for the original DFC method and the act-and-wait DFC method are shown in Figs. 4(a) and 4(b), respectively. It can be seen that the long-time delay can be used for the act-and-wait DFC method compared with the original one and the numerical estimation roughly agrees with the circuit experiment.

The time series data of the oscillator controlled by the act-and-wait DFC is shown in Fig. 5(a). The control current with (k_0, τ) corresponding to point A in Fig. 4(b) starts to flow into the circuit at time $t = 5$ ms. The act-and-wait controller experimentally fails to stabilize the fixed point. On the other hand, as shown in Fig. 5(b), the controller corresponding to point B succeeds in stabilizing the fixed point.

4. Conclusion

This paper provided the experimental verification of the act-and-wait DFC method. The act-and-wait controller was mainly implemented by the PIC device. The UFPs in the double-scroll circuit were experimentally stabilized by the controller. The stability regions on the numerical estimation roughly agree with those on the circuit experiments.

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