2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

A Method of Noise Generation with Cellular Automata for DT-CNN Annealing

Tomohiro FUJITA†, Shouta Mizoguchi†, Takeshi Kumaki†, and Takeshi OGURA†

†Ritsumeikan University 1-1-1 Noji-higashi, Kusatsu, Shiga, Japan
Email: {tfujita@se, kumaki@fc, togura@se}.ritsumei.ac.jp

Abstract—A method of random number (noise) generation is proposed. The random number is used for a kind of annealing to achieve global optimization on Discrete Time Cellular Neural Network (DT-CNN). For easy implementation on Cellular Automata (CA) and DT-CNN dedicated hardware engine, Cellular AutoMata on Content Addressable Memory (CAM^2), we utilize chaotic behavior of CA to generate noise. The average value of the noises generated by our proposed method is close to 0, and that contributes to improvement of the annealing performance.

1. Introduction

Cellular Neural Network [1] (CNN) is a kind of Neural Networks (NN). The CNN consists of lattice-shaped cells, and its special feature is having connection with only the nearest neighborhood cells.

The dynamics of the NN is governed by differential equations. In this dynamics Lyapunov function can be defined [2]. During the NN process the Lyapunov function continues to decrease, and the convergence of NN is interpreted as a minimal point of the Lyapunov function. This nature of neural networks is utilized to solve optimization problems. CNN also has Lyapunov function and it can solve quadratic assignment problem (QAP).

CNN have been applied to solve QAP in various fields to date. One of examples is solving the minimization problem of spin glass energy in statistical physics [3]. By Ising model [4] the spin glass energy is formularized by QAP. In Ising model atoms are connected with only neighbor atoms. So the spin glass energy can be mapped on the Lyapunov function of CNN. The minimization problem of spin glass energy can be solved with CNN. Ising model is applied to probabilistic information processing [5]. This class of problems which can be solved by CNN becomes a key in image processing field.

The drawback of the optimization with CNN is that in many cases the state of the network is trapped at a local minimum, and thus a global solution can not be found. To overcome this difficulty we proposed an annealing method on a CNN that realizes global optimization [6, 7]. In this scheme, noise is induced into network dynamics then gradually reduced. In this process, the state of the network is initially random but eventually becomes convergent. Due to the randomness of the noise, the network escapes from local minima.

In previous work [8] we proposed a hardware-oriented method of noise generation. The noise is generated using the chaotic behavior of class 3 cellular automata (CA) [9] on Cellular AutoMata on Content Addressable Memory (CAM^2). CAM^2 is a dedicated hardware for CA and CNN [10, 11], so that the noise can be generated easily. However, the average value of the noise generated by this method has deviation from 0, and that results with bad influence on the annealing performance. In this paper we will propose a noise generation method to improve the annealing performance.

2. DT-CNN Annealing

Discrete time CNN (DT-CNN) is a discretized time version of CNN. The DT-CNN model is illustrated by following equations.

$$x_{i,j}(t+1) = \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l)\, y_{i,j}(t) + \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l)\, u_{i,j} + I \quad (1)$$

$$y_{i,j}(t) = \frac{1}{2}\left(|x_{i,j}(t)+1| - |x_{i,j}(t)-1|\right) \quad (2)$$

where $x_{i,j}$, $y_{i,j}$ and $u_{i,j}$ are state, output and input variables, respectively, $N_r(i,j)$ is a set of neighborhood cells and $A(i,j;k,l)$ and $B(i,j;k,l)$ are parameters called templates. I is also a parameter called the threshold value. From Eq. (1) the DT-CNN model has only local connection with neighborhood cells in $N_r(i,j)$. For the DT-CNN model we can define Lyapunov function as we can do for general neural network models. The definition of the Lyapunov function is shown as follows:

$$E(t) = \frac{1}{2}\sum_{(i,j)} y_{i,j}^2(t) - \frac{1}{2}\sum_{(i,j)}\sum_{(k.l)} A(i,j;k,l)\, y_{i,j}(t)\, y_{k,l}(t) - \sum_{(i,j)}\sum_{(k.l)} B(i,j;k,l)\, y_{i,j}(t)\, u_{k,l} - \sum_{(i,j)} I\, y_{i,j}(t) \quad (3)$$

During CNN process, the Lyapunov function continue to decline at every moment. The state of CNN is converges at stationary point of the Lyapunov function. Therefore using this nature CNN can optimize functions represented by the Lyapunov energies. The Lyapunov function of CNN is a quadrature, so CNN can solve quadratic assignment problem (QAP).

The drawback of the optimization by this scheme is that the state is often captured by local minimum, and can not find global minimum. To overcome this difficulty, DT-CNN annealing method is proposed [6–8]. The DT-CNN annealing model is shown as follows:

$$\begin{aligned} x_{i,j}(t+1) &= \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l)\, y_{i,j}(t) \\ &\quad + \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l)\, u_{i,j} + I \\ &\quad + a(t)\, n_{i,j}(t) \qquad (4) \\ a(t+1) &= (1-\delta)\, a(t) \quad (0<\delta<1) \qquad (5) \end{aligned}$$

The noise term $a(t)\, n_{i,j}(t)$ is added to Eq. (1), here, $n_{i,j}(t)$ is a random number within the range from -1 to 1 and $a(t)$ is an amplitude of the noise term. $a(t)$ exponentially declines according to Eq. (5), so the noise term decreases. In early stage of the optimization process, the noise terms are large enough, so the state variables move randomly. In this situation, even if the state is captured by local minimum, the state could escape form it by the influence of the random noise. Gradually decaying the noise term, the system behaves rather deterministic than stochastic. The ability of escaping from local minima decline but the system tends to converge to minimum point. Therefore, it is expected that the ability of finding optimal point is improved by the DT-CNN annealing model.

In this method, the noise term plays an important role. Random numbers are required for every cell, so a lot of random numbers must be generated at once. Thus an efficient generation method is needed for this method. In next section, we introduce an efficient generation method using CAM^2.

3. Noise generation with CA

The DT-CNN annealing requires a noise generation method which makes a lot of random number at the same time. Our final goal is an implementation on our dedicated hardware engine, CAM^2. It is desired that the noise generation method can be easily implemented on CAM^2. We proposed a noise generation method based on class 3 CA [8]. The behavior of class 3 CA is chaotic [9], and we can use it for the noise generation.

CA are computational models proposed by Neumann [9] and consist of lattice-shaped cells. Since the architecture of CA is similar to that of the DT-CNN, we can implement CA on a universal CNN machine, CAM^2. Figure 1 shows the concept of the DT-CNN with noise, which has two layers: a DT-CNN layer and a CA layer.

The states of the cells vary on the basis of the state transition function F, which is represented by the rule number

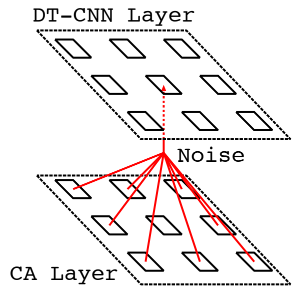

Figure 1: Hardware annealing on DT-CNN

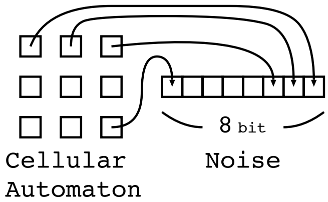

Figure 2: Noise generator

R as follows:

$$R = \sum_{n=0}^{N} \sum_{v_{i,j}=0}^{1} F(v_{i,j}, n) \times 2^{v_{i,j}+2n} \qquad (6)$$

where $v_{i,j}$ is the state of cell (i, j) and takes binary values, n is the number of neighborhood cells with state $v_{i,j}$ equal to 1, and N is the total numbers of neighborhood cells. Wolfram [9] sorted state transition functions into four classes. The functions in class 3 have chaotic behavior. Since we require disordered noise, we use CA in class 3 as the noise generator.

We generate noise $n_{i,j}$ from the CA as follows:

$$\begin{aligned} n_{i,j}(t) &= \left(\frac{v_{i-1,j-1}}{2^8} + \frac{v_{i-1,j}}{2^7} + \frac{v_{i-1,j+1}}{2^6} + \frac{v_{i,j-1}}{2^5} \right. \\ &\quad + \left. \frac{v_{i,j+1}}{2^4} + \frac{v_{i+1,j-1}}{2^3} + \frac{v_{i+1,j}}{2^2} + \frac{v_{i+1,j+1}}{2^1} \right) \times 2 \\ &\quad - 1 \qquad (7) \end{aligned}$$

The range of $n_{i,j}$ is $[-1, 1)$. Figure 2 shows the noise generation process.

The noise generation method by Eq. (7) is efficient on CAM^2. However, there is a problem on statistical property of it. Figure 3 is a histogram of the random number generated by Eq. (7). The rule number of CA was 143954. From Fig. 3, the distribution is asymmetry and has some structure. This results with the bias of distribution. The average value of these samples is -0.1548.

The deviation of the average value from 0 affects the annealing performance. We compared annealing performances with noises by Eq. (7) and with uniform random numbers [12]. From the comparison of two types of noise,

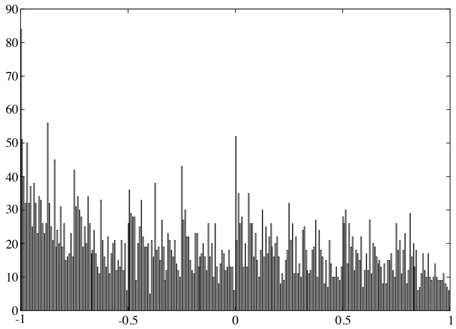

Figure 3: Histogram of random numbers generated by conventional method. Rule number 14395. The cell size is 50×50.

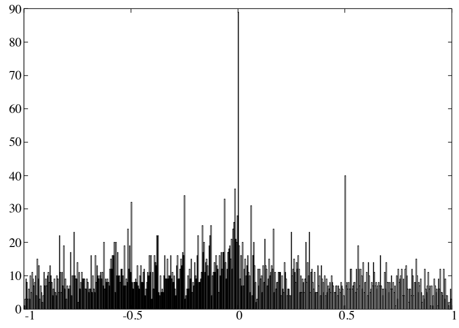

Figure 4: Histogram of random numbers generated by the proposed method. Rule number 14395. The cell size is 50×50.

we found the performance of the annealing with Eq. (7) was worse than with the uniform random number. With additional experimental fact, we found this degradation of the annealing performance is caused by the bias of the distribution [12]. To improve the annealing performance, non-biased random number is required.

4. Proposed Method

A random number generation method is proposed in this section. The method also uses CA to be easily implemented on CAM^2. The non-biased random number can be generated by this method. The proposed method is as follows:

$$\begin{aligned} n_{i,j}(t) &= (-1)^{v_{i,j}+1} \times \left(\frac{v_{i-1,j-1}}{2^8} + \frac{v_{i-1,j}}{2^7} + \frac{v_{i-1,j+1}}{2^6} + \frac{v_{i,j-1}}{2^5} \right. \\ &+ \left. \frac{v_{i,j+1}}{2^4} + \frac{v_{i+1,j-1}}{2^3} + \frac{v_{i+1,j}}{2^2} + \frac{v_{i+1,j+1}}{2^1} \right) \end{aligned} \quad (8)$$

The basic concept is same as Eq. (4), but the self-state $v_{i,j}$ determines the plus or minus of the generated number. If 0 and 1 appear with the same possibility in the state of CA cells, the distribution of this generated number is symmetric with the average value 0.

5. Experiments

Experiments of noise generation by proposed method is demonstrated in this section.

The distribution of random numbers was investigated. The random numbers were generated by CA with the rule number $R = 143954$. The cell size of CA was 50x50. The iteration of CA processes was 5000. The distribution of the generated noise is shown in Fig. 4. Comparing with the random numbers by the conventional method (Fig. 3), the distribution of random numbers generated by the proposed method has nearly symmetric structure.

Using various rule numbers of CA, we generated random numbers and evaluated the annealing performance. The rule numbers which belong to class 3 CA are shown in Tab. 1 with their index numbers.

Table 1: Index of rule number of CA

Index #	Rule #	Index #	Rule #
1	152822	12	22001
2	143695	13	63256
3	38999	14	56899
4	40053	15	40530
5	73546	16	163256
6	100728	17	173622
7	143953	18	186523
8	73298	19	144018
9	143954	20	58
10	201569	21	123657
11	56914		

The average values of the random numbers by CA with these rule numbers are shown in Fig. 5. We tried to generate both the conventional method with Eq. (7) and the proposed method with Eq. (8). The average values of the random numbers with both methods are shown in Fig. 5. The average values with the conventional method reveal larger deviation from 0. This figure also shows the average values with some rule numbers have larger deviation than others. It seems the states of the CA cells with these rule numbers do not take 0 or 1 with the same probability.

An experiment of the DT-CNN annealing with the CA based noises was conducted. We used two kinds of random noises generated by the conventional method (Eq. (7)) and the proposed method (Eq. (8)) for the annealing. In this experiment we used the following templates for the DT-CNN.

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 1.0 & 3.0 & 1.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix}, B = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}, I = 0 \quad (9)$$

The input variable $\boldsymbol{u}$ is randomly generated within the range from -4 to 4.

The optimal values of Lyapunov function obtained by

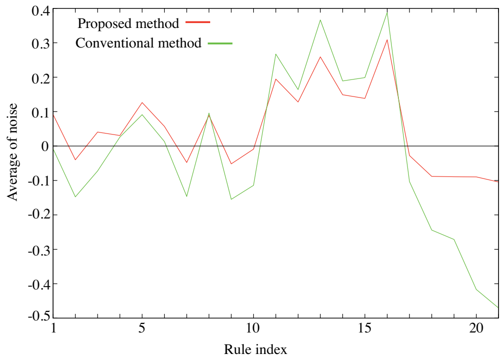

Figure 5: Average value of noise generated with CA

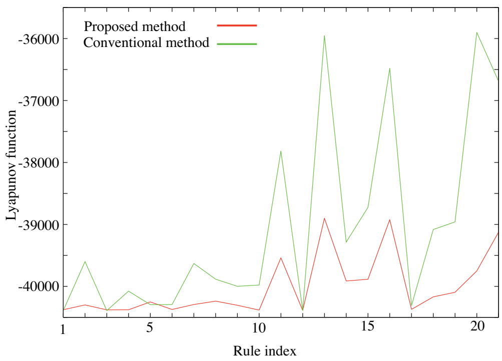

Figure 6: Optimal values of Lyapunov function by DT-CNN annealing

two methods are shown in Fig. 6. This figure shows the relation between the rule numbers of the CA used for the noise generation and the optimal values. In this figure, the green line shows the optimal values by the conventional method, and the red line shows the one by the proposed method. From this figure, we can see the optimal values by the proposed method are better than the one by the conventional method. From Fig. 5 and Fig. 6, we can also see the smaller deviation from 0 of the average value results with the better optimal value. From this fact, the small deviation from 0 of the average value is required for the good annealing performance. We can also confirm that the annealing performance depends on the rule number of the CA used for the noise generation.

6. Conclusion

In this study, we proposed a noise generation method with class 3 CA. In our method, the state of the CA cell determines plus or minus of the generated noise. The distribution of the noise is nearly symmetric. Comparing with the conventional method, the proposed method generates random numbers with smaller deviation from 0, and this nature results with good performance of DT-CNN annealing.

References

[1] L. Chua and L. Yang, "Cellular neural networks: theory," *IEEE Trans. Circuits Syst.*, vol. 37, no. 12, pp. 1257–1272, Oct. 1988.

[2] J. J. Hopfield, "Neural Networks and Physical Systems with Emergent Collective Computational Abilities," *Proceedings of the National Academy of Science*, vol. 79, pp. 2554–2558, Apr. 1982.

[3] M. Ercsey-Ravasz, T. Roska, and Z. Neda, "Cellular neural networks for NP-hard optimization," in *11th International Workshop on Cellular Neural Networks and Their Applications*, 2008, pp. 52–56.

[4] H. Nishimori, *Statistical Physics of Spin Glasses and Information Processing: An Introduction*. Oxford University Press, 2001.

[5] K. Tanaka, "Statistical-mechanical approach to image processing (topical review)," *J. Phys. A: Math. & Gen.*, vol. 35, no. 37, pp. R81–R150, 2002.

[6] T. Fujita, K. Sakomizu, and T. Ogura, "Hardware annealing on DT-CNN using CAM^2," in *Cellular Nanoscale Networks and Their Applications (CNNA), 2010 12th International Workshop on*, Feb. 2010.

[7] T. Fujita and T. Ogura, "Performance evaluation of DT-CNN annealing with CA noise," in *Proceedings of the 2010 International Workshop on Nonlinear Circuits, Communication and Signal Processing*, Mar. 2010, pp. 592–595.

[8] K. Sakomizu, T. Fujita, and T. Ogura, "Hardware annealing on discrete-time cellular neural network for object extraction," *J. Signal Processing*, vol. 13, no. 6, pp. 463–468, 2009.

[9] S. Wolfram, *Cellular automata and complexity : collected papers*. Westview Press, 1994.

[10] T. Ikenaga and T. Ogura, "CAM^2: A highly-parallel two-dimensional cellular automaton architecture," *IEEE Trans. Comput.*, vol. 47, no. 7, pp. 788–801, 1998.

[11] T. Fujita, T. Okamura, M. Nakanishi, and T. Ogura, "CAM^2-universal machine: A DTCNN implementation for real-time image processing," in *Cellular Neural Networks and Their Applications, 2008. CNNA 2008. 11th International Workshop on*, July 2008, pp. 219–223.

[12] T. Fujita and T. Ogura, "A global optimization method on dt-cnn using chaotic noise by ca," *IEICE Technical Report, NLP*, vol. 110, no. 335, pp. 37–41, dec 2010.