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A note on subthreshold signal reception using stochastic resonance receiver - Comparison of Dynamical and Non-Dynamical Devices -

Hiroya Tanaka†, Takaya Yamazato†, Yukihiro Tadokoro‡, and Shintaro Arai††

†Nagoya University, Nagoya 464-8603, Japan
‡Toyota Central R&D Laboratories, Inc., Aichi 480-1192, Japan
††National Institute of Technology, Kagawa College, Kagawa 769-1192, Japan
Email: tanaka@katayama.nuee.nagoya-u.ac.jp

Abstract—Stochastic resonance (SR) is an interesting phenomenon in that noise enhances system responses. Despite an attractive phenomenon of SR that noise enhances a weak signal below a receiver sensitivity, few researchers have addressed the SR effect in communication systems. Previously, we proposed an analysis method for the SR receiver with a device exhibiting SR and evaluated its bit error rate (BER) performance. However, the device was limited to a non-dynamical device, and there is room for improving the performance of the SR receiver by changing the device. In this paper, we use two typical devices exhibiting SR, i.e., a comparator as a non-dynamical device and a Schmitt trigger as a dynamical device. We evaluate the BER performances of these devices with analysis methods and numerical simulations. A performance comparison of these devices is also shown.

1. Introduction

Stochastic resonance (SR) is an interesting phenomenon in that noise enhances system response. Since its discovery by Benzi et al. in 1981 [1], this characteristic has been discussed in the context of nonlinear physics [2, 3], and its application in signal processing has spread to various fields, such as signal detection theory [4–6], wireless communication [7, 8], and imaging [9].

Despite attractive phenomenon of SR that noise enhances a weak signal below a receiver sensitivity, few researchers have addressed the SR effect in communication systems [7, 8]. So far, the tremendous amount of research into SR established for obtaining a weak signal in a strong noise environment and enhancement of the weak signal below the receiver sensitivity. The key is an intentional noise additionally applied at receivers. It is reported that a non-Gaussian intentional noise plays an important role in the weak signal reception. In fact, in [6], a nonlinear device designed for a non-Gaussian noise has exhibited better performance than linear filtering and achieves the Cramer-Rao lower bound. And also, in the subthreshold signal reception, an improvement of BER performances has been derived by adding Gaussian intentional noise [8].

This paper discusses the SR effect in a binary communication system for the reception of the subthreshold signal. We focus on the problem in which communication cannot be established when the received signal strength is below the receiver sensitivity. Overcoming receiver sensitivity introduces new and attractive challenges in wireless communication systems. If the receiver sensitivity can be improved, we can simultaneously reduce transmission power and interference to other users. Low-power wireless systems can provide solutions for both energy-efficient green wireless communications and wireless spectrum shortage.

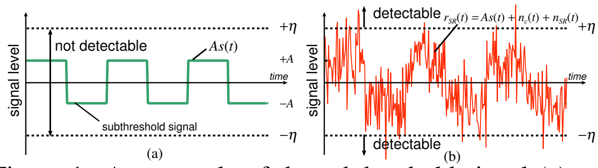

Figure 1: An example of the subthreshold signal (a) and both the channel noise and the intentional noise are added to it (b).

Previously, we proposed an analysis method of the SR receiver with a 3-level device as a non-dynamical device exhibiting SR, and presented an improvement of the bit error rate (BER) performance by the SR effect [8]. However, in general, a dynamical system can have better performance since it has a memory effect [5].

Stochastic resonance occurs in both dynamical systems and non-dynamical systems. In dynamical systems, a Schmitt trigger is known as a simple device exhibiting SR [2]. It has a memory and depends on the current input and state. While in non-dynamical systems, a threshold device such as a comparator is known as a simple device exhibiting SR [3]. It depends on only the current input. In this paper, we use a Schmitt trigger and a Comparator as typical devices in these systems. The distinction of these devices is whether they have a memory effect or not. A dynamical system can yield higher SNR than non-dynamical systems due to a memory effect [5]. For this reason, it is expected that the SR receiver with the Schmitt trigger has better performance than the Comparator for the reception of the subthreshold signal. In this sense, we propose an analysis method of the SR receiver with these devices and compare their BER performances. Reception sensitivity could be modeled as the threshold of a Schmitt trigger and a Comparator.

2. System model

In this paper, a received signal level is assumed to be below a receiver sensitivity. As shown in Fig. 1, when the received signal level is denoted by A, and the receiver sensitivity is denoted by η, we observe that $|A| < \eta$. A conven-

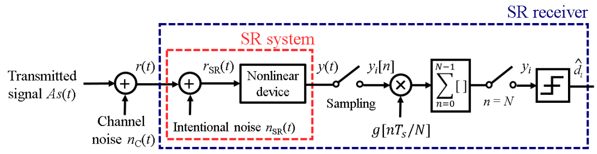

Figure 2: System model.

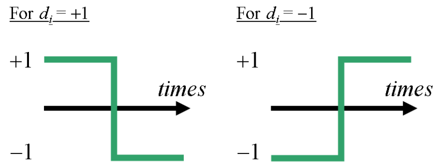

Figure 3: The desired signals $s(t)$ in the symbol duration: for $d_i = +1$ (left) and for $d_i = -1$ (right).

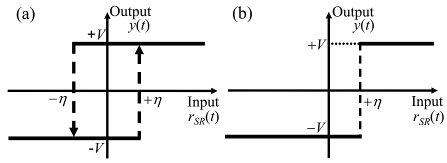

Figure 4: Input-Output characteristics of a Schmitt trigger (a) and a Comparator (b).

tional receiver cannot detect such a subthreshold signal, as shown in Fig. 1 (a).

Figure 2 shows the system model of the SR receiver. In Fig. 2, the SR receiver consisting of the SR system receives a desired signal $s(t)$ and channel noise $n_c(t)$. The desired signal $s(t)$ is expressed as follows.

$$s(t) = \sum_i d_i g(t - iT_s). \quad (1)$$

where d_i is a binary data sequence $\{\pm 1\}$ of the ith symbol, T_s is a symbol duration, and $g(t)$ is a rectangular pulse that $g(t) = +1$ in $0 \leq t < T_s/2$ and $g(t) = -1$ in $T_s/2 \leq t < T_s$. The desired signal $s(t)$ in the symbol duration is shown in Fig. 3. In each data, the signals have the same interval of positive and negative during symbol duration. This contributes to the decision rule. The data of $d_i = +1$ and $d_i = -1$ occur randomly.

In the receiver, the signal is received at the received signal level A and added to the channel noise $n_c(t)$. The received signal is expressed as follows.

$$r(t) = As(t) + n_c(t). \quad (2)$$

The channel noise $n_c(t)$ is the zero-mean white Gaussian noise with variance σ_c^2.

The channel noise is primarily dominated by the thermal noise that occurs in the receiver; thus, its power spectral density (PSD) is assumed to be uniform and is expressed as $N_0 = k_B T_0$, where k_B is the Boltzmann constant and T_0 is the noise temperature. In this paper, we set $T_0 = 300$ K, the noise PSD $N_0 \simeq 4.1 \times 10^{-21}$ W/Hz.

Unfortunately, the channel noise is always added to the received signal and cannot be separated from the signal component. The SR receiver differs from a conventional receiver in that it adds intentional noise at the receiver front end. The SR system is used for detecting a subthreshold signal which cannot be detected by conventional receivers. After that, the detected signal by the SR system can be operated by conventional receivers. Figure 1 (b) illustrates the detection of a subthreshold signal by the additional intentional noise. This shows a simple SR effect. The signal described in Fig. 1(b) is obtained by adding both the channel and the intentional noise to the subthreshold signal in Fig. 1(a). The intentional noise $n_{SR}(t)$ is assumed to be the zero-mean white Gaussian noise with variance σ_{SR}^2. The intentional noise should be optimally tuned to obtain the best SR receiver performance.

Reception sensitivity can be modeled as the threshold of the nonlinear devices which are simple nonlinear devices that exhibits the SR effect. The received signal, which is composed of $As(t)$, $n_c(t)$, and $n_{SR}(t)$, is fed into the nonlinear device. In this paper, we use a Comparator as a non-dynamical device and a Schmitt trigger as a dynamical device. Input-output characteristics of these devices are shown in Fig. 4. As shown in this figure, the Comparator has two outputs and one threshold. The output of the Comparator depends on the current input. While the Schmitt trigger has a hysteresis that exhibits a memory effect. If a subthreshold signal plus noise exceeds the threshold, the subthreshold signal can be detected when the noise is tuned optimally. This phenomenon is known as SR.

When the input signal $r_{SR}(t)$ is fed into the Comparator, the output of the Comparator is expressed as follows.

$$y(t) = \begin{cases} +V & (r_{\mathrm{SR}}(t) > \eta) \\ -V & otherwise \end{cases}. \quad (3)$$

While in the Schmitt trigger, the threshold is changed according to the current state as follows.

$$V_{th} = \begin{cases} -\eta & if \quad y(t) = +V \\ +\eta & if \quad y(t) = -V \end{cases}. \quad (4)$$

When the input signal $r_{SR}(t)$ is fed into the Schmitt trigger, the output of Schmitt trigger is expressed as follows.

$$y(t) = \begin{cases} -V & if \quad r_{SR}(t) < V_{th} \\ +V & if \quad r_{SR}(t) > V_{th} \end{cases}. \quad (5)$$

The η is the threshold of the Schmitt trigger or the Comparator and is assumed to be equivalent to the reception sensitivity of the conventional receiver.

These outputs of the nonlinear devices are sampled N times during the symbol and multiplied by $g(t)$ for detection. We represent the sample of the device output as $y_i[n]$ and the decision variable y_i as

$$y_i = \sum_{n=0}^{N-1} y_i[n] g[nT_s/N]. \quad (6)$$

In the detector, we restore the data using the decision variable with a decision threshold. We restore the data depending on whether the output is positive or negative as the

Table 1: Parameter settings for the computer simulation.

Parameter	Value
Symbol duration T_s [μsec]	1.0
Received signal level A [μV]	1.0
Sensitivity of the conventional receiver η [μV]	1.1
Numbers of samples per symbol N	10
Number of trials for simulation	10^5

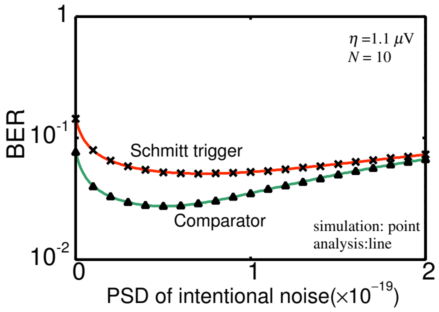

Figure 5: BER performance of the SR receiver with a Schmitt trigger and a Comparator.

following manner.

$$\hat{d}_i = \begin{cases} +1 & (y_i > 0) \\ -1 & otherwise \end{cases}. \quad (7)$$

This means that the detector performs the major decision. As N increases, the BER performance can be improved. In our system, sampling rates should be high for an enhancement of the subthreshold signal. One of the application scenario may be in the system with small bandwidth.

Note that the decision method comes from the characteristics of the device and the desired signal. In this paper, as shown in Fig. 4(b), the comparator does not have symmetrical thresholds. This causes asymmetry of $P[y(t) = +V]$ and $P[y(t) = -V]$ between the signal level $+A$ and $-A$. To avoid this asymmetry, we set that the desired signals in the data $d_i = \pm 1$ have the same interval of positive and negative during symbol duration.

3. Numerical results

3.1. BER performance of the SR receiver with a Schmitt trigger and a Comparator

Figure 5 shows the BER performance versus the PSD of the intentional noise in the SR receiver with a Schmitt trigger and a Comparator. The analytical and simulation results are shown by the solid line and points respectively. Parameter settings are shown in Table 1. As shown in Fig. 5, the BER performances are improved with increased PSD of the intentional noise. This is a typical phenomenon exhibiting SR. This figure also shows the analysis method is appropriate to the system model since the analytical results are perfectly consistent with the simulation results.

Figure. 5 also shows the minimum of BER of the Comparator is better than that of the Schmitt trigger. This is an unexpected result since a Schmitt trigger has a memory effect, and it should have better performance than a Comparator. Why is the performance of the Comparator better than that of the Schmitt trigger?

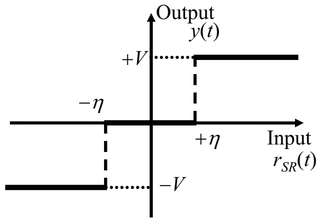

Figure 6: Input-Output characteristics of a 3-level device.

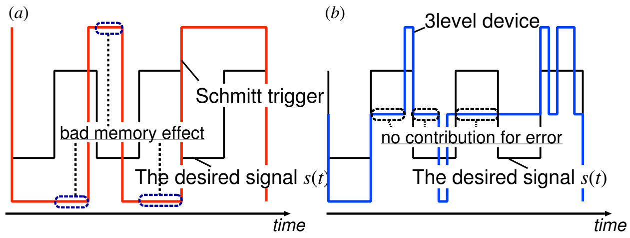

Figure 7: An output signal of the Schmitt trigger (a) and the 3-level device (b).

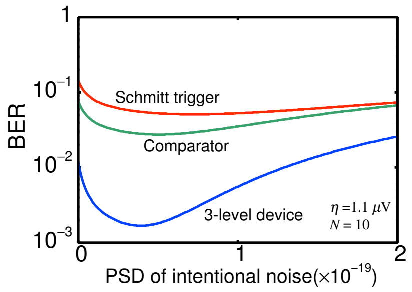

Figure 8: BER performance of the SR receiver with a 3-level device.

3.2. Comparison to the 3-level device

In this subsection, we discuss why the BER performance of the Comparator is better than that of the Schmitt trigger. We consider that a memory of the Schmitt trigger has a bad effect on the BER performance. To explain this effect, we use a 3-level device as a simple thresholds device without a memory shown as Fig. 6. As is the same as the Schmitt trigger, the 3-level device has two non-zero outputs and two thresholds but also has a zero output. The zero output has no contribution in the detection. This means 3-level device has no memory.

Fifure 7 shows an output signal of the Schmitt trigger and the 3-level device. In Fig. 7(a), the circled parts show the memory has a bad effect on the performance, while in Fig. 7(b), these parts has no contribution for error.

In our previous research, we proposed an analysis method of the SR receiver with the 3-level device [8]. Now we use the method for comparing the performances between these devices. Figure 8 shows the BER performance versus the PSD of the intentional noise in the SR receiver

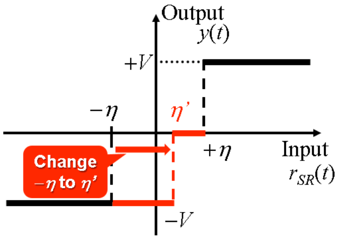

Figure 9: Input-Output characteristics of a 3-level device after changing the threshold.

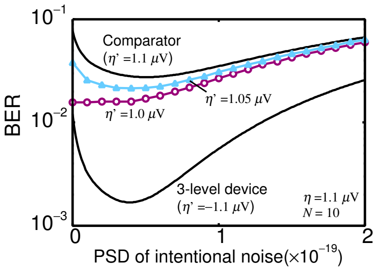

Figure 10: BER performances of the 3-level device after changing the threshold.

with the Comparator, the Schmitt trigger, and the 3-level device. Parameter settings are the same in Table 1. Figure. 8 shows the BER performance of the 3-level device has the best performance between these devices. This suggests that these BER performances depend on the threshold value, sampling rate, and noise bandwidth. The 3-level device has the good compatibility with binary communication systems.

3.3. Consideration

In this subsection, we compare the performances between the 3-level device and the Comparator by changing the threshold of the 3-level device. As shown in Fig. 9, we change the threshold $-\eta$ to η'. According to the characteristics, the output of 3-level device is changed as follows.

$$y(t) = \begin{cases} +V & (r_{\mathrm{SR}}(t) > \eta) \\ -V & (r_{\mathrm{SR}}(t) < \eta') \\ 0 & otherwise \end{cases} . \qquad (8)$$

When $\eta' = \eta$, the Input-Output characteristics of the 3-level device is coincident with that of the Comparator.

Fig. 10 shows the BER performances of the 3-level device after changing the threshold. Parameter settings other than the threshold are the same in Table 1. We change the threshold η' from $1.0\mu V$ to $1.1\mu V$. In this situation, the received signal level is below the threshold of the device. As the threshold η' approaches to η, the BER performance of the 3-level device approaches to that of the Comparator. This shows the BER performance of the 3-level device is superior to that of the Comparator.

4. Conclusion

We proposed an analysis method of SR receiver with a Schmitt trigger and a Comparator and evaluated BER performances. Numerical results show improvements of the BER performances by SR and an exact BER performances coincidence with simulated results. The performance comparison shows the performance of the Comparator is better than that of the Schmitt trigger. This is due to the memory effect of the Schmitt Trigger in the subthreshold signal reception. We also used a 3-level device as a simple threshold device without a memory for comparing BER performances to the Comparator and the Schmitt trigger. Numerical result shows that the 3-level device has the best performance between these devises and it has the good compatibility with binary communication systems.

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