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A calculation method of homoclinic points for periodic points with PSO

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Abstract— Homoclinic points are one of the essential indicators for chaotic orbits in dynamical systems. The study suggests a method to compute a homoclinic point using PSO.

1. Introduction

Homoclinic points can be computed using the Newton's method. However, it requires appropriate initial values. In contrast, we proposed a method to calculate the homoclinic points for a fixed point using particle swarm optimization (PSO). In this study, we extend our previous method to a l-periodic point.

2. Homoclinic points

Given a 2-dimensional discrete dynamical system described by

$$x_{k+1} = f_1(x_k, y_k), \qquad y_{k+1} = f_2(x_k, y_k), \tag{1}$$

where $k \in \mathbb{Z}$ denotes the discrete time, and $(x_k, y_k) \in \mathbb{R}^2$ corresponds to the state variables. A point $\left(x_p, y_p\right)$ is said to be an l-periodic point if:

$$\begin{cases} x_p = f_1^l\left(x_p, y_p\right) \\ y_p = f_2^l\left(x_p, y_p\right) \end{cases} \text{and} \begin{cases} x_p \neq f_1^i\left(x_p, y_p\right) \\ y_p \neq f_2^i\left(x_p, y_p\right) \end{cases} \text{for } 0 < i < l. \tag{2}$$

Suppose that μ_1 and μ_2 are the characteristic multipliers of an l-periodic point and $0 < |\mu_1| < 1 < \mu_2$, then the point has an unstable manifold $\alpha\left(x_p, y_p\right)$ and a stable manifold $\omega\left(x_p, y_p\right)$. Homoclinic points are the intersection of these manifolds. Suppose (x_α, y_α) and (x_ω, y_ω) are the homoclinic points located on the unstable and stable manifolds near the l-periodic point; another homoclinic point (x_h, y_h) is described with arbitrary integers M and N by follows:

$$x_h = f_1^{Ml}(x_\alpha, y_\alpha) = f_1^{-Nl}(x_\omega, y_\omega), \tag{3}$$

$$y_h = f_2^{Ml}(x_\alpha, y_\alpha) = f_2^{-Nl}(x_\omega, y_\omega). \tag{4}$$

3. Calculation of homoclinic points with PSO

We use two PSOs: PSO$_\alpha$ to operate on an $\alpha\left(x_p, y_p\right)$, and PSO$_\omega$ to operate on a $\omega\left(x_p, y_p\right)$. PSO$_\alpha$ and PSO$_\omega$ have different search spaces and objective functions. The best positions $\left(x_{g\alpha}, y_{g\alpha}\right)$ of PSO$_\alpha$ and $\left(x_{g\omega}, y_{g\omega}\right)$ of PSO$_\omega$ calculate the homoclinic point by alternately replacing them. Finally, when both objective functions satisfy the termination condition C, we obtain the homoclinic point. The objective function of PSO$_\alpha$ is defined by

$$F_\alpha(x_\alpha, y_\alpha) = \sqrt{\left(f_1^{Ml}(x_\alpha, y_\alpha) - f_1^{-Nl}\left(x_{g\omega}, y_{g\omega}\right)\right)^2 + \left(f_2^{Ml}(x_\alpha, y_\alpha) - f_2^{-Nl}\left(x_{g\omega}, y_{g\omega}\right)\right)^2} < C, \tag{5}$$

where the search space is (x_α, y_α), and $\left(x_{g\omega}, y_{g\omega}\right)$ is the best position of PSO$_\omega$ operated just before. We can define the objective function of PSO$_\omega$ in the same way.

4. Simulation result

We confirm the effectiveness of the proposed method with the Hénon map. We compute a homoclinic point corresponding to the 2-periodic point $\left(x_p, y_p\right) = (-0.46098, 0.32748)$ when the system parameters $a = 1.11$ and $b = 0.30$. We use the same parameters of PSO$_\alpha$ and PSO$_\omega$ as Reference[1], but change the mapping numbers into $M = 10$ and $N = 5$. Table 1 shows a homoclinic point (x_h, y_h), (x_α, y_α), and (x_ω, y_ω).

Table 1: Calculation results

(x_α, y_α)	$(-0.45919, 0.32722)$
(x_ω, y_ω)	$(-0.46098, 0.32748)$
(x_h, y_h)	$(0.18512, 0.21806)$

5. Conclusion

In this study, we extend the homoclinic points calculation method using PSO to the l-periodic point.

6. Acknowledgements

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References

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