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Reconstructing Connectivity of Nonlinear Dynamical Systems from Marked Point Processes

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Abstract—Complex phenomena are ubiquitous in the real world and these phenomena are often observed as marked point process, for example, transactions in stock markets, seismic events, neural systems, and so on. To investigate possible nonlinear dynamics that produce such complex phenomena, one of the important tasks is to understand how dynamic systems are connected. In this paper, we propose a method for reconstructing connectivity of nonlinear dynamical systems by transforming marked point processes to continuous time series.

1. Introduction

Interactions between nonlinear dynamical systems often produce complicated behavior. Such complicated behavior usually depend on connectivity in networks, that is, network topology. Thus, to analyze, model or predict complicated behavior produced from the networks, it is inevitable and essential to understand the network structures as well as the nonlinear dynamics.

Although it is not so easy to investigate the interactions directly, recent developments in measurement technologies makes it possible to observe multivariate time series data. Then, it is possible to estimate the network topology through the multivariate time series data.

If an observed time series is continuous and smooth, and sampled by a fixed interval, the network structure can be estimated through statistical measures [1–3] applied to the continuous time series. However, the nonlinear dynamical systems are often observed as event sequences, and it is difficult for us to directly apply the conventional statistical measures [1–3] to such event sequences. Then, it is an important issue to develop a method to estimate network structures in case that event sequences are observed. To resolve this issue, we have already proposed methods of estimating network structures only from the event sequences [4,5]. In Refs. [4, 5], we have treated observed event sequences as a point process, which means that the observed event sequences only have event timings. However, in the real world, additional information with event sequences can be essential for several phenomena, for example, financial systems and seismic events. Such sequences are often referred as a marked point process. If we use not only the information of the event timing but also the additional information, we can estimate more precisely the network structures. However, it is not so easy to treat such marked point processes directly. Then, in this paper, we proposed a method for transforming a marked point process data into a continuous time series to analyze such event sequences. After transforming the marked point processes into continuous time series, we applied the partialization analysis to the transformed continuous time series and estimated connectivity of nonlinear dynamical systems. We used two mathematical models: the coupled Rössler systems [6] and the coupled Lorenz systems [7].

2. Method

We used a kernel density estimator for transforming an event sequence into a continuous time series. To apply this method to a marked point process data, we modified the kernel density estimator by using the additional information. Let us define the lth event timing of the ith marked point process data as $t_i^l (l = 1, 2, \ldots, N)$, and the additional information at t_i^l as $h(t_i^l)$. Then, we use the following equations to transform the marked point process data $h(t_i^l)$ into the continuous time series $f_i(t)$:

$$f_i(t) = \sum_{\substack{l=1, \\ t-\frac{T}{2} \leq t_i^l \leq t+\frac{T}{2}}}^{N} K\Big(\frac{t - t_i^l}{T}\Big) h(t_i^l), \qquad (1)$$

$$K(t) = \frac{1}{2}(1 + \cos 2\pi t), \qquad (2)$$

where T is the bandwidth and $K(t)$ is the Hanning window function. Statistical characteristics of the transformed time series depend on the bandwidth T. Thus, it is important to decide the bandwidth T appropriately. In the simulations, we set the bandwidth when the correlation coefficient takes the highest value.

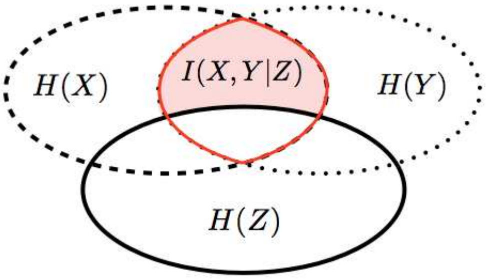

Figure 1: Mutual information $I(X, Y)$ is surrounded region by red. Partial mutual information $I(X, Y|Z)$ is pink region.

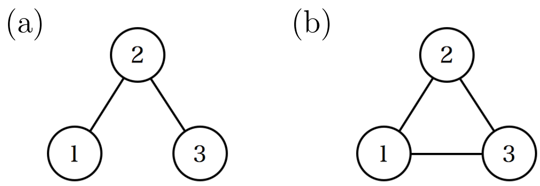

Figure 2: Coupling structures: (a) true structure and (b) misestimated results for the coupling structure by the mutual information.

To estimate the connectivity, we applied the partial mutual information analysis to the transformed continuous time series. The mutual information is a measure which identifies nonlinear statistical dependencies. The mutual information I between two variables X and Y is described by:

$$I(X, Y) = H(X) + H(Y) - H(X, Y), \tag{3}$$

where $H(X)$ is the marginal entropy of X, $H(X, Y)$ is the joint entropy between X and Y. However, the mutual information can be spuriously biased if two elements are indirectly connected. Partial mutual information is effective to remove such spurious bias [3]. The partial mutual information $I(X, Y|Z)$ is given by:

$$\begin{aligned} I(X, Y|Z) &= H(X, Z) + H(Y, Z) \\ &\quad - H(X, Y, Z) - H(Z). \end{aligned} \tag{4}$$

Namely, the partial mutual information $I(X, Y|Z)$ is the mutual information $I(X, Y)$ from which the effect of Z is removed (Fig. 1).

3. Simulations

To confirm the effectiveness of our method, we produced marked point process data from nonlinear dynamical systems.

3.1. Coupled Rössler Systems

The three coupled Rössler systems [6] are described by the following equations:

$$\left\{ \begin{array}{rcl} \dot{x}_i &=& -\omega_i y_i - z_i + \sum_{i \neq j} k_{ij}(x_j - x_i) \\ && +\sigma_i \eta_i, \\ \dot{y}_i &=& \omega_i x_i + a y_i, \\ \dot{z}_i &=& b + x_i z_i - c z_i, \end{array} \right. \tag{5}$$

with $i, j = 1, 2, 3$. We set the parameters $a = 0.15, b = 0.2, c = 10, \omega_1 = 1.03, \omega_2 = 1.01$ and $\omega_3 = 0.99$. These parameters lead to chaotic behavior of the systems. The term $\sigma_i \eta_i$ is noise. We set $\sigma_i = 1.5$ and η_i is Gaussian random number with a mean value and standard deviation of zero and unity, respectively. The coupling strengths are set to $k_{12} = k_{21} = k_{23} = k_{32} = 0.2$ and $k_{13} = k_{31} = 0$. The coupling structure of the systems is shown in Fig. 2(a). We defined the lth event timing t_i^l as the time when $|x_i(t)|$ takes the lth local maxima and the amplitude of the lth event as $|x_i(t_i^l)|$.

3.2. Coupled Lorenz Systems

In the coupled Lorenz systems, we consider the delays and unidirectional network. The three coupled Lorenz systems [7] are described by the following equations:

$$\left\{ \begin{array}{rcl} \dot{x}_i(t) &=& -\rho x_i(t) + \rho y_i(t), \\ \dot{y}_i(t) &=& -x_i(t) z_i(t) + r x_i(t) - y_i(t) \\ && + \sum_{j \neq i} k_{ij} y_j^2(t - \tau_{ij}), \\ \dot{z}_i(t) &=& x_i(t) y_i(t) - b z_i(t), \end{array} \right. \tag{6}$$

with $i, j = 1, 2, 3$. We set the parameters $\rho = 10, r = 28$, and $b = 8/3$ yielding a chaotic behavior. The coupling strengths are set to $k_{12} = k_{23} = 0.8$ and $k_{ij} = 0$ otherwise. The coupling structure of the systems is shown in Fig. 2(a). The delays are set to $\tau_{12} = 5$ and $\tau_{23} = 15$ time steps. We defined the lth event timing t_i^l as the time when $|y_i(t)|$ takes the lth local maxima and the amplitude of the lth event as $|y_i(t_i^l)|$.

4. Results

First, we estimated the connectivity of systems only from event timings (we set $h(t_i^l) = 1$ for $i = 1, 2, 3$ and $l = 1, 2, \cdots, N$). We show the results of the cross mutual information (Figs. 3 (a)–(c) and 5 (a)–(c)) and partial mutual information (Figs. 3 (d)–(f) and 5 (d)–(f)). From the results, it is difficult to detect coupling because no peaks are found. In contrast, if we use not only event timings t_i^l but also the additional information $h(t_i^l)$, the results show clear peaks (Figs. 4 and 6). From the results of the cross mutual information (Figs. 4 (a)–(c) and 6 (a)–(c)), we can identify sharp peaks. These results indicate the connectivity of the coupled Lorenz systems as Fig. 2(b). However, the connectivity between the systems 1 and 3 is misestimated because of a spurious bias. To remove the

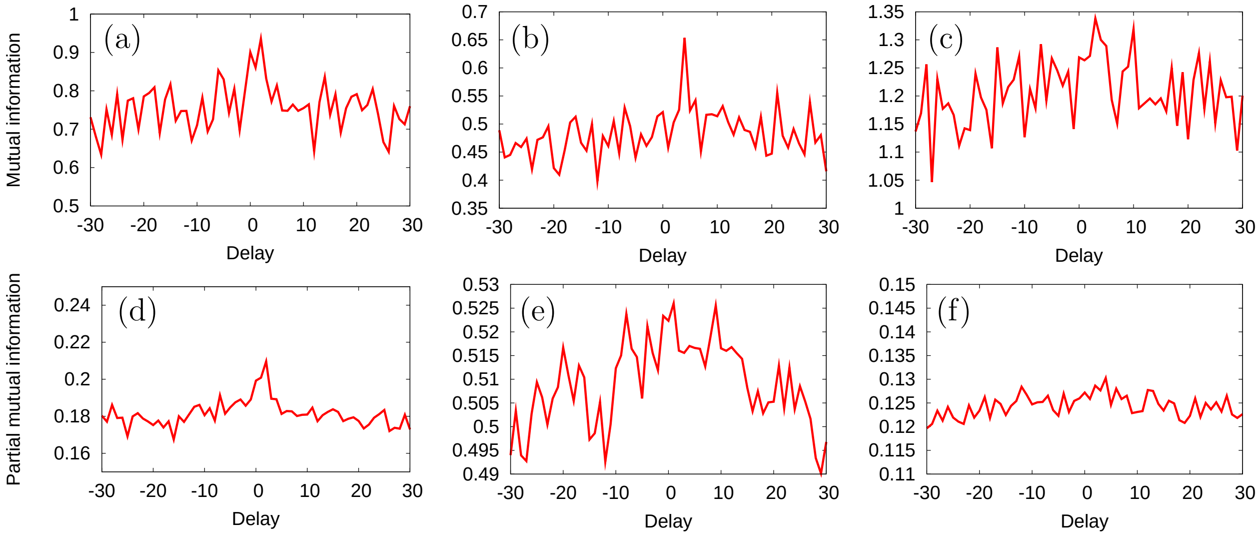

Figure 3: Averaged mutual information for 20 trials: (a) $I(f_1, f_2)$, (b) $I(f_1, f_3)$ and (c) $I(f_2, f_3)$, and averaged partial mutual information for 20 trials: (d) $I(f_1, f_2|f_3)$, (e) $I(f_1, f_3|f_2)$ and (f) $I(f_2, f_3|f_1)$ when we use only event timings t_i^l for the coupled Rössler systems.

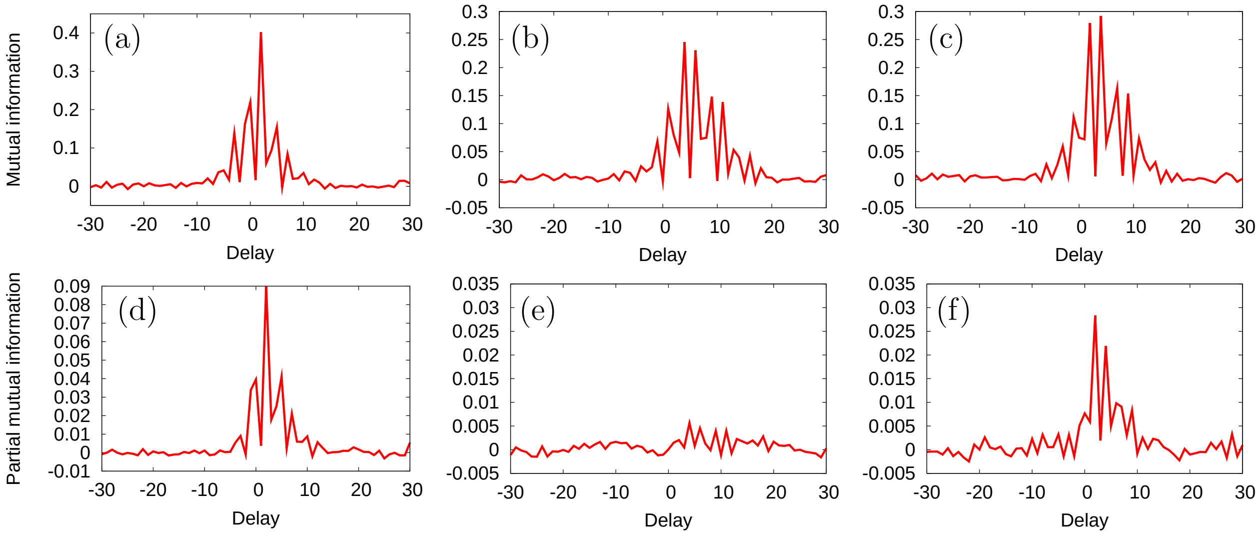

Figure 4: Averaged mutual information for 20 trials: (a) $I(f_1, f_2)$, (b) $I(f_1, f_3)$ and (c) $I(f_2, f_3)$. Averaged partial mutual information for 20 trials: (d) $I(f_1, f_2|f_3)$, (e) $I(f_1, f_3|f_2)$ and (f) $I(f_2, f_3|f_1)$ when we use not only event timings t_i^l but also the amplitude information $h(t_i^l)$ for the coupled Rössler systems.

spurious bias, we used the partial mutual information. In Figs. 4(d)–(f) and 6(d)–(f), we show the results of the partial mutual information. The results shown in Figs. 4(e) and 6(e) indicate that no connection exists between the systems 1 and 3. These results indicate that we can estimate the connectivity of the systems as Fig. 2(a) which is the true connectivity of the systems. In addition, we can clearly identify the delays $\tau_{12} = 5$ and $\tau_{23} = 15$ (Fig. 6(d)–(f)), because the peaks occur at these values.

5. Conclusion

In this paper, we proposed a new method for estimating connectivity of nonlinear dynamical systems from a marked point process. First, we transformed the marked point process data to continuous time series by the proposed method which is based on a kernel density estimator. Then, we applied the partialization analysis to the transformed time series. As a result, we can estimate the connectivity of the coupled Rössler systems and the coupled Lorenz systems.

As a future work, we have to optimize how to de-

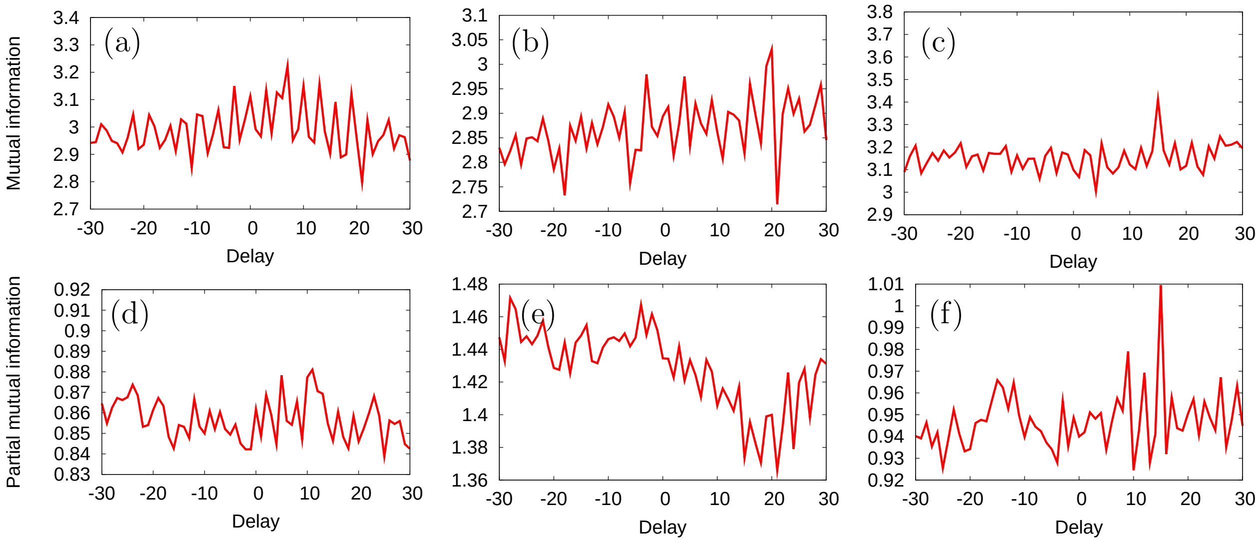

Figure 5: Averaged mutual information for 20 trials: (a) $I(f_1, f_2)$, (b) $I(f_1, f_3)$ and (c) $I(f_2, f_3)$ and averaged partial mutual information for 20 trials: (d) $I(f_1, f_2|f_3)$, (e) $I(f_1, f_3|f_2)$ and (f) $I(f_2, f_3|f_1)$ when we use only event timings t_i^l for the coupled Lorenz systems.

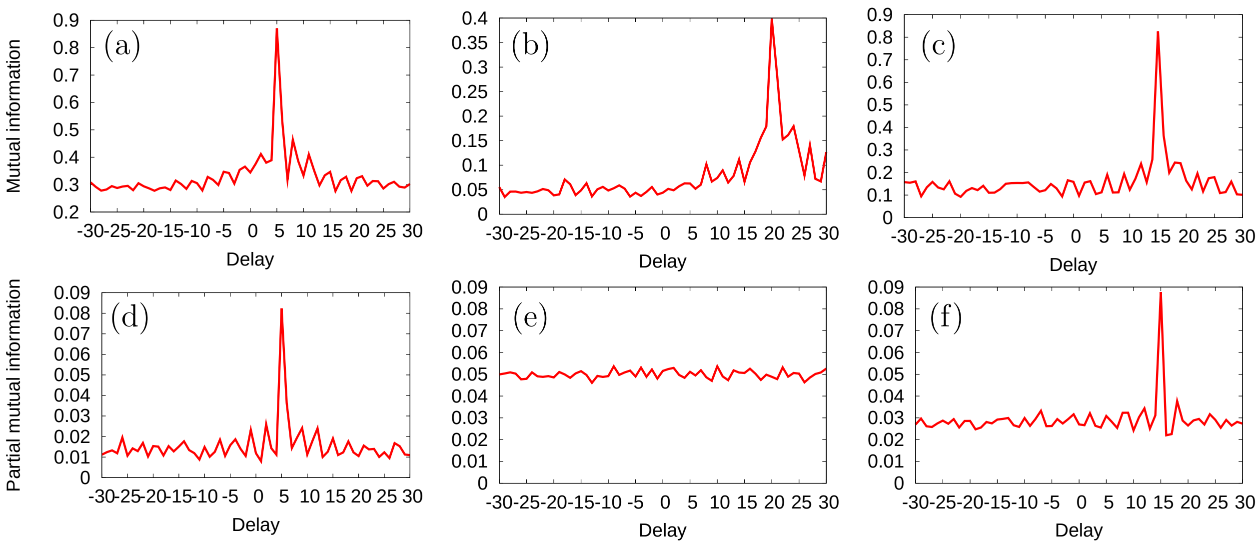

Figure 6: Averaged mutual information for 20 trials: (a) $I(f_1, f_2)$, (b) $I(f_1, f_3)$ and (c) $I(f_2, f_3)$, and averaged partial mutual information for 20 trials: (d) $I(f_1, f_2|f_3)$, (e) $I(f_1, f_3|f_2)$ and (f) $I(f_2, f_3|f_1)$ when we use not only event timings t_i^l but also the amplitude information $h(t_i^l)$ for the coupled Lorenz systems.

cide the bandwidth T only from observed marked point process data. Information of inter-spike interval distribution of event sequences and its related statistics could be important information to optimize the bandwidth.

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