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Changeover of Mobile to Stationary ILM on Excitation near a Free End of Finite Fermi-Pasta-Ulam Chains

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Abstract– Mobile localized oscillations in finite chains of the Fermi-Pasta-Ulam (FPU) of β-type are studied numerically. With both ends of the chains free, the localized oscillation is imposed initially near one end of the system. While localized near the end for a moment, it begins to be propagated down the system subject to reflections at both ends. In this process, there occurs a case in which the speed and the sense of propagation change irregularly. The mobile localized oscillation gradually loses its energy by radiation and is settled down into the stationary oscillation.

1. Introduction

It is now known that the intrinsic localized modes (ILMs) or the discrete breathers (DBs) are generic in spatially periodic, discrete and nonlinear systems (see, for example [1-5].) It is also known that in the system of celebrated FPU-β chains [6], which spatially extends infinitely, the stationary type and mobile type of ILMs can be excited [7]. There are two types of the stationary ILMs, the odd and even modes [1, 8], which have been shown that the former is linearly unstable while the latter stable in the FPU-β chains [9]. As for the mobile ILMs, however, the distinction between the odd and even modes is meaningless because they can take both modes and even their hybrid modes in the course of propagations [10].

In this paper, we consider finite chains of the FPU-β type and study numerically the localized oscillation imposed initially near one end of the system. Initial displacements of the masses are given in the form of the stationary, odd mode. The existence of the ends sets the localized oscillation in motion from its excited position and has effects on the propagation of the oscillation. Many calculations are carried out for the combinations of values of three parameters: the number of masses, N, the offset of the center position of initial localized displacements from the center of the chain, σ, and the maximum of initial displacement, A.

2. Formulation

We consider the FPU-β type chains which consist of N (>>2) identical masses numbered consecutively by integer j ($1 \le j \le N$), arranged along the x direction and interacted with the neighboring masses through the nonlinear potentials. The potential, V, is given by the form of a quadratic plus quartic function of relative displacement r between the adjacent mass:

$$V(r) = r^2 + \beta r^4 / 4, \tag{1}$$

where β is the parameter on the nonlinear coupling strength and to be set $\beta = 4$ in the following calculations in this paper. The motions of masses are restricted in the x-axis.

We denote the position from the equilibrium point of the j th mass by $x_j(t)$, t being the time. The equation of motion for each mass with the mass of unity is expressed as follows:

$$\begin{aligned} \ddot{x}_1 &= -r_{1+} - \beta r_{1+}^3, \\ \ddot{x}_j &= -\left(r_{j+} - r_{j-}\right) - \beta\left(r_{j+}^3 - r_{j-}^3\right), \\ \ddot{x}_N &= -r_{N-} - \beta r_{N+}^3, \end{aligned} \tag{2}$$

where the dot means the derivation with regard to t, $r_{j\pm}$ are the relative displacements between the j and $j \pm 1$ th masses:

$$\begin{aligned} r_{j-} &= x_j - x_{j-1}, \\ r_{j+} &= x_{j+1} - x_j, \end{aligned} \tag{3}$$

for $1 \le j \le N$. r_{1-} and r_{N+} are taken to vanish due to the free boundary conditions at the both ends of system. The N equations with the boundary and initial condition for positions are to be solved simultaneously in the next section. Linear analysis to these equations shows that all the angular eigenfrequencies ω_n ($n = 1, \cdots, N-1$) lie below the limit value $\omega_\infty = 2$.

3. Numerical Analysis and Results

3.1. Initial Condition

The initial conditions are given by values for x_j in the form of the type of odd mode:

$$\begin{aligned} x_{c-\sigma}(0) &= A, \\ x_{c-\sigma\pm1}(0) &= -A/2, \end{aligned} \tag{4}$$

where c, σ and A are constants, c=(N+1)/2 is the center of the system for N taken to be odd number (N = 65), and the integer σ ($0 \le |\sigma| \le (N+1)/2$) indicates the offset of the center position of deflection from c. Initial velocities of all masses are taken to vanish, $\dot{x}_j(0)=0$. The initial positions of masses for the special cases σ=0 and σ=31 are shown in Fig. 1.

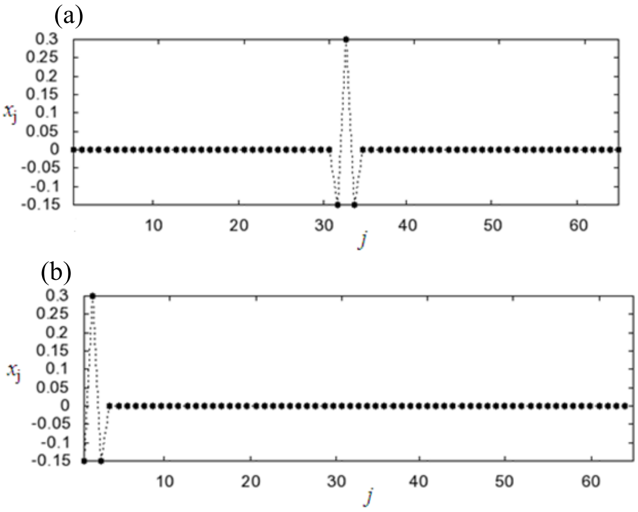

Fig.1. Initial positions of the masses for (a)σ=0, (b)σ=31 with A = 0.30.

3.2. Numerical Results

The nonlinear simultaneous equations (2) are solved by the standard Runge-Kutta method with the free boundary conditions at both ends of the system and appropriate initial conditions (4). The accuracy of numerical solutions is checked by monitoring the total energy to be conserved. In the following calculations, the relative error is found to be of order 10^{-12}.

In the case of σ = 0, the excitation of one stationary ILM of odd mode is confirmed. The masses in the localized part oscillate periodically in time with the frequency higher than 2. The excited ILM remains stationary at the center and preserves its initial profile for long-time calculations (Fig. 2). Even if the value of $|\sigma|$ is not much smaller than c ($|\sigma| \le 27$), the excited ILM remains stationary at its excited position unless the excited position is not so close to the end of the system that the border of localized part touches the end.

In contrast, in the case of the large value of $|\sigma|$ ($28 \le |\sigma| \le 32$), the excited localized oscillation is set in motion from its excited position and propagates the system. Typical result for σ=31 is shown in Fig. 3. The localized part is propagated toward the other end at a constant speed and is reflected at the free end (j=65) to return back to the initial position. Then it is reflected at the free end (j=1) and propagated toward the other end. This regular process repeats for a long time. Our detail observations of the ILMs show that the localized parts are neither decayed nor dispersed by reflections at ends (see Fig. 4.). The propagating speed of localized oscillation is higher as the larger value of $|\sigma|$ is taken.

In the case that the value of σ is fixed at 31, the ILM is excited for A > 0.18. As the value of A becomes larger, the frequency of ILM becomes higher. For 0.18 < A <0.36, the localized oscillation is propagated with a constant speed with reflections at both ends, which becomes smaller inversely proportional to the value of A. For the value of A beyond 0.36, the propagating speed is no longer constant (Fig. 5(a)). For A > 0.48, moreover, not only the speed but also the sense of propagation changes irregularly (Figs. 5(b) and (c)). It is observed in our calculations that such localized oscillation is finally trapped at somewhere in the system and the mobile ILM is settled down into the stationary one.

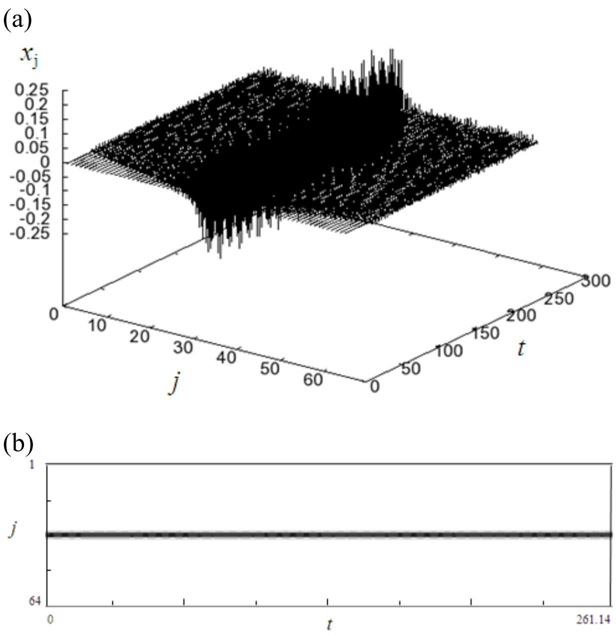

Fig.2. (a)Spatio-temporal profile and (b) the trace of the maximum amplitude in the envelope of the stationary ILM in the j-t plane for σ=0 and A=0.30.

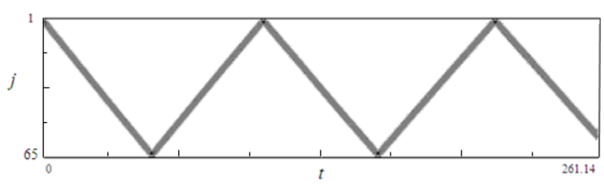

Fig.3. Trace of the maximum amplitude in the envelope of the mobile ILM in the j-t plane for σ=31 and A=0.30.

Fig.4. Snapshots of reflection of ILM at the free end (A =0.30)

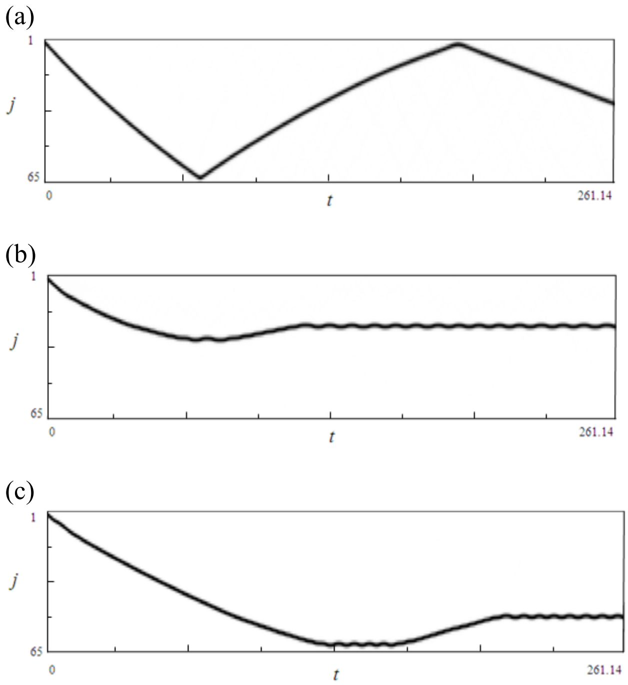

Fig. 5. Trace of the maximum amplitude in the envelope of the mobile ILM excited for σ=31 with (a) A=0.44, (b) A=0.60 and (c) A=0.67.

3. Conclusions

The propagations of nonlinear localized oscillation in the finite chains of the FPU-β type with free ends have been numerically studied. The oscillation excited initially near one end of system is set in motion by the interaction with the end of system. The existence of the ends triggers the localized oscillation to set in motion, while the localized oscillations are reflected at the free ends without any losses. It has been shown, however, that although total energy of the system is conserved, the energy of localized oscillation is lost gradually by radiation, when it moves the position from one mass to another [10]. Specially for large-amplitude case, the localized oscillation irregularly moves in the system. Such mobile ILM is finally settled down into the stationary one.

Acknowledgments

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References

[1] A. J. Sievers, S. Takeno, “Intrinsic localized modes in anharmonic crystals,” Phys. Rev. Lett., vol.61 (8), pp.970–973, 1988.
[2] S. Aubry, “Breathers in nonlinear lattices: existence, linear stability and quantization,” Physica D, vol.103, pp.201-250, 1997.
[3] S. Flach, C. R. Willis, “Discrete breathers,” Phys. Rep., vol.295, pp.181–264, 1998.
[4] Y.S. Kivshar, S. Flach (Eds.), Focus issue: “Nonlinear localized modes: physics and applications,” Chaos, vol.13, pp.586–799, 2003.
[5] D. K. Campbell, S. Flach, Y. S. Kivshar, “Localizing energy through nonlinearity and discreteness,” Phys. Today, vol.57, pp.43-49, 2004.
[6] G. Gallavotti, “The Fermi-Pasta-Ulam problem,” Springer, Berlin Heidelberg, 2008.
[7] S. Takeno, K. Hori, “A propagating self-localized mode in a one-dimensional lattice with quartic anharmonicity,” J. Phys. Soc. Jpn.,59, pp.3037-3040, 1990.
[8] J. B. Page, “Asymptotic solutions for localized vibrational modes in strongly anharmonic periodic system,” Phys. Rev. B41, pp.7835-7838, 1990.
[9] K. W. Sandusky, J. B. Page, K. E. Schmidt, “Stability and motion of intrinsic localized modes in nonlinear periodic lattices,” Phys. Rev. B46, pp.6161-6168, 1992.
[10] K. Yoshimura, Y. Doi, “Moving discrete breathers in nonlinear lattice: resonance and stability,” Wave Motion, 2007, 45, 83–99.