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Quartic Double-well Bistable SR-Based Multilateration Positioning Method Used in WSN

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Abstract – In this paper, a novel wireless positioning approach combining the traditional multilateration method and the dynamic stochastic resonance (SR) system is proposed, which can be used in the wireless sensor networks (WSN) node positioning. According to the fact that the wireless signal will attenuate through the wireless channel, which will lead to the degradation of received signal-to-noise ratio (SNR), the traditional quartic double-well bistable SR is analyzed. And by choosing the appropriate SR noise and corresponding driving parameters, the SNR gain of the received signal can be obtained, then the positioning accuracy will be enhanced. Computer simulation results show the advantages over the traditional multilateration poisoning method especially under low SNR circumstances.

1. Introduction

Nowadays the requirements of wireless positioning have increased sharply in various kinds of commercial applications, including the indoor and outdoor environments. Within many positioning applications, the wireless signal is often used to realize the positioning objective, such as the application in the wireless sensor networks (WSN). While in the wireless positioning methods, there are many different wireless transmission signals which can be used, for example the received signal strength indicator (RSSI), direction of arrival (DOA), angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and so on [1]. Although the principles of these approaches are simple and clear, the positioning accuracies of these methods are influenced by the received signal-to-noise ratio (SNR) seriously. Especially under low SNR, the positioning accuracy may be degraded drastically.

In this paper, a novel positioning approach used in WSN is introduced, which combines the traditional multilateration positioning method and the dynamic stochastic resonance (SR) technique. By analyzing the SNR in the traditional quartic double-well bistable SR system, the corresponding optimal driving parameters can be derived, which can guarantee the SNR gain of the proposed approach, and correspondingly the positioning accuracy of the improved multilateration can then be enhanced. The computer simulation results also show the advantages of the proposed approach especially under low SNR circumstances.

The remains of the paper are organized as follows: The traditional WSN positioning method of multilateration is first explained in Section II, together with its positioning performance deficiency. Section III introduces the proposed novel SR-based improved multilateration positioning approach, focusing on the optimization and the SNR gain of the proposed SR system. Section IV gives the computer simulation results, which reveals the improvement of the positioning accuracy of the proposed approach. And Section V concludes the whole paper.

2. Traditional Multilateration Positioning Method

In the various wireless sensor networks positioning methods, it can be found that the multilateration method is frequently discussed and used [2]. Fig.1 shows the schematic diagram of the traditional multilateration method. So in the following, we give a brief explanation to this wireless positioning technique.

To simplify the analysis, we assume that all the nodes (including the beacon nodes, reference nodes and the target node) are located in the same 2-dimensional coordinate plane. As shown in Fig.1, L_1, L_2, …, L_M are M beacon nodes with fixed two dimensional coordinates (x_1,y_1), (x_2,y_2), …, (x_M,y_M), respectively; and L_0 is the target node with coordinate (x,y). Suppose the distances from the target node L_0 to each beacon nodes L_1, L_2, …, L_M are d_1, d_2, …, d_M, respectively. Then we can get

$$\begin{cases} d_1 = \sqrt{\left(x-x_1\right)^2 + \left(y-y_1\right)^2} \\ d_2 = \sqrt{\left(x-x_2\right)^2 + \left(y-y_2\right)^2} \\ \quad \vdots \\ d_M = \sqrt{\left(x-x_M\right)^2 + \left(y-y_M\right)^2} \end{cases} . \tag{1}$$

When the distances d_1, d_2, …, d_M can be measured correctly, the target node coordinate (x,y) can then be estimated unbiasedly.

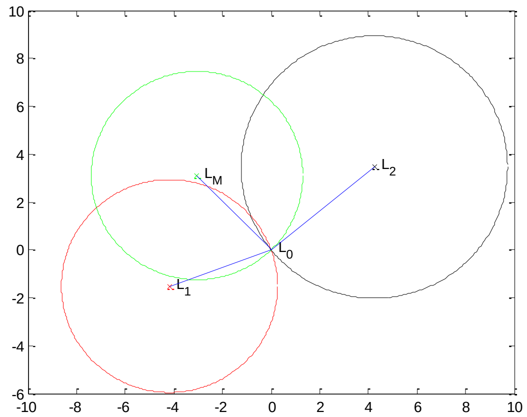

Fig.1. Traditional Multilateration Method

While in many applications, the distances information cannot be achieved accurately, so there may exist some measurement error between $\left(d_1, d_2, \cdots, d_M\right)$ and their estimates $\left(\hat{d}_1, \hat{d}_2, \cdots, \hat{d}_M\right)$. By introducing the maximal likelihood (ML) estimation method [2], the final multilateration poisoning result can be expressed as

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_{ML} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \hat{\mathbf{b}}, \qquad (2)$$

where $\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_{ML}$ is the ML estimation vector of the target node coordinate $\begin{bmatrix} x \\ y \end{bmatrix}$, the matrix $\mathbf{A} = \begin{bmatrix} 2\left(x_1 - x_M\right) & 2\left(y_1 - y_M\right) \\ 2\left(x_2 - x_M\right) & 2\left(y_2 - y_M\right) \\ \vdots & \vdots \\ 2\left(x_{M-1} - x_M\right) & 2\left(y_{M-1} - y_M\right) \end{bmatrix}$, and $\hat{\mathbf{b}} = \begin{bmatrix} x_1^2 - x_M^2 + y_1^2 - y_M^2 + \hat{d}_M^2 - \hat{d}_1^2 \\ x_2^2 - x_M^2 + y_2^2 - y_M^2 + \hat{d}_M^2 - \hat{d}_2^2 \\ \vdots \\ x_{M-1}^2 - x_M^2 + y_{M-1}^2 - y_M^2 + \hat{d}_M^2 - \hat{d}_{M-1}^2 \end{bmatrix}$ is an estimate vector corresponding to the vector $\mathbf{b} = \begin{bmatrix} x_1^2 - x_M^2 + y_1^2 - y_M^2 + d_M^2 - d_1^2 \\ x_2^2 - x_M^2 + y_2^2 - y_M^2 + d_M^2 - d_2^2 \\ \vdots \\ x_{M-1}^2 - x_M^2 + y_{M-1}^2 - y_M^2 + d_M^2 - d_{M-1}^2 \end{bmatrix}$.

The positioning error of the above ML-based multilateration estimation can be calculated by

$$\left\|\mathbf{e}_{ML}\right\| = \left\| \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_{ML} - \begin{bmatrix} x \\ y \end{bmatrix} \right\| = \left\| \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \left(\hat{\mathbf{b}} - \mathbf{b}\right) \right\|, \qquad (3)$$

where $\mathbf{e}_{ML} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_{ML} - \begin{bmatrix} x \\ y \end{bmatrix}$ is the ML estimation positioning error vector of the multilateration method, and $\|\cdot\|$ is the modulo operator.

It can be found clearly in (3) that the positioning precision of the multilateration method depends on the deviation of $\hat{\mathbf{b}} - \mathbf{b}$ for the matrix A can be regarded as a fixed matrix. In other words, the measurement error between $\left(d_1, d_2, \cdots, d_M\right)$ and $\left(\hat{d}_1, \hat{d}_2, \cdots, \hat{d}_M\right)$ will influence the positioning precision performance seriously. In order words, if $\hat{d}_m$ ($m=1,2,\ldots,M$) is written as the summary of d_m ($m=1,2,\ldots,M$) and a measurement noise or additive noise, the signal-to-noise ratio (SNR) of $\hat{d}_m$ ($m=1,2,\ldots,M$) will be a critical factor to the traditional multilateration positioning method. Obviously, the higher the SNRs of $\hat{d}_m$ ($m=1,2,\ldots,M$) are, the smaller the positioning error of the ML-based multilateration positioning method will be.

In many real WSN positioning applications, the distance information or other positioning information is transmitted through the wireless channel, which may bring lots of channel fading, multi-user interference, multi-path effect, inter-symbol interference and many other uncertainties. And they will lead to the attenuation of the receiving SNR and reduce the SNR of $\hat{d}_m$ ($m=1,2,\ldots,M$) at the receiver. So it restricts the real applications of the traditional multilateration method seriously.

3. Proposed Multilateration Positioning Approach Based on Stochastic Resonance

Due to the deficiency of the traditional multilateration method stated above, a novel WSN positioning approach based on the dynamic stochastic resonance technique is proposed, which can improve the positioning accuracy of the traditional multilateration method.

In the WSN positioning methods, the received signal strength indicator (RSSI) signal is often used to measure the distance between the beacon node and the target node, which can be expressed by [2]

$$P_r(d) = P_0(d_0) - 10\eta \log_{10}\left(\frac{d}{d_0}\right) + X_\sigma, \qquad (4)$$

where $P_r(d)$ is the received signal strength at the target node, d is the distance between the target node and the beacon node, $P_0(d_0)$ is the reference signal strength at certain reference node, d_0 is the corresponding reference distance from the reference node to the beacon node, η is the a signal fading parameter within the range [2,4], X_σ is the additive channel noise which can always be supposed to obey the white Gaussian distribution with mean 0 and variance σ_n^2.

Under the real application of WSN positioning circumstance, suppose that many RSSI time samples can be obtained at certain target node, then (4) can be changed to

$$r(t) = s(t) + n(t), \quad (t = 1, 2, \cdots, T) \tag{5}$$

where $r(t)$ is the received signal, $s(t)$ is the transmission signal, $n(t)$ is the additive channel noise which is the time sample of X_σ and it is independent to $s(t)$, and T is the total sampling number. For simplicity, we assume σ_s^2 and σ_n^2 are the variances of $s(t)$ and $n(t)$, respectively.

When the locations of the beacon nodes and the target node are all determined, the signal $s(t)$ in (5) can be regarded as a direct-current signal with mean value $\bar{s}$, so the RSSI signal $r(t)$ also possesses the white Gaussian distribution with mean $\bar{s}$ and variance σ_n^2. According to the analyses in the last Section, when the SNR of $r(t)$ is low, the positioning accuracy will be very poor.

So to improve the positioning accuracy, an apparent methodology in the multilateration is to increase the SNR of the received signal. While in the dynamic system theory, stochastic resonance (SR) is a kind of interesting dynamic system which can help to increase the signal SNR to some extent under some certain conditions. Here we introduce the idea of SR to improve the positioning accuracy of the traditional multilateration method.

Without loss of generality, a traditional quartic double-well bistable SR system is used in the following analysis, whose dynamic differential equation can be expressed by [3]

$$\frac{z(t+\Delta t) - z(t)}{\Delta t} = 2z(t) - z^3(t) + g_1 \cdot \alpha(t) + g_2 \cdot \beta(t), \tag{6}$$

where $z(t)$ is the state variable, Δt is the sampling period, $\alpha(t)$ is the driving signal, $\beta(t)$ is the inner SR noise, g_1 and g_2 are the driving parameters corresponding to $\alpha(t)$ and $\beta(t)$, respectively.

In the proposed approach, the normalized received signal $r(t)$ in (5) is used as the driving signal $\alpha(t)$ in (6), and an inner additive white Gaussian noise $\beta(t)$ with mean 0 and variance 1 is used as the inner SR noise. So (6) can be changed to

$$\begin{aligned}\frac{z(t+\Delta t) - z(t)}{\Delta t} &= 2z(t) - z^3(t) + g_1 \cdot \frac{r(t)}{\|r(t)\|} + g_2 \cdot \beta(t) \\ &= 2z(t) - z^3(t) + g_1 \cdot \frac{s(t)}{\|r(t)\|} + g_1 \cdot \frac{n(t)}{\|r(t)\|} + g_2 \cdot \beta(t).\end{aligned} \tag{7}$$

If we define

$$g_3 = g_1 / \|r(t)\|, \tag{8}$$

and

$$g_4 = \sqrt{\frac{g_1^2}{\|r(t)\|^2} \cdot \hat{\sigma}_n^2 + g_2^2}, \tag{9}$$

where $\hat{\sigma}_n^2$ is the estimate of σ_n^2, which can be calculated by

$$\hat{\sigma}_n^2 = E\left[\left(r(t) - E[r(t)]\right)^2\right], \tag{10}$$

according to the previous study, it is found the output SNR of the traditional quartic double-well bistable SR system can be approximated by [4]

$$SNR_0 \approx \frac{\sqrt{2} a g_3^2 \varepsilon^2 c^2}{g_4^4 - 2 g_3^2 \varepsilon^2 c^2} e^{-\frac{2U_0}{g_4^2}}, \tag{11}$$

where $a = 2$, $b = 1$, $c = \sqrt{a/b}$ and $U_0 = a^2/4b$ are constants; $\varepsilon^2 = 2\bar{s}^2$ is a parameter related to the signal power.

To ensure the SNR gain through the SR processing, it requires that

$$SNR_o > SNR_i = \bar{s}^2 / \sigma_n^2, \tag{12}$$

where SNR_i is the input SNR of the SR system, or the SNR of the received signal. While it can be deduced from (12) that

$$\left(\sqrt{2} U_0 + \frac{\varepsilon^2 c^2}{\sigma_n^2} e^{\frac{2U_0}{g_4^2}}\right) g_3^2 > \frac{g_4^4}{2\sigma_n^2} e^{\frac{2U_0}{g_4^2}}. \tag{13}$$

Considering a low SNR circumstance, (13) can be simplified as

$$g_3^2 > \frac{g_4^4}{2\sqrt{2} U_0 \sigma_n^2} e^{\frac{2U_0}{g_4^2}}. \tag{14}$$

In (14), it can be found that the right side of the inequality can reach its maximum when $g_4 = \sqrt{U_0}$, so if the following condition can be fulfilled

$$g_3^2 > \frac{U_0}{2\sqrt{2}\sigma_n^2} e^2, \tag{15}$$

the SNR gain of the SR system can be obtained. For example, when $SNR_i < 10dB$, we can choose

$$g_3^2 > \frac{11 \cdot U_0}{2\sqrt{2} \cdot E\left[r^2(t)\right]} e^2. \tag{16}$$

By taking $g_4 = \sqrt{U_0}$ and a big enough value of g_3 which fulfills (15) back into (8) and (9), the driving parameters g_1 and g_2 can be defined.

At last, take the values of $z(t)$ into the traditional multilateration positioning to calculate $\hat{d}_m$ ($m=1,2,\ldots,M$) again. The positioning accuracy of the proposed approach can then be improved based on the increasing of SNR_o corresponding to SNR_i in the SR system.

4. Computer Simulations

In this section, we give some computer simulation results to show the exact positioning performance of the proposed approach, which are compared with those of the traditional multilateration method in the WSN positioning applications.

In the computer simulations, the traditional discrete overdamped bistable oscillator SR system presented in (6) is used, while the initial value $z(0)$ is chosen randomly within $(-1,+1)$, and the sampling period is $\Delta t = 0.0195$.

The simulation scene is as follows: suppose that there are ten beacon nodes and one target node within a 10×10 square meters area. The total sampling number is $T=10^5$, the signal fading parameter in (4) is $\eta=2$.

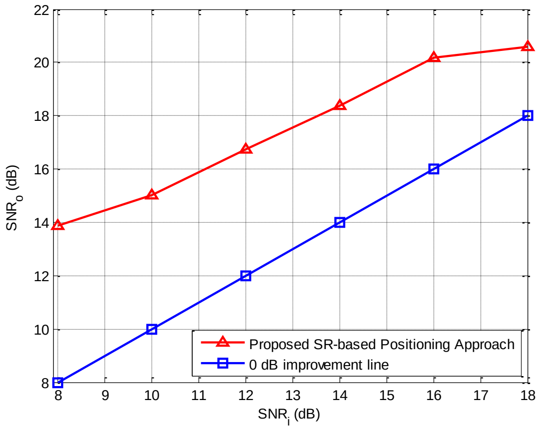

Fig.2. SNR Improvement of the SR System

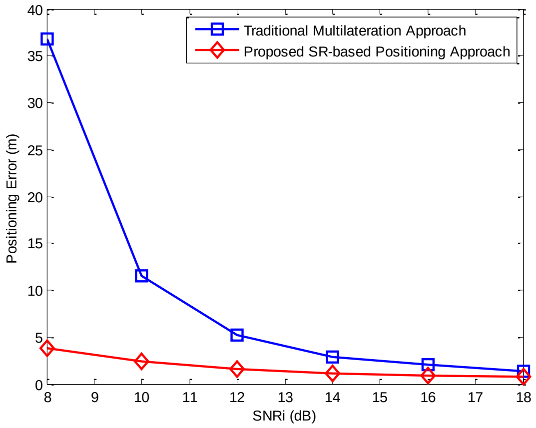

Fig.3. Average Positioning Error Performance Comparison

Fig.2 firstly gives the SNR improvement of the proposed SR system while SNR_i varies from 8dB to 18dB. The SNR gains can be observed clearly in the figure. And Fig.3 presents the average poisoning error performance comparison results of the proposed approach and the traditional multilateration positioning approach within the same SNR range. It can be found that the positioning accuracy is improved significantly, especially under low SNR circumstances, which shows the positioning advantage of the proposed SR-based approach.

5. Conclusions

In this paper, an improved multilateration positioning approach used in WSN is proposed, which is based on the SR technique. Theoretical analyses indicate the SNR improvability can be achieved by using the proposed SR-based approach. And the computer simulations also certify the advantages of this novel positioning approach. And it can certainly be extended to some other wireless positioning applications such as indoor LED localization and so on.

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