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Experimental Study of Gaussian Noise Generation Using a CMOS PLL

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Abstract– There have been reports on generation of white noise as a chaotic oscillation in phase-locked loops(PLLs). This method is interesting in the view point of simplicity compared to the other conventional methods: amplification of electronic noise and digitally synthesized random sequences. However, the characterization of the noise from the PLL except for the power spectrum analysis has not been made enough in the previous studies so far. In this study amplitude probability as well as the power spectral density of the wideband noise obtained form a CMOS PLL is discussed.

1. Introduction

It has been known that chaotic oscillations often employ wide-band frequency components due to their own complex nature. One can regard these oscillations as noise. So far surveys of the parameters yielding chaotic oscillations in PLL systems such as the forced pendulum[1] and the rf-driven Josephson junctions[2] have been done. Empirically the frequency components of the chaotic noise have been known to be restricted in individual frequency bands. However, it has found based on the heuristic numerical study that the white noise can be generated from a chaotic PLL[3]. And a followed analog PLL circuit experiment successfully demonstrated to generate the white noise. Then the upper limit of frequency band defined by means of the flat power spectrum was restricted to below the frequency of 500Hz due to relatively slow switching speeds of the devices used in the circuit [4,5]. The other experiment under similar PLL dynamics was made with the CMOS PLL (CD4046) when sinusoidally driven[6]. Then the upper limit of the frequency band of white noise exceeded the audio frequency. In this paper the statistical property of the waveform of the white noise for the same CMOS PLL is studied when both a single power supply and a square-shaped input signal are adopted into the circuit which are mainly different points from the previous work[6].

2. Dynamics and Circuit Configuration of PLL

2.1. Dynamics of PLL

There have been two types of the PLL dynamics associated with white noise generation reported so far. The first type is represented as,

$$\frac{d^2\phi}{dt^2} + \gamma\frac{d\phi}{dt} + \sin\phi = a\sin\omega t, \qquad (1)$$

where ϕ and t are the phase and the normalized time, respectively. For chaotic operations of PLL the system should be in the underdamped regime, and then the required effective damping factor γ is less than unity. The term on the right hand side of Eq.(1) is corresponding to the external drive force. For this system the dynamics can also be interpreted with the *washboard model* in which a particle with coordinate ϕ travels in the tilted washboard potential U giving a force $-\mathrm{d}U/\mathrm{d}\phi$ in the presence of viscous damping. Then the generation of the white noise can be considered as the random walk on a particle on the washboard tilted alternatively. There have been many reports on parameters resulting chaos in this system.

The second type of the dynamics is represented as,

$$\frac{d^2\phi}{dt^2} + \beta\frac{d\phi}{dt} + h(\phi) = a\sin\Omega_m t, \qquad (2a)$$

where,

$$h(\phi) = \begin{cases} (2/\pi)\phi & \text{for } -\pi/2 < \phi < \pi/2 \\ (2/\pi)(\pi - \phi) & \text{for } \pi/2 < \phi < 3\pi/2 \end{cases} \qquad (2b)$$

The symmetry of the function $h(\phi)$ with respect to ϕ is similar to that for $\sin\phi$ in Eq.(1). So it seems reasonable to expect the similar results associated with the broad band noise generation even for the partly different dynamics compare to Eq.(1). We report the experimental results associated with noise generation from this type of dynamics which is carried out by using a CMOS PLL with a frequency modulated(FM) input signal. The chaotic behavior for this system has been studied[7].

2.2.Circuit Configuration and Parameters

Fig.1 shows the circuit configuration of the CMOS PLL (CD4046) used in this study. The complete circuit consists of an EXOR phase detector(PD), the lag filter used as a low-pass filter and a voltage controlled oscillator(VCO). The phase transfer function of the PLL is primary determined by the natural frequency ω_n being represented as,

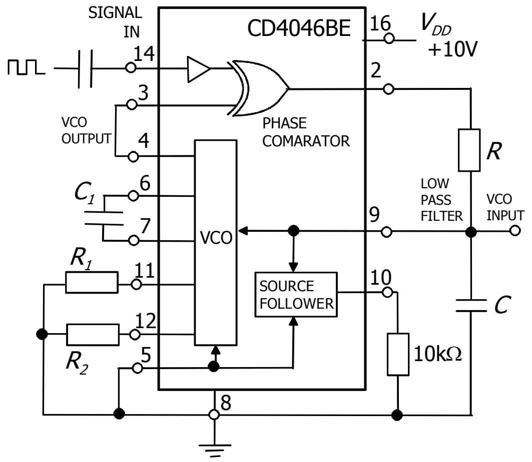

Fig.1 Circuit configuration.

$$\omega_n = \sqrt{K_0 / \tau_1}\ , \qquad (3)$$

where K_0 is the loop gain which is the product of PD gain K_{PD} and VCO gain K_{VCO}, and τ_1 is the time constant of the filter.

In this study the sinusoidally-modulated FM signal having square shape was firstly used as an input reference signal in the PLL. Then the phase of FM signal can be represented as $\omega_C t+(\Delta\omega/\omega_m)\cos\omega_m t$, where ω_C, $\Delta\omega$ and ω_m are the carrier frequency, the maximum frequency deviation and the modulation frequency, respectively. The ratio $\Delta\omega/\omega_m$ indicates the modulation index. Using the normalized parameters: $\beta(\equiv\omega_n/K_0)$, $\Omega_m(\equiv\omega_m/\omega_n)$, m $(\equiv\Delta\omega/\omega_n)$ and the redefined parameter: $a(\equiv m[\beta+\Omega_m]^{1/2})$ we obtain the same equation as Eqs.(2) for the circuit operation when ω_C is chosen to be consistent with the center frequency of phase lock[8]. For this case one should note that ϕ in Eqs.(2) is the phase error defined as the phase difference between the input signal and the VCO output as long as the linear characteristics for both PD and VCO maintained.

In our experiment the VCO characteristics are firstly determined by the capacitor C and the resistor R_1 for a fixed supply voltage V_{DD} which also determine K_{PD}. The additional frequency offset for the VCO is eliminated for $R_2=\infty$ though the finite R_2 value is also taken into account later. At beginning the circuit parameters were chosen as R_1=6.8kΩ, C_1=0.001μF, $R_2=\infty$, R=1kΩ, and C=0.01μF.

Before using the FM input signal a periodic square signal was applied to the input terminal of PLL in order to reveal the values of K_{PD} and K_{VCO} under phase locked condition. Fig.2 shows the plot of the averaged PD output voltage, in other words, the VCO averaged input voltage as the function of the phase error ϕ. A linear characteristic is obtained except for a plot located near $\phi=-\pi/2$. The origin of this unfitted point might be due to the relatively high K_0 value. K_{PD} is calculated to be 2.71V/rad from the slope of the solid line in Fig.2. Further, Fig.3 shows the oscillation frequency as a function of the VCO average input voltage. From this figure the value of K_{VCO} is determined to be 156krad/sV, corresponding to 24.8kHz/V. The center frequency of the phase lock obtained for the above circuit configuration is 92.0kHz which can be confirmed by π/2 phase difference between the input signal and the VCO output. Furthermore, the phase lock takes place over the frequency range from 33kHz to 180kHz. Consequently ω_n=205krad/s, K_0=422krad/s, τ_1=10μs, and β=0.49 are determined.

3. Results and discussions

3.1. FM input signal

As mentioned above the circuit configuration determines β, meaning the parameter on the left-hand side of Eq.(2a) fixed. In order to fix the values of residual normalized parameters on the right-hand side of this equation, Ω_m and m as FM conditions are adjusted for the generation of wideband noise. Figure 4 shows the power

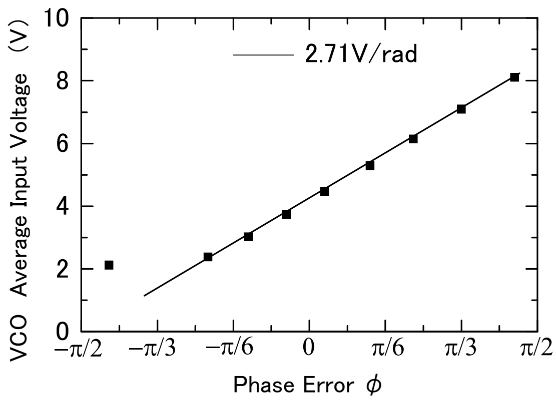

Fig.2. Plot of VCO averaged input voltage (averaged PD output voltage) versus phase error ϕ.

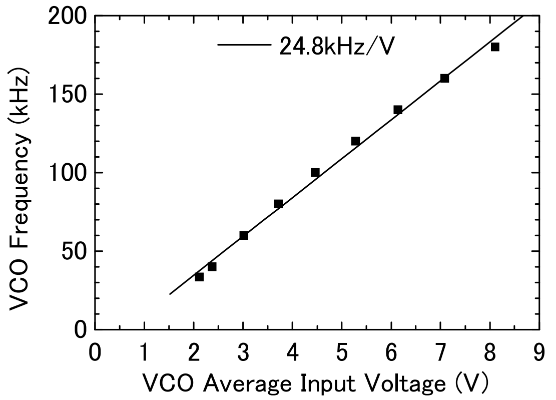

Fig.3. VCO's frequency versus average voltage characteristic.

spectral density of the VCO input signal for $\omega_m=2\pi\times$ 20.0krad/s and $\Delta\omega=2\pi\times$57.0krad/s. These conditions give Ω_m=0.67 and m=1.9. The spectrum in Fig.4 was obtained as the average of 16 samples and could be flatten out for the larger sample numbers. As shown in this figure the white noise generation can be confirmed as the flat spectrum below the frequency of 2kHz. It is remarkable that the similar spectral density can be obtained even when $\Delta\omega$ is independently increased up to $2\pi\times$92.0krad/s

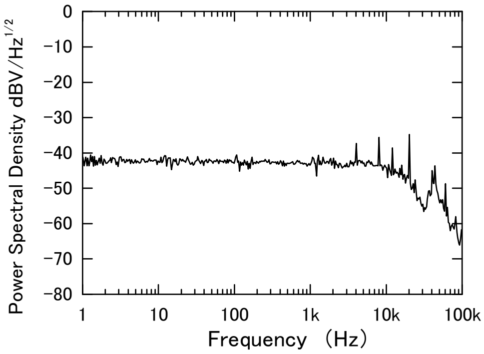

Fig.4. Power spectral density of the VCO input signal for the square FM signal used as a reference input. The system parameters are β=0.49, Ω_m=0.67 and m=1.9.

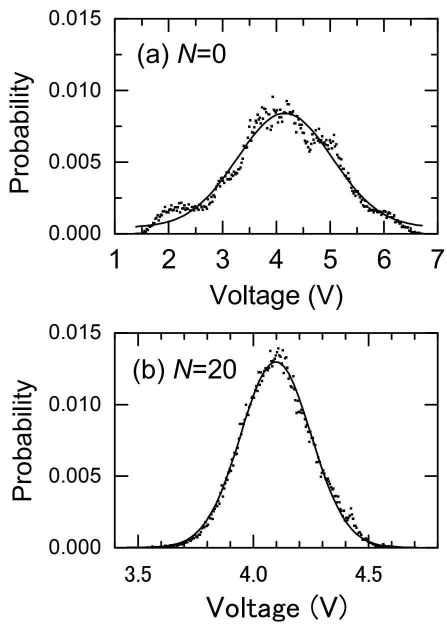

Fig.5. Amplitude probability distributions of VCO input voltage.(a) and (b) show the results obtained *without filter* and *with moving average filter*, respectively. The solid curve shows the Gaussian distribution function calculated for the best fitting. N denotes the number of samples. The system parameters are β=0.49, Ω_m=0.67 and m=1.9.

with the frequency increment of $2\pi\times$ 1krad/s. This maximum value of $\Delta\omega$ is the capable maximum frequency deviation for the fixed carrier frequency of 92.0kHz. Thus, for the normalized maximum frequency deviation the range indicated by $1.9<m\leq3.1$ can be allowed to induce the white noise generation for Ω_m=0.67 and β=0.49.

Next the statistical characteristics of the noise waveform are considered. According to our knowledge, the amplitude histogram of the white noise obtained as the chaotic oscillation in a practical PLL has not been reported so far in spite of flat power spectrum observed experimentally by a few authors[4-6]. The largest peak in the spectral density appeared at 20kHz is identical with the modulation frequency, and not the carrier frequency. Another visible peak has not been indentified yet. Figure 5 shows the amplitude probability density for the noise corresponding to the spectrum shown in Fig.4. The solid line for each case shows the Gaussian distribution function with which the mean value and the variance are chosen to obtain the best fitting. In this figure (a) and (b) show the results obtained *without filter* and *with moving average filter*, respectively. Taking account the sampling interval $2.5\tau_l$ as well as the sampling number N=20, the cutoff frequency of this digital low pass filter is estimated to be approximately 2kHz.

Concerning the Gaussian noise generation from the system governed by Eq.(2) one can expect a mechanism similar to that of washboard model that is used for interpretation of the dynamics of Eq.(1). Therefore the Brownian motion which is also known as the Wiener process is expected to result in generation of the Gaussian noise for not only Eq.(1) but Eq.(2) .

3.2. Periodic input signal

The nature of frequency pull-in process in PLLs has been known to be complex and sometimes chaotic. We tried to drive the PLL with a square periodic signal instead of an FM signal, while no appropriate dynamic model has been available for this scheme. Then the signal frequency was set to be out of the lock range. Figure 6 shows the power spectral density of the obtained noise for the drive frequencies of 13kHz and 230kHz. Evidently the white spectral density is obtained below about 2kHz although there is a large difference in noise level between the two spectra for the white region in Fig.6. It is interesting to note that the upper frequency limit of 2kHz is similar to that for the FM input signal. On the other hand the visible peaks for frequency region higher than 2kHz in Fig.6 are seemed to be independent of each drive frequency. Fig.7 shows the corresponding amplitude probability densities for the noise. Then the plotted data is obtained with a moving average filter. The solid curve shows the Gaussian distribution function calculated for the best fitting. As shown in this figure the low frequency components of noise obtained for the periodic square signal drive also obey the Gaussian distribution. These results suggest that

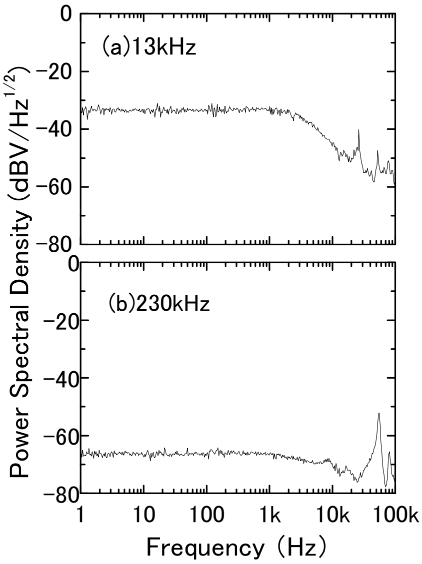

Fig.6. Power spectral densities of the VCO input signal obtained for the periodic square signal as a reference input with the variation of signal frequency. (a)13kHz and (b)230kHz.

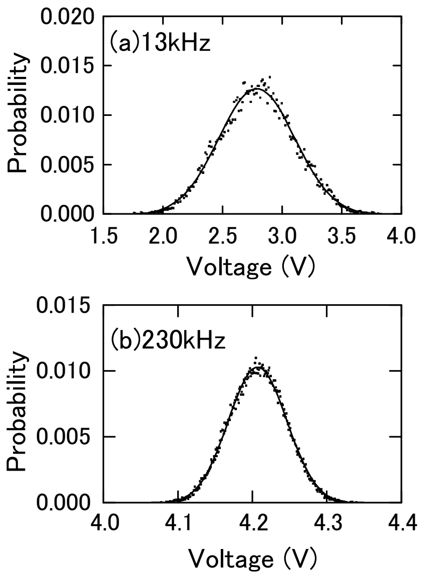

Fig.7. Amplitude probability distributions of the VCO input voltage obtained with a moving average filter. The frequency of the periodic square signal is (a) 13kHz and (b) 230kHz, respectively. The solid curve shows the Gaussian distribution function calculated for the best fitting.

chaotic oscillation for the drive frequency outside of the lock range can also be the Gaussian noise even though the dynamics is expected to be different from Eq.(2). This heuristics might open another way to generate the white Gaussian noise.

As shown in Figs.5 and 7 the obtained white noise has non-zero mean. This finite mean value is the voltage offset which is approximately equals to $V_{DD}/2$ in volt and determines a center frequency for the circuit driven with a single power supply. So this is not a component of the noise itself. In this reason the presence of this offset voltage is excluded in our arguments.

4.Concluding remarks

The wideband noise obtained by adjusting the system parameters for a CMOS PLL circuit operating with a single power supply was studied and confirmed to be the white Gaussian noise. The other parameter values in addition to the finite frequency offset for the VCO were also adopted to survey another conditions yielding wideband noise generation, and then it was found that the white noise could be easily obtained whenever the broadband noise was obtained. This study as well as the previous works suggests that a PLL can be used as a white Gaussian noise generator. There are two terminal voltages can be used as candidates for white noise source: the chaotic PD output voltage and the chaotic VCO input voltage, as long as their high frequency components are filtered out. This means that the origin of the Gaussian noise is the low frequency components resulted by the phase jitter of the square PD output.

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