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Application of Lévy Flight to Differential Evolution Algorithm

Takuya Shindo† and Nobuhiko Itoh†

†Nippon Institute of Technology
4–1 Gakuendai, Miyashiro, Minamisaitama, Saitama 345-8501, Japan

Email: t-shindo@nit.ac.jp, itoh@nit.ac.jp

Abstract— Cuckoo Search (CS) and Differential Evolution (DE) are multipoint search metaheuristics proposed for continuous optimization problems. Both methods show relatively high performance among similar methods. In addition, it is easy to use because there are few preset parameters by the user. Moreover, they have the common feature of being easy to implement because they are simple algorithms. On the other hand, CS is a global search type method, and DE is a local search type method. In this study, we proposed a hybrid method that utilizes generation update of DE and Lévy flight of CS. We also performed numerical experiments using the benchmark test suite, compared the performance with conventional CS and DE, and confirmed the effectiveness of the proposed method.

1. Introduction

In the real world, there are many problems that are modeled as optimization problems. Meta-heuristics is one of the techniques for solving such optimization problems. In this research, among meta-heuristics, we focus on simple and high-performance CS[1] and DE[2] compared with other methods.

CS and DE have performed well compared to other similar methods in several optimization studies and competitions[3][4]. There are also examples of its adoption as an optimization solver for real problems. For example, an improved version of CS is used as a solver in simulation software for stability analysis. The CS achieves an average of 3 times reduction in calculation time while maintaining the same accuracy as the calculation result by Simulated Annealing [5]. DE is also used as an optimization solver in SciPy, a scientific computing library provided in Python[6]. CS and DE have such high performance that they are put to practical use in the optimization of real problems. Furthermore,, they have common features that the number of parameters that need to be preset by the user is small and the algorithms are simple. On the other hand, CS is a global search type method, and DE is a local search type method.

In this study, using CS and DE, which are easy to implement and modify due to these features, we propose a CS-based hybrid method with DE that improves the local search performance of CS. In addition, we conduct numerical experiments using benchmark functions to evaluate the performance of the proposed method.

ORCID iDs First Author: 0000-0002-0693-3245, Second Author: 0009-0006-9677-0983, :

2. Cuckoo Search

CS was proposed in 2009 by X. S. Yang and S. Deb, inspired by the reproductive behavior of cuckoos and their associated relationships with other bird species. It is a multi-point search type meta-heuristics that guarantees global convergence. Efficient global search is possible because Lévy Flight is used for the movement method of the cuckoo, which is the search agent.

2.1. Lévy Flight

Movement patterns such as flight and foraging of some organisms have been observed to be Lévy Walks. For example, in the foraging of animals, it is possible to search for food efficiently by searching mainly in the vicinity and occasionally searching in the distance. In CS, efficient global search can be performed by using Lévy Flight, which applies Lévy distribution to the step length, for the cuckoo's movement method.

Figure 1 shows the trajectory of Random Walk. Figure 2 shows the trajectory of the Lévy Walk. Applying random numbers based on the Lévy distribution to Random walk gives Lévy Walk. Figure 3 shows the Lévy distribution. The probability density function of the Lévy distribution is Eq. 1.

$$f(x;\mu,\sigma) = \begin{cases} \sqrt{\frac{\sigma}{2\pi}} \frac{e^{-\frac{\sigma}{2(x-\mu)}}}{(x-\mu)^{-\frac{3}{2}}} & (\mu < x) \\ 0 & (\mu \geq x) \end{cases} \quad (1)$$

μ is the location parameter and σ is the scale parameter.

2.2. Algorithm of CS

The pseudocode of CS is shown in Algorithm 1. In CS, the three parameters that need to be set in advance are the total number of nests $\mathrm{N} = \{1, \ldots, \mathrm{N}\}$, the probability of finding eggs $\mathrm{p_a} \in [0, 1]$, and the Lévy Flight coefficient $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_D)$ (each component $\alpha_i > 0$). N corresponds to the population size, which is a common parameter for multipoint search methods. Therefore, there are two parameters specific to the method.

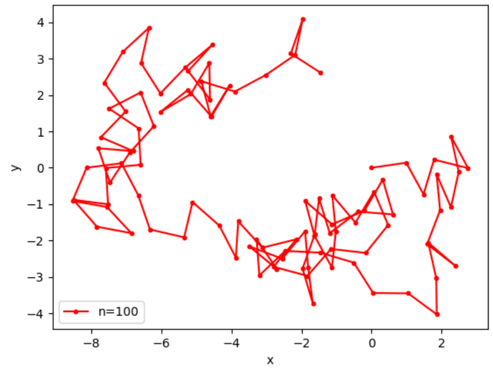

Figure 1: An example of a 100-step Random Walk trajectory in a two-dimensional space.

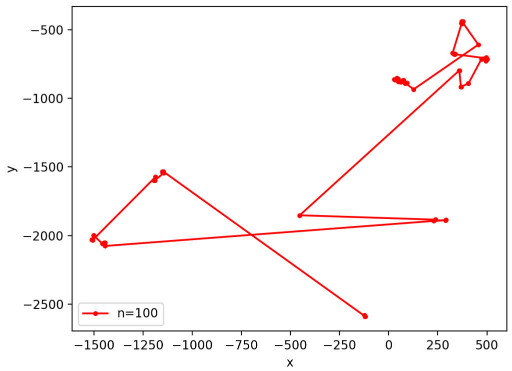

Figure 2: An example of a 100-step Lévy Walk trajectory in a two-dimensional space.

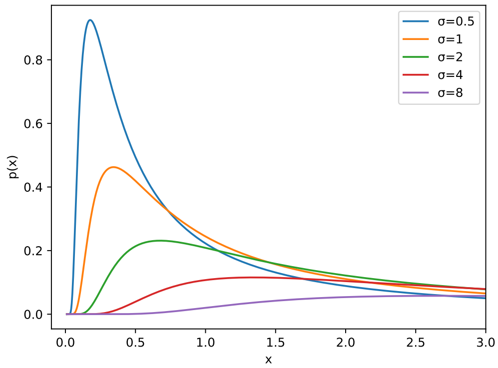

Figure 3: Probability density function of the Lévy distribution

Algorithm 1 Pseudo code of CS

Randomly generate N nests $\boldsymbol{x}_{\mathrm{N}}$ in the search space
while Search end condition Not reached **do**
  From the nest $\boldsymbol{x}_i$, which is uniformly randomly selected from $\boldsymbol{x}_{\mathrm{N}}$, the cuckoo moves to xnew with Lévy Flight according to Eq. 2 and lays eggs $f(\boldsymbol{x}_i^{\mathrm{new}})$).
  Evaluate $(f(\boldsymbol{x}_j))$ a nest $\boldsymbol{x}_j$ uniformly randomly selected from $\boldsymbol{x}_{\mathrm{N}}$.
  if $f(\boldsymbol{x}_i^{\mathrm{new}})$ is better than $f(\boldsymbol{x}_j)$ **then**
    Replace $\boldsymbol{x}_j$ with $\boldsymbol{x}_i^{\mathrm{new}}$
  end if
  All nests are evaluated $(f(\boldsymbol{x}_{\mathrm{N}}))$ and sorted, and the nests with good evaluation values are saved.
  Delete the number of $\mathrm{N} \times \mathrm{p_a}$ in order from the nest with the worst evaluation value and recreate nest.
end while

In the CS algorithm, the next position (temporary, new nest) $\boldsymbol{x}_i^{\mathrm{new}}$ of the cuckoo that performed Lévy Flight from a uniform randomly selected nest $\boldsymbol{x}_i$ is determined by Eq. (2).

$$\boldsymbol{x}_i^{\mathrm{new}} = \boldsymbol{x}_i + \boldsymbol{\alpha} \odot \boldsymbol{s} \tag{2}$$

where $\boldsymbol{s} = (s_1, \ldots, s_D)$ is a random step length based on the Lévy distribution and $\odot$ represents the Hadamard product. $\boldsymbol{\alpha}$ is the coefficient of Lévy Flight and the step size according to the problem. As with $\mathrm{p_a}$, $\boldsymbol{\alpha}$ can be applied without changing the value of each component $\alpha_i = 1.0$ in many problems [1].

3. Differential Evolution

Differential Evolution (DE) [2] was proposed by R. Storn and K. Price in 1995. DE is a multi-point search type meta-heuristics. In DE, the population updates the solution through a process of mutation, crossover, and selection. In addition, the three parameters that need to be set in advance are the number of individuals $\mathrm{N} = \{1, \ldots, \mathrm{N}\}$, the scale factor $\mathrm{F} \in [0, 2]$, and the crossover probability $\mathrm{CR} \in [0, 1]$. As with CS, N corresponds to the population size, so there are two parameters specific to DE. Each individual $\boldsymbol{x}_i (i \in \{1, \ldots, \mathrm{N}\})$ is represented by a D-dimensional solution vector $\boldsymbol{x}_i = (x_{i,1}, \ldots, x_{i,D})$. In this study, we add the process of mutation and crossover in DE to the algorithm of CS.

4. Proposed Method

Conventional methods of local search by groups of individuals, such as DE, have the characteristic that the search results depend on the initial placement of each individual. This feature is similar to simple local search methods such as the hill-climbing method. In addition, in the solution search, methods that do not have a mechanism to deteriorate the solution do not perform a global search when they converge to a certain solution. Therefore, in methods with these characteristics, such as conventional DE, the final so-

Algorithm 2 Pseudocode of CSDE

Randomly generate N nests $\boldsymbol{x}_{\mathrm{N}}$ in the search space
while Search end condition Not reached **do**
  From the nest $\boldsymbol{x}_i$, which is uniformly randomly selected from $\boldsymbol{x}_{\mathrm{N}}$, the cuckoo moves to xnew with Lévy Flight according to Eq. (2) and lays eggs $(f(\boldsymbol{x}_i^{\mathrm{new}}))$.
  Create nest $\boldsymbol{v}_i$ by mutation
  A secondary candidate $\boldsymbol{u}_i^{\mathrm{new}}$ is generated by crossover using $\boldsymbol{x}_i^{\mathrm{new}}$ and $\boldsymbol{v}_i$, and evaluated $(f(\boldsymbol{u}_i^{\mathrm{new}}))$.
  Evaluate $(f(\boldsymbol{x}_j))$ a nest $\boldsymbol{x}_j$ uniformly randomly selected from $\boldsymbol{x}_{\mathrm{N}}$.
  if $f(\boldsymbol{x}_i^{\mathrm{new}})$ is better than $f(\boldsymbol{x}_j)$ or $f(\boldsymbol{u}_i^{\mathrm{new}})$ is better than $f(\boldsymbol{x}_j)$ **then**
    replace $\boldsymbol{x}_j$ with $\boldsymbol{x}_i^{\mathrm{new}}$ **or** $\boldsymbol{u}_i^{\mathrm{new}}$, whichever is better evaluated
  end if
  All nests are evaluated $(f(\boldsymbol{x}_{\mathrm{N}}))$ and sorted, and the nests with good evaluation values are saved.
  Delete the number of $\mathrm{N} \times \mathrm{p_a}$ in order from the nest with the worst evaluation value and recreate nest.
end while

lution is often a local optimum.

On the other hand, in a method that applies a search method based on RandomWalk, such as Lévy Flight, if the algorithm does not use information other than the previous search position for the next search, it becomes a probabilistic search method. Such methods are suitable for global solution search. However, the ability to search locally is low. In addition, compared to the local solution search method, the method takes longer time to converge to the solution.

These two methods are polar opposites. By appropriately combining these methods, we can consider a hybrid method that complements each other's weaknesses. In this study, we propose Cuckoo Search with built-in Differential Evolution (CSDE) as a hybrid method that incorporates the DE process into CS.

In the proposed method, the process of mutation and crossover is incorporated into the algorithm of CS from DE.

In CSDE, one secondary candidate $\boldsymbol{u}_i^{\mathrm{new}}$ is generated from the next position (temporary, new nest) $\boldsymbol{x}_i^{\mathrm{new}}$ of the individual that performed Lévy Flight, and three $\boldsymbol{x}_i^{\mathrm{new}}$, $\boldsymbol{u}_i^{\mathrm{new}}$, $\boldsymbol{x}_j$ are compared.

Algorithm 2 shows the pseudocode of the proposed method. All solution vectors are regarded as nests instead of individuals. In addition, The scale factor F, which is a parameter of conventional DE, is not changed. Here, $\boldsymbol{v}_i$ is regarded as a temporary nest. Then, the three nests $\boldsymbol{x}_{\mathrm{r1}}$, $\boldsymbol{x}_{\mathrm{r2}}$ and $\boldsymbol{x}_{\mathrm{r3}}$ are uniformly randomly selected from all nests $\boldsymbol{x}_{\mathrm{N}}$. However, r1 $\neq$ r2 $\neq$ r3.

Table 1: Experimental conditions

Number of trials	51
Dimension D	10
Max FEs	$10000 \times D$
Population size N	D
$\mathrm{p_a}$	0.25
α_i	1.0
F	0.7
CR	0.9

5. Numerical Simulations

To evaluate the performance of the proposed method, we used the CEC2014 benchmark[7]. In numerical experiments, conventional DE, conventional CS, and CSDE (proposed method) were performed for comparison. In addition, we also evaluated the proposed method (PCSDE: Perturbed Cuckoo Search by Differential Evolution) in which CS and DE search solutions affect each other. Table 1 shows the experimental conditions.

6. Results

Figure 4 shows an example of numerical experiment results. The proposed method CSDE obtained the best results in 21 functions among 28 benchmark functions. Compared to the conventional CS, the performance improvement rate of all benchmark functions was 19% on average. In particular, the improvement of solution search performance for unimodal functions is considered to be the effect of hybridization.

7. Conclusions

In this study, we proposed a CS-based hybrid method with DE in order to improve the local search performance of CS. In addition, we performed numerical experiments using the CEC2013 benchmark to evaluate the performance of the conventional CS and DE and the proposed method. As a result of numerical experiments, we confirmed that the CSDE of the proposed method improves the performance by an average of 19% compared to the conventional CS, confirming the effectiveness of the proposed method.

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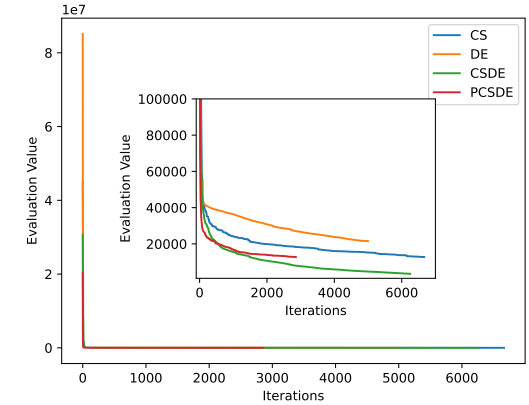
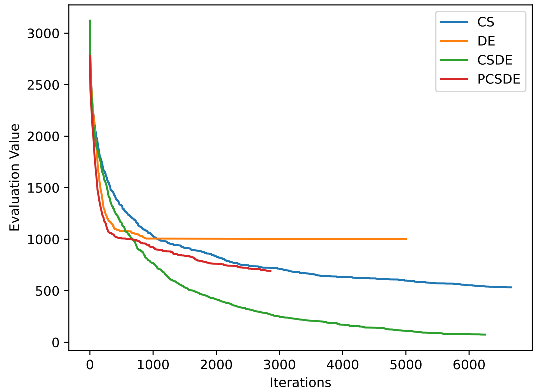
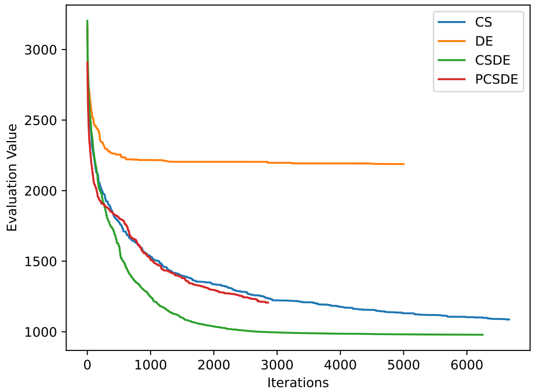


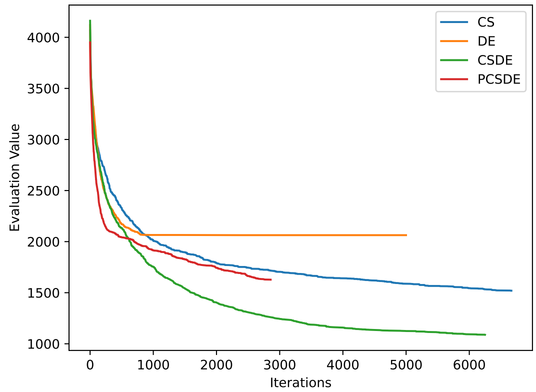
(a) f_4: Rotated Discus Function



(b) f_{14}: Schwefel ' s Function



(c) f_{15}: Rotated Schwefel ' s Function



(d) f_{22}: Composition Function 2

Figure 4: An example of numerical experiment results.

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