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# Experimental Investigation of Reinforcement Learning Using Optical Spatiotemporal Dynamics

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**Abstract–** In this work, we focus on the parallel nature of optical information processing and apply a scalable photonic hardware to reinforcement learning. Experimental reinforcement learning was achieved by spatiotemporal dynamics generated using a semiconductor laser and a spatial light modulator.

## 1. Introduction

Reinforcement learning has been used in various research fields, and the multi-armed bandit problem [1] is an example of reinforcement learning. The goal of the multi-armed bandit problem is to maximize the total rewards in which a player repeatedly selects multiple slot machines with different unknown hit probabilities. To solve the multi-armed bandit problem, two actions of "exploration" and "exploitation" must be balanced, and the two actions are in a trade-off relationship. Exploration is an action to search for the slot machine with the highest hit probability by selecting multiple slot machines randomly. On the contrary, exploitation is an action to repeatedly play the slot machine with the highest hit probability estimated by the exploration to increase the total rewards.

On the other hand, research that takes advantage of the parallel nature of spatial light has been active in recent years [2~5]. We have also studied the use of a spatial light modulator and have achieved experimental decision making for the multi-arm bandit problem in many slot machines [6]. However, in that work, the dynamics of each macro pixel was assumed to be independent, and spatial coupling has not yet been discussed.

In this study, we add spatial coupling to the spatiotemporal dynamics of a semiconductor laser and a spatial light modulator in a software approach, and experimentally demonstrate the investigation of the dynamics and its implementation in decision making.

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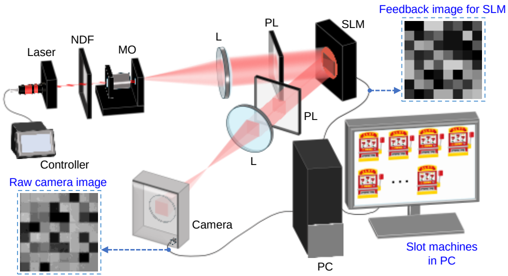


Fig. 1 Experimental setup for spatiotemporal dynamics using optoelectronic feedback system with semiconductor laser, camera, and spatial light modulator.

## 2. Methods

### 2.1 Experimental setup

Our experimental setup is shown in Fig. 1. The laser beam emitted from a collimator becomes a spherical wave by passing through a spatial filter, and is injected into a spatial light modulator. The spatial light modulator modulates the phase of the incident light by the input voltage of the pixels on the spatial light modulator. Intensity modulation can be achieved by combining two polarizers and the spatial light modulator. The intensity-modulated laser output is captured by a CMOS camera and transmitted to a computer. The post processing of the detected image is performed in the computer and the image signal is fed back to the spatial light modulator. Then, the spatial light modulator modulates the phase of the laser light with a new input values. Thus, spatiotemporal dynamics can be generated by repeating the detection and feedback of the image in the optoelectronic feedback system.

### 2.2 One-dimensional map of the feedback system

The intensity modulation characteristics of the spatial light modulator obtained experimentally are shown in Fig. 2. Since the intensity modulation characteristics are sinusoidal, the intensity dynamics of this feedback system for each pixel can be modeled as follows.

$$I^{CAM} = a \cdot \cos(2\pi f I^{SLM}) + b \tag{1}$$

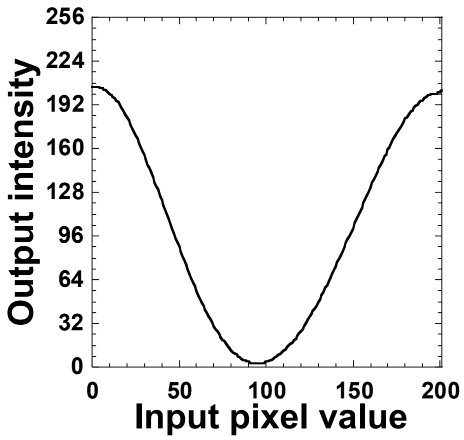

Fig. 2 The intensity modulation characteristics of the spatial light modulator

where $I^{CAM}$ is the value of the macro pixel on the camera, $I^{SLM}$ is the value of the macro pixel on the spatial light modulator. $a$ is the amplitude of the intensity modulation, $f$ is the frequency of the intensity modulation, and $b$ is the bias of the intensity modulation.

The feedback signal from the computer to the spatial light modulator can be described as follows.

$$I^{SLM}(t+1) = \beta \cdot I^{CAM}(t) \qquad (2)$$

where $\beta$ is the feedback strength and it is the parameter that determines the spatiotemporal dynamics.

From Eqs. (1) and (2), one-dimensional map of the feedback system for each pixel can be written as follow.

$$I^{CAM}(t+1) = a \cdot \cos\left(2\pi f \beta I^{CAM}(t)\right) + b \qquad (3)$$

Equation (3) shows that the spatiotemporal dynamics generated by the feedback system is based on a sinusoidal function.

### 2.3 Spatial coupling

Next, software coupling is applied between spatially adjacent macro pixels. The coupling method is shown in Fig. 3. The coupled value can be described as follows.

$$\tilde{I}_{center} = (1-\kappa) I_{center} + \frac{\kappa}{n}\left(I_{top} + I_{bottom} + I_{left} + I_{right}\right) \qquad (3)$$

where $\kappa$ is the coupling strength.

The larger $\kappa$, the greater the influence of the surrounding macropixels' dynamics, and the smaller $\kappa$, the greater the influence of its own dynamics. Basically, the coupling is added from the four macro pixels located above, below, left, right and center of the pixel.

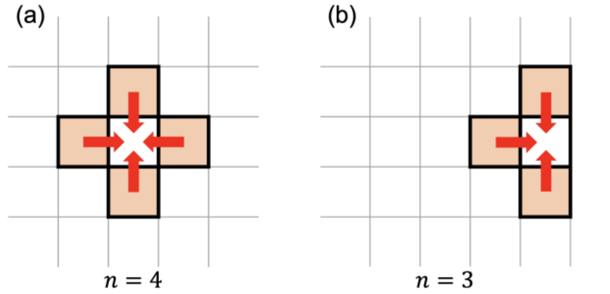

Fig. 3 Coupling method (a) when the macro pixel is located in the center (b) when the macro pixel is located at the edge or corner.

If a macro pixel is located at the edge or corner, only those present from the macro pixels on the top, bottom, left, and right are coupled.

## 3. Generation of spatiotemporal dynamics

In this section, we observe spatiotemporal dynamics generated in the experimental setup. The chaotic dynamics can be generated by setting the feedback strength to an appropriate value, and here we set the feedback strength $\beta = 3.2$.

First, the spatiotemporal dynamics when no spatial coupling is added (i.e., coupling strength $\kappa = 0$) is shown in Fig. 4. Irregular spatiotemporal patterns are observed at the different number of iterations in Fig. 4. Fig. 6 shows the time waveforms of two neighboring macro pixels, showing no correlation between them.

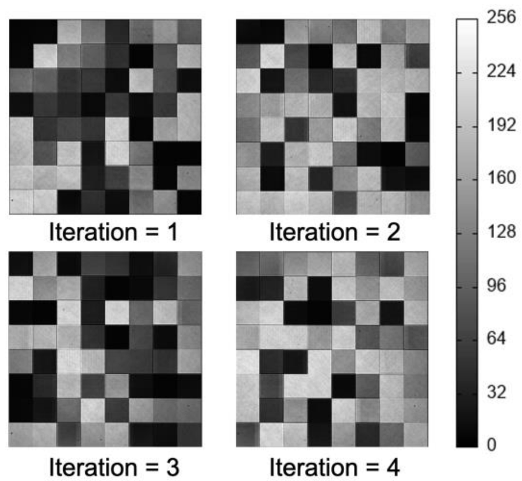

Fig. 4 Spatiotemporal patterns of spatial light modulator at four successive iterations at the coupling strength $\kappa = 0$.

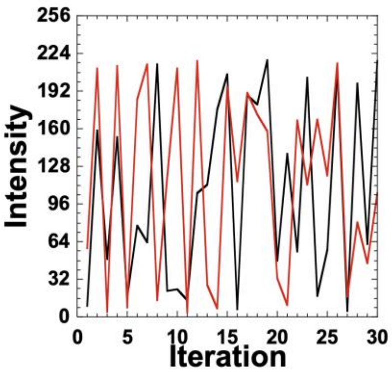

Fig. 5 Time waveforms of two neighboring macro pixels at the coupling strength $\kappa = 0$.

Next, Figure 6 shows the spatiotemporal dynamics with spatial coupling. The case with coupling strength $\kappa = 0.6$ is shown here as an example. In Figure 6, each macro pixel shows different chaotic dynamics, but neighboring macro pixels show similar changes. In fact, Figure 7, which shows the time waveforms of two neighboring macro pixels, shows similar changes for both.

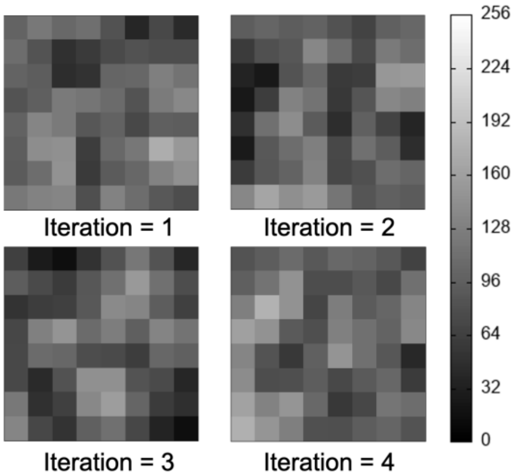

Fig. 6 Spatiotemporal patterns of spatial light modulator at four successive iterations at the coupling strength $\kappa = 0.6$.

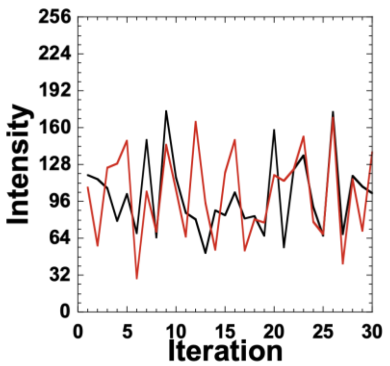

Fig. 7 Time waveforms of two neighboring macro pixels at the coupling strength $\kappa = 0.6$.

## 4. Analysis of spatially coupled spatiotemporal dynamics

### 4.1 Evaluation index

In this chapter, we introduce the cross-correlation function and the order parameter [7] for the analysis of coupled spatiotemporal dynamics. The cross-correlation function is a function used to evaluate the similarity between two signals and is defined as follows.

$$C = \frac{\langle (I_{a,i} - \overline{I_a})(I_{b,i} - \overline{I_b}) \rangle}{\sigma_a \sigma_b} \tag{4}$$

where, $I_{a,i}$ and $I_{b,i}$ are the i-th sample point in the time waveform of the light intensity of two different macro pixels. $\overline{I_a}$ and $\overline{I_b}$ are the means of $I_{a,i}$ and $I_{b,i}$, $\sigma_a$ and $\sigma_b$ are the standard deviations of $I_{a,i}$ and $I_{b,i}$ $\langle \cdot \rangle$ is the time mean.

The closer the cross-correlation function is to 1, the stronger the positive correlation, the closer to 0, the uncorrelated, and the closer to -1, the stronger the negative correlation. In practice, the dynamics is evaluated with the mean value of all cross-correlation function of neighboring macro pixels.

The order parameter is a measure of the degree of synchronization and is defined as follows.

$$z = re^{i\varphi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} = \frac{1}{N}\sum_{j=1}^{N} (\cos\theta_j + i\sin\theta_j)$$
$$|z| = \sqrt{\left(\frac{1}{N}\sum_{j=1}^{N}\cos\theta_j\right)^2 + \left(\frac{1}{N}\sum_{j=1}^{N}\sin\theta_j\right)^2} \tag{5}$$

where $\theta_j$ is the light intensity of the j-th macro pixel $I_j$ converted to radians, and is expressed as $\theta_j = \pi f \times I_j$ using the experimentally obtained frequency $f$ of the intensity modulation characteristic.

The order parameter $|z|$ takes values between 0 and 1, with values closer to 0 indicating a state in which the entire image is oscillating discretely, and values closer to 1 indicating a state in which the entire image is synchronized.

### 4.2 Result

First, the results of the average cross-correlation function between neighboring macro pixels for different coupling strengths $\kappa$ are shown in Fig. 8. The cross-correlation function increased up to a coupling strength of κ=0.6~0.7 and then decreased. It reached a maximum value of 0.542 when the coupling strength κ=0.7.

Next, the order parameter results for different coupling strengths $\kappa$ are shown in Fig. 9. The order parameter increased roughly monotonically with increasing coupling strength, reaching a maximum value of 0.863 at a coupling strength of κ=0.8.

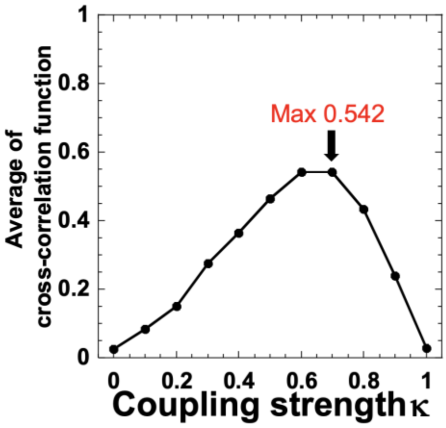

Fig. 8 the average cross-correlation function between neighboring macro pixels for different coupling strengths $\kappa$

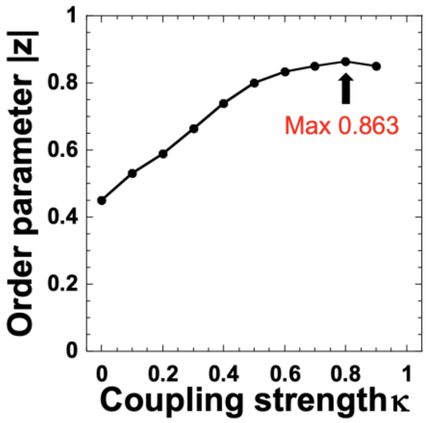

Fig. 9 the order parameter results for different coupling strengths $\kappa$

## 5. Conclusion

We generated spatially coupled spatiotemporal dynamics in an experimental setup with a semiconductor laser and a spatial light modulator. By varying the coupling strength $\kappa$, the time waveforms of neighboring macro pixels were correlated. We introduced the cross-correlation function and the order parameter to analyze spatially coupled spatiotemporal dynamics. The cross-correlation function showed a maximum value of 0.542 when the coupling strength $\kappa$ =0.7. The order parameter showed a maximum value of 0.863 when the coupling strength $\kappa$ =0.8.

In addition to the contents of this paper, the presentation will show the results of adapting spatially coupled spatiotemporal dynamics to a decision-making experiment.

## Acknowledgement
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