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Acceleration Technique for Shielding Current Analysis in Superconducting Film with Cracks

Teruou Takayama† and Atsushi Kamitani†

†Faculty of Engineering, Yamagata University
4–3–16, Johnan, Yonezawa Yamagata 992-8510, Japan
Email: takayama@yz.yamagata-u.ac.jp

Abstract—Acceleration technique for analyzing the shielding current density in a high-temperature superconductor with multiple cracks has been proposed. In this technique, it is necessary to solve the simultaneous linear equations obtained from the discretization of the finite element method and the Backward Euler method. By applying the linear equations to the GMRES(k) method with $\mathcal{H}$-matrix method, the results of computations show that the CPU time of the GMRES(k) with $\mathcal{H}$-matrix method is much faster than that of the LU decomposition.

1. Introduction

As a contactless and a high-speed measurement method of a critical current density j_C in a high-temperature superconductors (HTSs) film, Hattori *et al.* proposed the scanning permanent magnet method [1]. In the method, a permanent magnet is moved along a HTS surface. They found that a spatial distribution of j_C can be estimated from the distribution of an electromagnetic force acting on the film.

In order to simulate a scanning permanent magnet method, a numerical code was developed for analyzing the time evolution of a shielding current density in an HTS film with multiple cracks. After an initial-boundary-value problem of the shielding current density is spatially discretized with the finite element method (FEM), the resulting ordinary differential system is solved by using the backward Euler method and the Runge-Kutta (R-K) method with an adaptive step-size algorithm. In particular, it is found that the R-K method is a useful tool with increasing the number of nodes for the FEM. However, it is necessary to consider the further high-speed of the shielding current analysis for a large-scale problem.

The purpose of the present study is to propose a further high-speed method for analyzing the time evolution of the shielding current density in an HTS film containing multiple cracks. In addition, we investigate the performance of the proposed method.

2. Governing equations

In Fig. 1, we show a schematic view of a scanning permanent magnet method. In the method, a cylindrical magnet of radius R and height H is located just above an HTS thin film, and a distance between a magnet bottom and film surface is denoted by L. We assume a rectangle-shaped HTS film of length l, width w, and thickness b, and the square cross-section of the film is denoted by Ω. If a crack is contained in the film, Ω has not only the outer boundary C_0 but the inner boundaries C_j ($j = 1, 2, \cdots, m$).

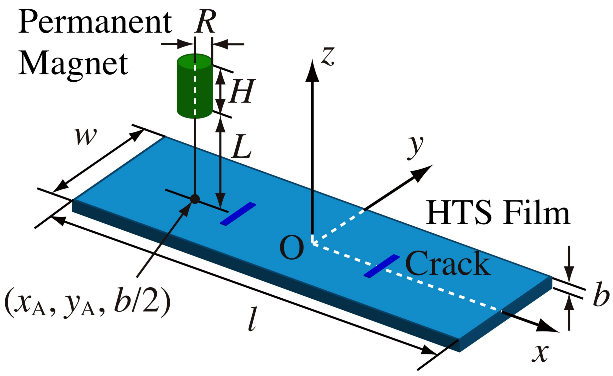

Figure 1: A schematic view of a scanning permanent magnet method.

In the present study, we use the Cartesian coordinate system $\langle O : \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\rangle$, where the z-axis is parallel to the thickness direction and the origin O is the centroid of the film. In terms of the coordinate system, the symmetry axis of the coil is determined as $(x, y) = (x_M, y_M)$. For characterizing the magnet flux density at $(x, y, z) = (x_M, y_M, b/2)$ for $v = 0$ mm/s. The movement of the magnet is assumed as $x_M(t) = vt - l/2$, where v is the magnitude of the scanning velocity.

As is well known, a shielding current density $\boldsymbol{j}$ in an HTS is closely related to the electric field $\boldsymbol{E}$. The relation can be written as $\boldsymbol{E} = E(|\boldsymbol{j}|)\boldsymbol{j}/|\boldsymbol{j}|$. As a function $E(j)$, we assume the power law $E(j) = E_C[j/j_C]^N$, where E_C is the critical electric field and N is a constant.

By using the thin-layer approximation [3], the shielding current density $\boldsymbol{j}$ can be expressed as $\boldsymbol{j} = (2/b)\nabla S \times \boldsymbol{e}_z$, and the time evolution of the scalar function $S(\boldsymbol{x}, t)$ is governed by the following integro-differential equation:

$$\mu_0 \frac{\partial}{\partial t}(\hat{W}S) + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = -\frac{\partial}{\partial t}\langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle. \qquad (1)$$

where a bracket $\langle\ \rangle$ is an average operator over the thickness of the HTS film. Also, $\hat{W}$ is defined by

$$\hat{W}S = \iint_{\Omega} Q(|\boldsymbol{x}-\boldsymbol{x}'|)S(\boldsymbol{x}',t)d^2\boldsymbol{x}' + \frac{2}{b}S(\boldsymbol{x},t), \tag{2}$$

and an explicit function $Q(r)$ in (2) is expressed as follows:

$$Q(r) = -\frac{1}{\pi b^2}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+b^2}}\right). \tag{3}$$

The initial and boundary conditions to (1) are assumed as follows:

$$S = 0, \quad \text{on} \quad C_0 \tag{4}$$

$$\frac{\partial S}{\partial s} = 0, \quad \text{on} \quad C_i \tag{5}$$

$$h(\boldsymbol{E}) \equiv \oint_{C_i} \boldsymbol{E}\cdot\boldsymbol{t}dt = 0, \tag{6}$$

Here, s and $\boldsymbol{n}$ are an arclength along C_i and a normal unit vector on C_i, respectively.

3. Discretization

In the present study, we discritize the initial-boundary-value problem of (1) in time and space using the finite element method (FEM) and the Backward Euler method, respectively. A region Ω is divided into a number N_{e} of square elements Ω_{e}:

$$\Omega \simeq \bigcup_{e=1}^{N_{\mathrm{e}}} \Omega_e,$$

where Ω_e is eth element. On the other hand, a crack is given by a line segment.

By using the Backward Euler method, the initial-boundary-value problem of (1) is transformed into the nonlinear boundary-value problem as follows:

$$(*1)\begin{cases} G(S^{(n+1)}) & = 0, \quad \text{in} \quad \Omega & \text{(7a)} \\ S^{(n+1)} & = 0, \quad \text{on} \quad C_0 & \text{(7b)} \\ \dfrac{\partial S^{(n+1)}}{\partial s} & = 0, \quad \text{on} \quad C_i & \text{(7c)} \\ \oint_{C_i} \boldsymbol{E}^{(n+1)}\cdot\boldsymbol{t}ds & = \phi_i, & \text{(7d)} \\ N(\boldsymbol{E}^{(n+1)}) & = 0. & \text{(7e)} \end{cases}$$

Here, a superscript (n) is a value at time $t = t^n (\equiv n\Delta t)$. In addition, $G(\boldsymbol{S}^{(n+1)})$ is defined by

$$G(S^{(n+1)}) \equiv \mu_0 \hat{W}S^{(n+1)} + \Delta(\nabla\times\boldsymbol{E}^{(n+1)})\cdot\boldsymbol{e}_z - u, \tag{8}$$

where u is

$$u = \mu_0\hat{W}S^{(n)} - (\langle\boldsymbol{B}^{(n+1)}\cdot\boldsymbol{e}_z\rangle - \langle\boldsymbol{B}^{(n)}\cdot\boldsymbol{e}_z\rangle). \tag{9}$$

Note that numerical solution contains the error due to which a boundary condition (9) is not satisfied. In order to resolve this problem, we propose the virtual voltage

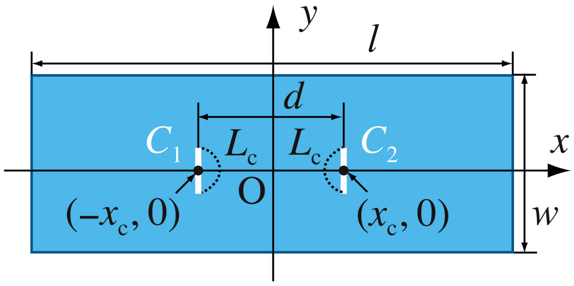

Figure 2: Two cracks in an HTS film.

method. In the method, a virtual voltage ϕ is added to a numerical boundary condition (7e). Here, $N(\boldsymbol{E})$ is evaluated from $h(\boldsymbol{E})$. Therefore, the solution of the problem (∗1) include not only $S^{(n+1)}$ but also ϕ_i.

When the FEM and the Newton method are applied to the problem (∗1), we get simultaneous equations system:

$$\begin{bmatrix} A(\boldsymbol{S}) & C & F(\boldsymbol{\phi}) \\ C^T & 0 & \boldsymbol{0} \\ D^T(\boldsymbol{S}) & \boldsymbol{0}^T & O \end{bmatrix}\begin{bmatrix} \delta\boldsymbol{S} \\ \boldsymbol{\lambda} \\ \delta\boldsymbol{\phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{g}(\boldsymbol{S},\boldsymbol{\phi}) \\ \boldsymbol{0} \\ -\boldsymbol{h}^*(\boldsymbol{S}) \end{bmatrix}, \tag{10}$$

at each time step of the Newton method. Here, $\boldsymbol{S}$ is the n-vector, and the n is a number of nodes for the FEM. The $(m-n)$-by-n matrices $A(\boldsymbol{S})(\equiv W + J(\boldsymbol{S}))$ is determined from $\hat{W}$, where $J(\boldsymbol{S})$ is the Jacobian matrix. The matrix C, $F(\phi)$, $D(\boldsymbol{S})$, and the vector $\boldsymbol{g}(\boldsymbol{S},\phi)$ are obtained by the cracks. As a result, the initial-boundary problem of (1) is reduced to the simultaneous equations system at each time step of the Newton method. The procedure for the system is as follows:

1. By solving the simultaneous equations, we get the correction $\delta\boldsymbol{S}$ and $\delta\boldsymbol{\phi}$.
2. We update an approximate solutions by means of $\boldsymbol{S}^{(j)} \Leftarrow \boldsymbol{S}^{(j-1)} + \delta\boldsymbol{S}$ and $\boldsymbol{\phi}^{(j)} \Leftarrow \boldsymbol{\phi}^{(j-1)} + \delta\boldsymbol{\phi}$.

Two procedures are repeated until the termination condition:

$$\max\left(\frac{\|\delta\boldsymbol{S}\|}{\|\boldsymbol{S}\|}, \frac{\|\delta\boldsymbol{\phi}\|}{\|\boldsymbol{\phi}\|}\right) \le \varepsilon, \tag{11}$$

is satisfied. Here, ε is a constant, and $\|\quad\|$ denotes the maximum norm. Under the numerical method, we develop the numerical code for analyzing the time evolution of the shielding current density in an HTS film with multiple cracks.

4. High-speed method for shielding current analysis

In this section, we propose a high-speed analysis of a shielding current density in an HTS film. To this end, we use the GMRES(k) method as a solver of simultaneous equations (10), and we measure the CPU time for the

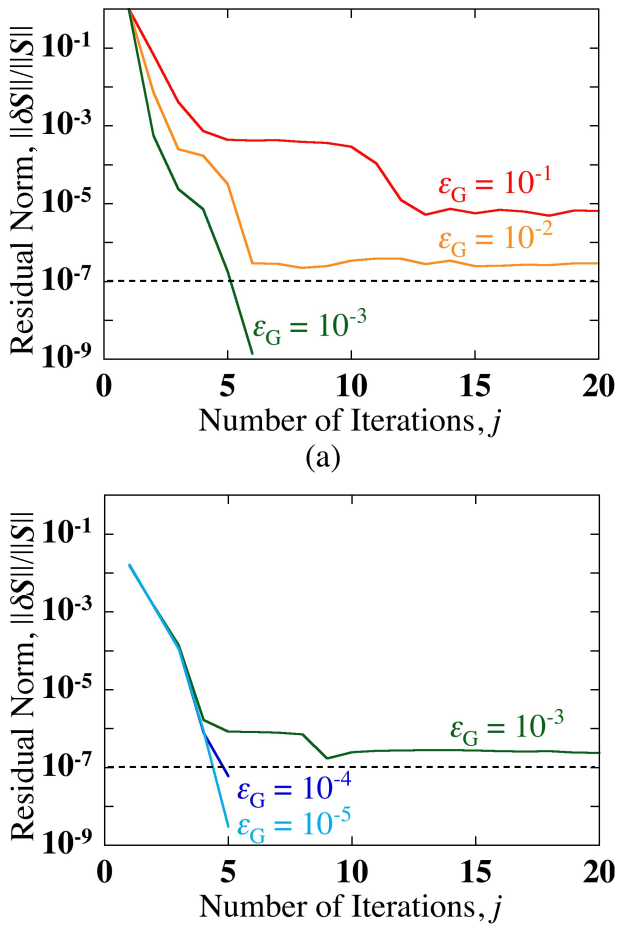

Figure 3: Residual norm history of the Newton method for the case with (a) $x_{\mathrm{M}} = -17.9$ mm and (b) $x_{\mathrm{M}} = -8.8$ mm. Here, $k = 100$ and $n = 3816$.

simulation of a scanning permanent magnet method Here, a convergence criterion of the GMRES(k) method is denoted by ε_{G}.

Throughout the present study, the physical and geometrical parameters are fixed as follows: $w = 12$ mm, $l = 36$ mm, $b = 1$ μm, $j_{\mathrm{C}} = 1$ MA/cm^2, $E_{\mathrm{C}} = 1$ mV/m, $N = 20$, $L = 0.5$ mm, $R = 0.8$ mm, $H = 2$ mm, $B_{\mathrm{F}} = 0.1$ T, $v = 10$ cm/s, $m = 2$, $d = 12$ mm, $L_{\mathrm{c}} = 2$ mm, $\varepsilon = 10^{-7}$. In Fig. 2, we show the two cracks, C_1 and C_2, in the HTS film. The crack distance d and the crack size L_{c} are given by $d = 12$ mm and $L_{\mathrm{c}} = 2.4$ mm.

4.1. Implementation of GMRES(k) method

Firstly, we investigate the influence of the Newton method on the convergence determinant of GMRES(k) method. In Figs. 3, we show the residual norm history of the Newton method. We see from Fig. 3 (a) that, for $\varepsilon_{\mathrm{G}} = 10^{-1}$ and $\varepsilon_{\mathrm{G}} = 10^{-2}$, the residual norm of the Newton method becomes constant, whereas the Newton method converges in 6 iterations. Also, we cannot obtain the solution at this step for $\varepsilon_{\mathrm{G}} = 10^{-3}$ (see Fig. 3 (b)). On the other hand, for $\varepsilon_{\mathrm{G}} = 10^{-4}$ and $\varepsilon_{\mathrm{G}} = 10^{-4}$, the Newton method converges in 5 iterations. Consequently, it is necessary that the convergence determinant is $\varepsilon_{\mathrm{G}} \leq 10^{-4}$. In the following, the value of ε_{G} is fixed to $\varepsilon_{\mathrm{G}} = 10^{-5}$.

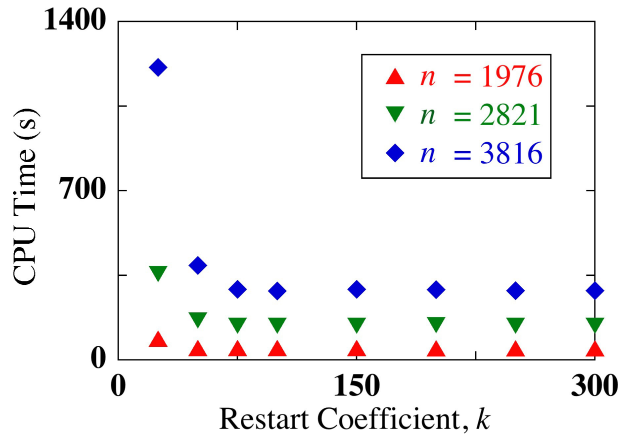

Figure 4: Dependence of the CPU time on the restart coefficient k.

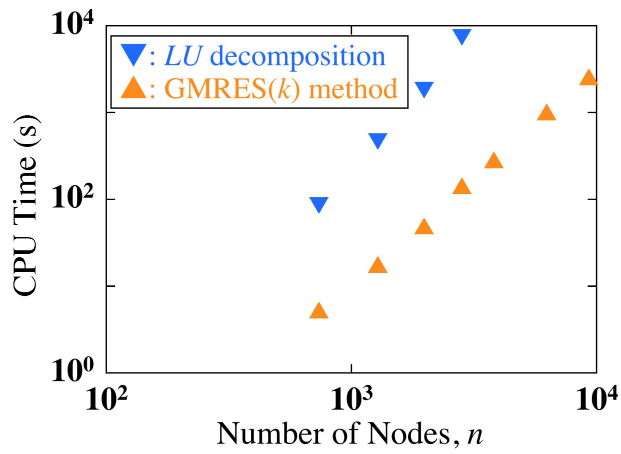

Figure 5: Dependence of the CPU time on the number n of nodes.

Next, we investigate the influence of the restart coefficient k on the CPU time. In the following, note that we measure the CPU time from $t = 0$ s to $t = 1.2 \times 10^{-2}$ s. In Fig. 4, we show the dependence of the CPU time on the restart coefficient k. From this figure, the CPU time decreases with increasing the value of k monotonously, and it almost becomes a constant. In addition, the CPU time drastically increases with the number n of nodes for $25 \leq k \leq 75$. As a result, it is found that the CPU time decreases by using the large value of k. Hereafter, we use $k = n/5$ as the the value of the restart coefficient.

Finally, we compare the CPU time of the two solvers for the simultaneous equations. In Fig. 5, we show the dependence of the CPU time on the number n of nodes.

The results of the computations show that the CPU time of the GMRES(k) method is much faster than that of the LU decomposition. Especially, it is found that, for $n = 2821$, the GMRES(k) method is about 52 times faster than the LU decomposition.

4.2. Implementation of $\mathcal{H}$-matrix method

As is well known, a matrix-vector multiplication is the dominant operation in the GMRES(k) method. In the present study, we adopt the $\mathcal{H}$-matrix method for calculating the matrix-vector multiplication with a high-speed.

In this method, firstly, a coefficient matrix is transformed into a set of a block matrices (a hierarchical matrix) by using the clustering from the node information of the FEM. Note that the matrix is hierarchized until the number of the node is less than $N_{\min}$ in the cluster.

Next, an admissibility condition, $\min(\mathrm{diam}(\tau), \mathrm{diam}(\sigma)) \leq \eta \mathrm{dist}(\tau, \sigma)$, is applied to the all block matrices. Here, τ and σ are bounding boxes, and η is a constant. In addition, $\mathrm{diam}(\tau)$ and $\mathrm{dist}(\tau, \sigma)$ are diagonal of the bounding box t and a distance between the bounding boxes τ and σ, respectively. If the condition is satisfied, a block matrix becomes a low lank matrix by using the adaptive cross approximation (ACA). In other words, the block matrix $B \in \mathbb{R}^{t \times s}$ is approximated by $B \simeq UV^T$, where the matrices U and V are t-by-r matrix and r-by-s matrix, respectively. On the other hand, if the condition is nod satisfied, the block matrix B is stored as the general matrix. In Fig, 6, the matrix W is applied to the $\mathcal{H}$-matrix method and the ACA. We see from this figure that the row lank matrices are shown in the rainbow color.

In Fig. 7, we show the dependence of the speedup ratio $\tau_{\mathrm{N}}/\tau_{\mathrm{H}}$ on the number n of the nodes. Here, τ_{N} and τ_{H} are the CPU time of the GMRES(k) method and the GMRES(k) method with the $\mathcal{H}$ matrix method, respectively. From this figure, the speedup ratio monotonously increases with n. In this sense, the GMRES(k) method with the $\mathcal{H}$-matrix method is a powerful tool for analyzing the shielding current density in the HTS film.

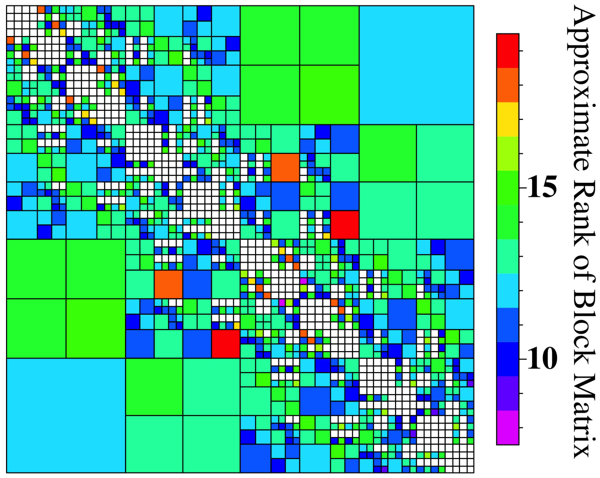

Figure 6: The approximation rank of block matrix for the case with $n = 3816$ and $N_{\min} = 15$.

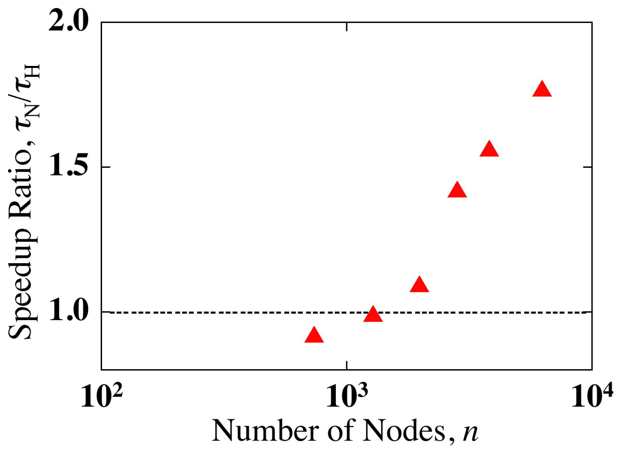

Figure 7: Dependence of the speedup ratio $\tau_{\mathrm{N}}/\tau_{\mathrm{H}}$ on the number n of nodes for the case with $N_{\min} = 15$.

5. Conclusion

For the purpose of a high-speed analysis of a shielding current density in an HTS film with cracks, we apply the resulting linear equations to the GMRES(k) method with $\mathcal{H}$-matrix method. The results of computations show that the CPU time of the GMRES(k) with $\mathcal{H}$-matrix method is much faster than that of the LU decomposition. Therefore, this method is a powerful tool for analyzing the shielding current density in the HTS film for a large-scale problem.

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