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An Error Bound of a Solution for Linear Passive DC Circuits Without Constructing Circuit Equations

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Abstract—This paper gives a method to evaluate an error bound on an approximate solution of a circuit composed of linear passive resistors and dc soruces. The error bound is easily calculated from circuit topology and conductance values and is obtained without constructing circuit equations.

1. Introduction

When we solve a circuit equation by computer, the accuracy of an obtained solution $\tilde{x}$ is an important problem, in particular in circuits containing active elements [1]. Accuracy-guaranteed algorithms, which have recently been rapidly developed mainly in Japan and Germany, give an answer to the problem. There are two main methods for accuracy-guaranteed algorithms;

(i) Oishi-Rump algorithm utilizing the control of rounding mode of calculation

(ii) Method utilizing the fixed point theory

In the case of linear simultaneous equations, $Ax = b$, the Oishi-Rump algorithm seems very powerful and gives very good error bound. To derive the error bound, the Oishi-Rump algorithm requires an approximate inverse matrix, R, of A.

When we use a general-purpose circuit simulator such as the SPICE, however, we cannot get necessary information to apply the Oishi-Rump algorithm. In the case of SPICE we cannot get internal data, such as the coefficient matrix A of the circuit equation, the dimension n of A, the approximate inverse matrix of A, etc. We do not know even what kind of equations (a loop equation, a nodal equation, a mixed equation, a tabuleau equation, etc.) is used in the SPICE, though it may be guessed.

Even in such a situation can we estimate the error bound for a given circuit? In this paper we give an answer to this problem when the circuit is composed of linear passive resistors and dc sources. Fundamentally we utilize the formula used in the Oishi-Rump method, but we need neither A nor R.

If the solution is correct, then the total current flowing into each node is exactly 0, and conversely, if the total current flowing into a node is 0 at every nodes, the the solution is correct. So voltage and current error bounds may be represented by using a column vector c, of which the i-th element represents the total current flowing into the node i.

In this paper we give an error bound for nodal voltages by a product of magnification factors (defined later) and c. Here the magnification factors can be obtained in terms of circuit topology. The amount of required computation for the error bound is very few (about $O(n^2)$ additions and $O(n)$ multiplications) and the result is obtained without constructing a circuit equation.

The discussion depends on the property of hyperdominancy of nodal matrices. In the case where all dominance conditions are satisfied with inequality, an error bound is easily obtained. Most of this paper is devoted to the cases where dominance conditions hold with equality for some rows of a nodal admittance matrix.

[1] An example in which $\tilde{x}$ may considerably differ from the true solution will be presented at the conference. In this paper however we treat only a passive resistive circuit as a preliminary work.

2. Fundamental formula of accuracy-guaranteed algorithm and its circuit-theoretic interpretation

2.1. Equation and a residual vector

Let an $n \times n$ matrix be A and let

$$Ax = b \tag{1}$$

be an equation to be solved. When we solve Eq.(1) by LU decomposition etc., we can obtain not only an approximate solution $\tilde{x}$ but also an appoximate inverse matrix $R \approx A^{-1}$ in the process of calculation.

Let

$$r = [r_i] \equiv b - A\tilde{x} \tag{2}$$

be a residual vector [2].

If A is symmetric and satisfies

$$a_{ii} \geq \sum_{j=1; j\neq i}^{n} |a_{ij}| \quad (i = 1, 2, \cdots, n) \tag{3}$$

then we say that A is a hyperdominant matrix. If in particular all "$\geq$" in Eq.(3) are replaced by the inequality "$>$", then A is said to be "strictly hyperdominant. We call the conditions in Eq.(3) "dominance conditions".

2.2. Fundamental theorem

Since we study the maximum error bound, we use the infinity norm $\|\cdot\|$ for vectors and matrices. Let x_* be the exact solution of Eq.(1). The following is well-known:

Theorem 1: Assume that

$$\|RA - I\| < 1 \tag{4}$$

is satisfied. Then we have

$$\|x_* - \tilde{x}\| < \frac{\|Rr\|}{1 - \|RA - I\|} \tag{5}$$

Note that Theorem 1 holds for any R satisfying Eq.(4), that is, R need not be an approximate inverse matrix of A.

As was described in Introduction, we utilize Eq.(5) without use of both A and an approximate inverse R of A.

[2] In this paper the notation "r_i" does not mean the resistance value.

2.3. Property of linear passive resistive circuits

Let N denote a linear passive resistive circuit with $(n+1)$ nodes. Let the nodal equation for N be Eq.(1). We consider a nodal equation since it contains the necessary and sufficient information on N.

Lemma 1: The nodal matrix A of N is a hyperdominant matrix (See Eq.(3)).

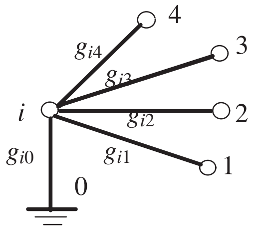

Fig. 1 Bold lines denote resistors(conductances)

Fig. 1 shows the node i of N and the resistors connected to this node, where the ground node is denoted by the node "0", the bold lines denote resistors and g_{ij} represents the conductance between nodes i and j. The nodal matrix A is given in terms of g_{ij} as

$$A = \begin{bmatrix} \sum_{j=0}^{n} g_{1j} & -g_{12} & -g_{13} & \cdots & -g_{1n} \\ -g_{21} & \sum_{j=0}^{n} g_{2j} & -g_{23} & \cdots & -g_{2n} \\ \cdots & \cdots & \cdots & \cdots & \\ -g_{n1} & -g_{n2} & -g_{n3} & \cdots & \sum_{j=0}^{n} g_{nj} \end{bmatrix} \quad (6)$$

$$g_{ii} = 0, \quad g_{ij} = g_{ji} \ \ (i, j = 1, \cdots, n) \quad (7)$$

$$a_{ii} = \sum_{j=0}^{n} g_{ij} (\geq 0), \quad a_{ij} = -g_{ij} (\leq 0) \quad (i \neq j) \quad (8)$$

As seen from Eqs.(7) and (8), we have:

Lemma 2: The sum of the elements of the i-th row of A is 0, if and only if $g_{i0} = 0$.

Lemma 3 : In the case of a strictly hyperdominant matrix we have:

$$g_{i0} \neq 0 \quad (i = 1, \cdots, n) \quad (9)$$

Lemma 4 : If the dominance conditions are satisfied with equality at all nodes, then $g_{i0} = 0 (i = 1, \cdots, n)$. This means that the node 0 and other nodes are separated each other and then $|A| = 0$. We therefore see that some circuits can be extremely ill-conditioned when all g_{i0} are very small.

In this paper we assume:

Assumption 1 : The solution $\tilde{x}_i$ is the node potential.

The i-th element of $r(= b - A\tilde{x})$ represents the total current flowing into node i. For Fig. 1 we have:

$$r_i = \sum_{j=0}^{4} g_{ij}(\tilde{x}_j - \tilde{x}_i) \quad (\tilde{x}_0 = 0) \quad (10)$$

Thus we easily calculate r from both $\tilde{x}$ and the circuit configuration without A and b in Eq.(1).

As another example of circuits having a hyperdominant matrix, we have:

Lemma 5: The coefficient matrix of a loop equation of a planar circuit is hyperdominant, if we choose loop currents appropriately.

2.4. Error bound for a strictly hyperdominant case

If A is strictly hyperdominant, then we can verify that

$$R = \mathrm{diag}\left[\frac{1}{a_{11}}, \frac{1}{a_{22}}, \cdots, \frac{1}{a_{nn}}\right] \quad (11)$$

satisfies Eq.(4).

We define the **margin figure** δ_i of the i-th row of A as:

$$\delta_i \equiv 1 - \frac{\sum_{j \neq i} |a_{ij}|}{a_{ii}} \quad (12)$$

From Eq.(4) we easily see that $0 \leq \delta_i \leq 1$. Then we have by simple calculation:

$$\|I - RA\| = \max_i \{1 - \delta_i\} \quad (13)$$

$$1 - \|I - RA\| = \min_i \delta_i \equiv \delta_{min} \quad (14)$$

$$\|x_* - \tilde{x}\| < \frac{\|Rr\|}{1 - \|RA - I\|} = \frac{1}{\delta_{min}} \|Rr\| = \frac{1}{\delta_{min}} \max_i \left[\frac{|r_i|}{a_{ii}}\right] \quad (\text{if } \delta_{min} \neq 0) \quad (15)$$

Here we call $\frac{1}{a_{ii}\delta_{min}}$ a **magnification factor**.

Lemma 6 : We can evaluate an error bound of the approximate solution $\tilde{x}$ by Eq.(15), if $\delta_{min} > 0$ and a_{ii} are known (Note that a_{ii} is given by Eq.(8)).

The amount of computation required to calculate δ_{min} and a_{ii} is about n^2 additions and $2n$ multiplications, and is called the **basic calculation**.

Circuit theoretic interpretations of these equations are as follows:

$$\frac{1}{\delta_{min}} = \frac{1}{1 - \|I - RA\|} = \max_i \frac{\sum_{j=0}^{n} g_{ij}}{g_{i0}} = \max_i \frac{a_{ii}}{g_{i0}} = \max_i \frac{\text{sum of conductances at the node } i}{\text{conductance between nodes } i\text{-}0} \quad (16)$$

$$\frac{r_i}{a_{ii}} = \frac{\text{total current flowing into node } i}{\text{Sum of conductances at the node } i} \quad (17)$$

In case of Eq.(9) we have $0 < \delta_i < 1$ $(i = 1, \cdots, n)$ and $\delta_{min} > 0$, and therefore we have:

Lemma 7 : In the case of Eq.(9) we can verify the accuracy of a solution $\tilde{x}$ **without a circuit equation** by Eqs.(15) – (17). Required computation is about n^2 additions and $2n$ multiplications.

3. The case where some rows of A satisfy dominance conditions with equality

Suppose in this section that $g_{i0} = 0$ at some nodes i.

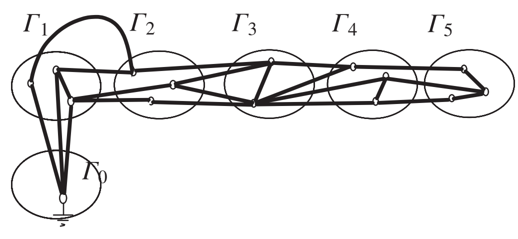

Fig. 2 General configuration ($m = 5$)

We can classify all nodes of N into Γ_k $(k = 0, 1, \cdots, m)$ as follows: First let $\Gamma_0 = \{0\}$ and let Γ_1 be a set of nodes such that each node in Γ_1 is directly connected (via a bold line) to the node in Γ_0. Similarly Γ_k is a set of nodes such that each node in Γ_k is directly connected to the node in

Γ_{k-1} but not directly connected to a node in Γ_l $(l < k-1)$. Thus Γ_k $(k = 0, 1, \cdots, m)$ are uniquely determined and any circuit can be drawn as in Fig. 2.

For simplicity we let $m = 5$ in this paper. Then the conductance matrix A of N can be written as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 \\ 0 & A_{32} & A_{33} & A_{34} & 0 \\ 0 & 0 & A_{43} & A_{44} & A_{45} \\ 0 & 0 & 0 & A_{54} & A_{55} \end{bmatrix} \quad \left\{ \begin{matrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \end{matrix} \right\} \tag{18}$$

Then from the definition of Γ_k we see that the following important lemma holds:

Lemma 8:

(i) The dominance conditions of the rows of Γ_1 are satisfied with inequality.

(ii) The dominance conditions of the rows of $\Gamma_2 \sim \Gamma_m$ are satisfied with equality.

(iii) Each row of the submatrices $A_{k+1,k}$ $(k = 1, 2, m-1)$ in Eq.(18) always has at least one nonzero element.

That is,

$$a_{ii} - \sum_{j \neq i}^{n} |a_{ij}| = a_{ii} + \sum_{j \neq i}^{n} a_{ij} = g_{i0} (> 0) \quad (i \in \Gamma_1) \tag{19}$$

$$a_{ii} - \sum_{j \neq i}^{n} |a_{ij}| = a_{ii} + \sum_{j \neq i}^{n} a_{ij} = 0 \quad (i \in \Gamma_2 \sim \Gamma_m) \tag{20}$$

3.1. Operation 1: Multiply each column of A in Eq.(18) by positive constants

We multiply the columns of A by ρ's as follows:

$$\hat{A} = \begin{bmatrix} \rho_1 A_{11} & \rho_2 A_{12} & 0 & 0 & 0 \\ \rho_1 A_{21} & \rho_2 A_{22} & \rho_3 A_{23} & 0 & 0 \\ 0 & \rho_2 A_{32} & \rho_3 A_{33} & \rho_4 A_{34} & 0 \\ 0 & 0 & \rho_3 A_{43} & \rho_4 A_{44} & \rho_5 A_{45} \\ 0 & 0 & 0 & \rho_4 A_{54} & \rho_5 A_{55} \end{bmatrix} \tag{21}$$

$$\hat{A} = AU \tag{22}$$

$$U = \mathrm{diag}\,[\rho_1, \cdots, \rho_1, \rho_2, \cdots, \rho_2, \cdots, \rho_m, \cdots, \rho_m] \tag{23}$$

where we assume:

$$\rho_1 < \rho_2 < \rho_3 < \rho_4 < \rho_5 \tag{24}$$

and ρ's are values to be determined later.

Assumption 2: Suppose that by choosing ρ_k $(k = 1, 2, \cdots, m)$ appropriately we can make all rows of $\hat{A}$ in Eq.(21) satisfy the dominance conditions with **inequality**.

Then we can apply Lemma 7 in the previous section. So for

$$R = R_{Dom}(\hat{A}) = \mathrm{diag}\,[\frac{1}{\hat{a}_{11}}, \frac{1}{\hat{a}_{22}}, \cdots, \frac{1}{\hat{a}_{nn}}] \tag{25}$$

$||R\hat{A} - I|| < 1$ is satisfied and therefore Theorem 1 can be applied for $\hat{A}$. Here Theorem 1 should be modified slightly as:

Theorem 2: If for $\exists U$ and $\exists R$

$$||RAU - I|| < 1 \tag{26}$$

is satisfied, then we have

$$||x_* - \tilde{x}|| < \frac{||U|| ||Rr||}{1 - ||I - RAU||} \tag{27}$$

3.2. Determination of ρ's in Operation 1

We have the following:

Theorem 3: We can choose ρ_k $(k = 1, 2, \cdots, m)$ appropriately so that Assumption 2 is satisfied.

Proof of Theorem 3)

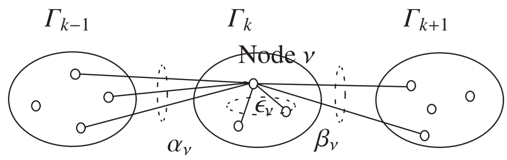

Fig. 3 Definition of α_ν and β_ν

Let ν be a node in Γ_k. Referring to Fig. 3, we define α_ν, β_ν and ϵ_ν as follows:

$$\alpha_\nu \equiv \sum_{j \in \Gamma_{\nu-1}} g_{\nu,j}, \quad \beta_\nu \equiv \sum_{j \in \Gamma_{\nu+1}} g_{\nu,j}, \quad \epsilon_\nu \equiv \sum_{j \in \Gamma_\nu} g_{\nu,j} \tag{28}$$

$$a_{\nu,\nu} \equiv \alpha_\nu + \beta_\nu + \epsilon_\nu, \quad \alpha'_\nu \equiv \frac{\alpha_\nu}{a_{\nu,\nu}}, \quad \beta'_\nu \equiv \frac{\beta_\nu}{a_{\nu,\nu}}, \tag{29}$$

Thus α_ν (or β_ν or ϵ_ν, respectively) means the sum of conductances connected from the node ν to the the nodes in Γ_{k-1} (Γ_{k+1} or Γ_k), and α'_ν (resp., β'_ν) are defined as the ratio of α_ν (resp., β_ν) to total conductances connected to the node ν.

We want to determine ρ's such that not only all of the dominance conditions in $\hat{A}$ are satisfied with inequality but also δ_{min} defined for $\hat{A}$ may be largest. Unfortunately it is difficult to find the optimum ρ's without much computations. So here we give formulae obtained heuristically. The followings are conditions for dominance conditions to hold with inequality.

$$\rho_1 g_{i_1,0} - (\rho_2 - \rho_1) \sum_{j \in \Gamma_2} g_{i_1 j} > 0 \quad (i_1 \in \Gamma_1) \tag{30}$$

$$(\rho_2 - \rho_1) \sum_{j \in \Gamma_1} g_{i_2,j} - (\rho_3 - \rho_2) \sum_{j \in \Gamma_3} g_{i_2,j} > 0 \quad (i_2 \in \Gamma_2) \tag{31}$$

$$(\rho_3 - \rho_2) \sum_{j \in \Gamma_2} g_{i_3,j} - (\rho_4 - \rho_3) \sum_{j \in \Gamma_4} g_{i_3,j} > 0 \quad (i_3 \in \Gamma_3) \tag{32}$$

$$(\rho_4 - \rho_3) \sum_{j \in \Gamma_3} g_{i_4,j} - (\rho_5 - \rho_4) \sum_{j \in \Gamma_5} g_{i_4,j} > 0 \quad (i_4 \in \Gamma_4) \tag{33}$$

$$(\rho_5 - \rho_4) \sum_{j \in \Gamma_4} g_{i_5,j} > 0 \quad (i_5 \in \Gamma_5) \tag{34}$$

Since "$\sum_{j \in \Gamma_{\nu+1}} g_{i_\nu,j} > 0$ for $\exists i$, we see that Eqs.(30)–34) are satisfied for choosing $\rho_i - \rho_{i-1} (> 0)$ sufficiently small.

Since

$$\delta_i = 1 - \eta_i = \frac{\hat{a}_{ii} - \sum_{j \neq i} |\hat{a}_{ij}|}{\hat{a}_{ii}} = \frac{\Delta_i}{\hat{a}_{ii}} \tag{35}$$

the problem is to find

$$\max_{\rho_i} \left[\min_i \left\{ \frac{\Delta_1}{\hat{a}_{11}}, \frac{\Delta_2}{\hat{a}_{22}}, \frac{\Delta_3}{\hat{a}_{33}}, \cdots, \frac{\Delta_n}{\hat{a}_{nn}} \right\} \right] \tag{36}$$

Let

$$\rho'_1 = \frac{\rho_2}{\rho_1}, \quad \rho'_2 = \frac{\rho_3}{\rho_2}, \quad \rho'_3 = \frac{\rho_4}{\rho_3}, \quad \rho'_4 = \frac{\rho_5}{\rho_4} \tag{37}$$

$$g'_{i_1,0} = \frac{g_{i_1,0}}{a_{i_1,i_1}}, \quad \text{etc} \tag{38}$$

Then from Eqs.(28), 29), (38) we have $\alpha'_\nu = \sum_{j\in\Gamma_{\nu-1}} g'_{i_\nu,j}$ and $\beta'_\nu = \sum_{j\in\Gamma_{\nu+1}} g'_{i_\nu,j}$, and from Eqs.(30)–(34) we have

$$h_1 \equiv \min_{i_1}(g'_{i_1,0} - (\rho'_1 - 1)\beta'_1 \quad (i_1 \in \Gamma_1) \tag{39}$$

$$h_2 \equiv \min_{i_2}(1 - (\rho'_1)^{-1})\alpha'_2 - (\rho'_2 - 1)\beta'_2 \quad (i_2 \in \Gamma_2) \tag{40}$$

$$h_3 \equiv \min_{i_3}(1 - (\rho'_2)^{-1})\alpha'_3 - (\rho'_3 - 1)\beta'_3 \quad (i_3 \in \Gamma_3) \tag{41}$$

$$h_4 \equiv \min_{i_4}(1 - (\rho'_3)^{-1})\alpha'_4 - (\rho'_4 - 1)\beta'_4 \quad (i_4 \in \Gamma_4) \tag{42}$$

$$h_5 \equiv \min_{i_5}(1 - (\rho'_4)^{-1})\alpha'_5 \quad (i_5 \in \Gamma_5) \tag{43}$$

Though the condition $h_1 = h_2 = h_3 = h_4 = h_5$ is desirable, it need not be satisfied rigorously for our purpose. Let

$$\epsilon_i \equiv \rho'_i - 1 \quad\Rightarrow\quad 1 - (\rho'_i)^{-1} = \frac{\epsilon_i}{1+\epsilon_i} \tag{44}$$

So Eqs.(37)–(41) can be rewritten as (In the following we omit "$\min_i$" for a while).

$$\alpha'_1 - \epsilon_1\beta'_1 = h_1 > 0 \tag{45}$$

$$\frac{\epsilon_1}{1+\epsilon_1}\alpha'_2 - \epsilon_2\beta'_2 = h_2 > 0 \tag{46}$$

$$\frac{\epsilon_2}{1+\epsilon_2}\alpha'_3 - \epsilon_3\beta'_3 = h_3 > 0 \tag{47}$$

$$\frac{\epsilon_3}{1+\epsilon_3}\alpha'_4 - \epsilon_4\beta'_4 = h_4 > 0 \tag{48}$$

$$\frac{\epsilon_4}{1+\epsilon_4}\alpha'_5 = h_5 > 0 \tag{49}$$

Noting that $h_1 < \alpha'_1$ and $h_i < \frac{\epsilon_{i-1}}{1+\epsilon_{i-1}}\alpha'_i$, we introduce new parameters p_i $(0 < p_i < 1)$ such that

$$h_1 \equiv (1-p_1)\alpha'_1, \quad h_i \equiv (1-p_i)\frac{\epsilon_{i-1}}{1+\epsilon_{i-1}}\alpha'_i, \ (i = 2, \cdots, 4) \tag{50}$$

and for convenience we set $\gamma_i \equiv \alpha'_i/\beta_i$ $(i = 1, 2, 3, 4)$. Then we have from Eqs.(45)–(49)

$$\epsilon_1 = p_1\gamma_1 \tag{51}$$

$$\epsilon_2 = p_2\gamma_2\frac{\epsilon_1}{1+\epsilon_1} = \frac{p_1p_2\gamma_1\gamma_2}{1+p_1\gamma_1} \tag{52}$$

$$\epsilon_3 = \frac{p_1p_2p_3\gamma_1\gamma_2\gamma_3}{1+p_1\gamma_1+p_1p_2\gamma_1\gamma_2} \tag{53}$$

$$\epsilon_4 = \frac{p_1p_2p_3p_4\gamma_1\gamma_2\gamma_3\gamma_4}{1+p_1\gamma_1+p_1p_2\gamma_1\gamma_2+p_1p_2p_3\gamma_1\gamma_2\gamma_3} \tag{54}$$

Then we have

$$\frac{\epsilon_i}{1+\epsilon_i} = \frac{\prod_{j=1}^k(p_j\gamma_j)}{1+\sum_{k=1}^i\left(\prod_{j=1}^k(p_j\gamma_j)\right)} \quad (i = 1,2,3,4) \tag{55}$$

From Eqs.(45)–(49) and Eqs.(51) –(54) we have

$$\begin{aligned} h_1 &= \min_{i\in\Gamma_1}(1-p_1)\alpha'_1 \\ h_{i+1} &= \min_{i\in\Gamma_{i+1}}(1-p_i)\alpha'_i\frac{\prod_{j=1}^k(p_j\gamma_j}{1+\sum_{k=1}^i\left(\prod_{j=1}^k(p_j\gamma_j)\right)} \quad (i = 1,\cdots,4) \\ h_5 &= \min_{i\in\Gamma_5}\alpha'_5\frac{\prod_{j=1}^4(p_j\gamma_j}{1+\sum_{k=1}^i\left(\prod_{j=1}^4(p_j\gamma_j)\right)} \end{aligned} \tag{56}$$

The original problem was to find $\max_{p_i}\min\{h_i\}$.

Lemma 9 : δ_i can be determined by $\gamma_i(=\alpha_i/\beta_i)$ and α'_i. Required computation =basic calculation+ (about n additions and $5n$ multiplications).

Important Remark: Since all calculations in Eqs.(51)–(56) are the sum or the multiplication of positive values, the cancellation of significant digits does not occur.

Example 1: For the circuit in Fig. 4 we choose $p = 0.5$. Then we have the following results.

$$\begin{array}{lll} \alpha'_{1_1} = 1/3 & \alpha'_{1_2} = 1/6 & \alpha'_{1_3} = 1/3 \\ \beta'_{1_1} = 1/3 & \beta'_{1_2} = 2/6 & \beta'_{1_3} = 1/3 \\ \alpha'_{2_1} = 2/2 & \alpha'_{2_2} = 1/2 & \alpha'_{2_3} = 2/3 \end{array}$$

$\delta_1 = 0.833,\ \delta_2 = 0.307,\ \delta_{min} = 0.307,\ 1/\delta_{min} \approx 3.26$

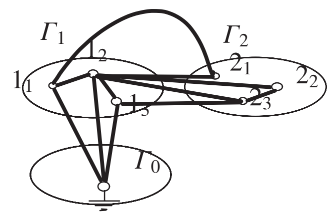

Fig. 4 Example 1 where all conductances are 1[S]

Conclusion

We gave an error bound of a solution $\tilde{x}$ without constructing circuit equations. In addition to the method in Section 3 we can also give another method which performs additions of rows on A. Due to the lack of space we omitted it. In both methods the required calculation for error bound is $O(n)$ additions and $O(n)(\sim 8n)$ multiplications.

Most of the results do not require the symmetry of the coefficient matrix A and therefore is useful for more general type of hyperdominancy.

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