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Sequential Monte Carlo framework for simultaneously estimating and controlling nonlinear neuronal dynamics

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Abstract— Estimating and controlling nonlinear neuronal system are crucial for understanding the neuronal system and brain functions. However, it is challenging to determine appropriate time-series input for the nonlinear system including unobservable state and unknown dynamics. We propose a framework for estimating and controlling an individual neuron by leveraging the sequential Monte Carlo method. For estimating the hidden state and its dynamics, we derive an online algorithm based on the sequential Monte Carlo method and the expectation-maximization algorithm. In addition, we constitute the feedback control law by employing the Monte Carlo method based model predictive control. We verify the effectiveness of the proposed method using simulation environments. The results suggest that with the proposed method we can simultaneously estimate the latent variables and the parameters and control neuronal state toward the desired firing pattern.

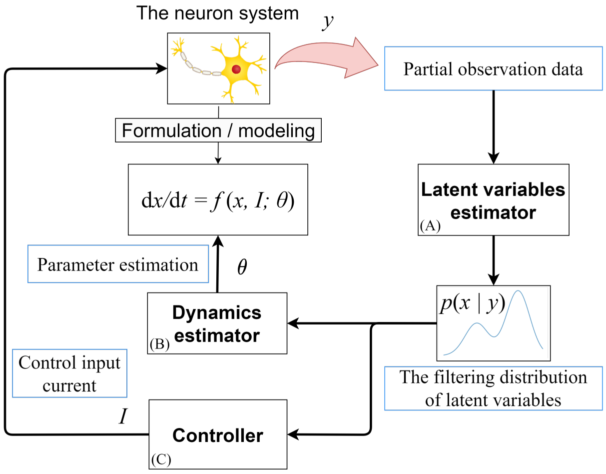

Figure 1: The overview of the proposal method

1. Introduction

Estimating and controlling nonlinear neuronal system are crucial for understanding the neuronal system and brain functions [1, 2]. However, it is still challenging to constitute a feedback control law for neurons since we cannot directly observe neuronal multi-dimensional state; we can only obtain a part of the entire neuronal activities. For instance, only noisy membrane potentials are observable [3, 4, 5]. Therefore, it is essential to establish a method for simultaneously estimating and controlling the nonlinear neuronal dynamics from only noisy membrane potentials.

To control the neurons, it is required to have accurate information about the state and the dynamics of the neurons. Some methods have been proposed to estimate the state and the dynamics of the neurons from lower dimensional observation data based on the statistic machine learning [6, 7]. A previous method has been proposed to control neurons based on the modern control theory [8]. However, many previous works assume either the state or the dynamics to be known. This is unrealistic assumption in neuronal systems, since several neuron models, like the Morris-Lecar model [9], have unobservable latent variables. Moreover, imaging technique provides only lower dimensional variables compared with high dimensional latent variables. In this study, we propose a statistical machine learning based framework for simultaneously estimating and controlling the nonlinear individual neuronal state and dynamics under a realistic situation where only noisy membrane potentials can be obtained. We focus on a probabilistic framework to estimate and control neuronal dynamics, in particular, the state-space model and the sequential Monte Carlo (SMC) method as well as model predictive control theory. In order to verify the effectiveness of the proposed method, we estimate and control the neuron model under simulation environments.

2. Proposed Method

We propose a statistical machine learning based framework for estimating and simultaneously controlling the nonlinear dynamics of individual neurons. The overview of the the framework is shown in Fig. 1. Our method consists of three parts; (A) estimating the filtering distribution of the hidden state from partial observation data, (B) estimating the parameter governing the neuronal dynamics, (C) formulating the optimal control problem as a Bayesian statistic problem.

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2.1. Morris-Lecar Neuron Model

Here we formulate a nonlinear state-space model based on the Morris-Lecar neuron model. The Morris-Lecar neuron model is defined as follows:

$$\begin{aligned} C_m \frac{dv}{dt'} &= -g_L(v - E_L) - g_{Ca} m_\infty(v)(v - E_{Ca}) \\ &\quad - g_K n(v - E_K) + I, \\ \frac{dn}{dt'} &= -\phi \cosh\Big(\frac{v - V_3}{2V_4}\Big)(n - n_\infty(v)), \end{aligned} \tag{1}$$

where t' means continuous time, v is a membrane voltage, n is a normalized channel variable, and I is an applied current. $m_\infty(v)$ and $n_\infty(v)$ are nonlinear functions. The Morris-Lecar model can be expressed by the state vector $x = \begin{bmatrix} v & n \end{bmatrix}^T$ and the parameter $\theta = \begin{bmatrix} g_L & g_{Ca} & g_K \end{bmatrix}^T$ as follows:

$$\frac{dx}{dt'} = f(x, I; \theta) = \begin{bmatrix} \frac{1}{C_m}(I - V_{\text{ion}}(x)^T \theta) \\ -\phi \cosh\Big(\frac{v - V_3}{2V_4}\Big)(n - n_\infty(v)) \end{bmatrix}, \tag{2}$$

where $V_{\text{ion}}(x) = \begin{bmatrix} v - E_L & m_\infty(v)(v - E_{Ca}) & n(v - E_K) \end{bmatrix}^T$.

Here, we derive system model and observation model for the state-space model. At first, we discretize the continuous-time model given by Eq. (2) with the time step Δt. Then, we consider a fluctuating system noise governed by an additive Gaussian noise. The system model of the state-space model is formulated as follows:

$$p(x_{t+1} \,|\, x_t, I_t, \theta) = \mathcal{N}(x_{t+1} \,|\, x_t + f(x_t, I_t; \theta)\Delta t, \Sigma_x), \tag{3}$$

where t denotes a discrete time index, x_t the state value at time $t' = t\Delta t$, respectively.

Next, we consider the observation model for the state-space model. We assume we can obtain only a one-dimensional time series of noisy membrane potentials $\{y_t\}_{t \geq 0}$. By introducing the additive Gaussian noise with its variance $\sigma_y^2 \in \mathbb{R}$, we formulate the observation model of the state-space model as follows:

$$p(y_t \,|\, x_t) = \mathcal{N}(y_t \,|\, v_t, \sigma_y^2). \tag{4}$$

2.2. Sequential Monte Carlo Method for Estimating Latent State

To estimate the latent state x_t from noisy time-series membrane potentials $Y_t = \{y_1, y_2, \ldots, y_t\}$ online, we apply the sequential Monte Carlo (SMC) method [10].

Here, we consider estimating the latent state x_t by using predictive distribution $p(x_t \,|\, Y_{t-1})$ and the filtering distribution $p(x_t \,|\, Y_t)$. In the SMC method, both distributions are approximated by using particles, which represent the value of the latent state. The SMC method is divided into two steps; the prediction step and filtering step.

In the predictive step, we sample the particles with the system model Eq. (3) as follows:

$$x_t^{(i)} \sim p(x_t \,|\, x_{t-1}^{(i)}, I_{t-1}, \theta) \quad \text{for } i = 1, \ldots, N, \tag{5}$$

where i is the index of the particle and N is the number of the particles. In the filtering step, by using the observation model Eq. (4), the particles are resampled with a probability proportional to their weights $w_t^{(i)}$. Here, the weight of i th particle $w_t^{(i)}$ is obtained by $w_t^{(i)} = p(y_t \,|\, x_t^{(i)})$. By using resampled particles, we obtain an approximated filtering distribution as follows:

$$p(x_t \,|\, Y_t, \theta) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_t - x_t^{(i)}), \tag{6}$$

where $\delta(\cdot)$ is a Dirac's delta function. The obtained particles $\{(x_t^{(i)}, w_t^{(i)})\}_{i=1}^N$ also are used not only to estimate the parameters in section 2.3, but also to constitute the feedback law introduced in section 2.4.

2.3. Online EM Algorithm for Estimating Model Parameter

To estimate the model parameter θ online, we adapt the original offline-based expectation-maximization (EM) algorithm into an online one with the SMC.

In our study, we consider the parameter θ_k is updated at times $\{k \times T_{\text{interval}}\}_{k=1,2,\cdots}$, where k represents the renewal number and T_{interval} denotes the interval of the update time. In addition, the expectation with respect to the previous parameter θ_{k-1} is replaced with the filtering distribution obtained online via SMC. The procedure is summarized:

$$\hat{\theta}_k = \arg\max_{\theta} E[\log p(Y_{\text{win}(t)}, X_{\text{win}(t)} \,|\, \theta)]_{\text{SMC}_{\text{win}(t)}(\theta_{k-1})}, \tag{7}$$

$$\theta_k = (1 - \eta_k)\theta_{k-1} + \eta_k \hat{\theta}_k, \tag{8}$$

where $X_{\text{win}(t)} = X_{t-L(t):t}$, $\text{SMC}_{\text{win}(t)}(\theta_{k-1}) = p(X_{\text{win}(t)} \,|\, Y_{\text{win}(t)}, \theta_{k-1})$ is the filtering distribution obtained in section 2.2, $0 < \eta_k < 1$ is a number that satisfies $\eta_k \to 0$ as $k \to \infty$, $L(t)$ is a window function.

2.4. Model Predictive Control Driven by the Sequential Monte Carlo Method

To constitute the feedback control law under a practical situation where the state and the parameters are unknown, we combine the SMC method-based model predictive control (SMC-MPC) [11] with our dynamics estimator. Note that the conventional methods [11] assumed either the state or the dynamics to be known. In the SMC-MPC, the optimal control problem for the naive MPC is replaced with a filtering problem based on the state-space model. First, to formulate the filtering problem, we define the extended system model in the horizon. We consider the control input I_t is regarded as the part of the extended state $\zeta_t = (x_t, I_t, \tilde{I}_t)$, where $\tilde{I}_t$ is a dummy input to preserve the initial control input. Hence we formulate the extended system model in the horizon as follows:

$$\begin{cases} x_{\tau+1} \sim p(x_{\tau+1} \,|\, x_\tau, I_\tau; \theta), \\ I_{\tau+1} \sim p(I_{\tau+1} \,|\, \zeta_\tau), \quad I_t \sim p(I_t \,|\, I_{t-1}), \\ \tilde{I}_{\tau+1} = \tilde{I}_\tau, \quad \tilde{I}_t = I_t, \end{cases} \tag{9}$$

where $\tau \in \{t, t+1, \ldots, t+T_H\}$ is the discrete time in the horizon and T_H is a length of the horizon. In our study, the control input transition $p(I_{\tau+1} \mid \zeta_\tau)$ and the initial distribution for the control input $p(I_t \mid I_{t-1})$ are designed to ensure that the input time-series $\{I_t, \ldots, I_{t+T_H}\}$ always remain within a certain range.

Next, we formulate the extended observation model in the horizon. We consider controlling the state towards the given reference state trajectory $\{r_\tau\}_{\tau=t}^{t+T_H}$. In the SMC, we regard the reference trajectory as a realization of the Markov chain with the extended system model in the horizon. Therefore, we formulate the extended observation model (referred as a setpoint equation in [11]) as follows:

$$r_\tau \sim p(r_\tau \mid \zeta_\tau) = \mathcal{N}(r_\tau \mid x_\tau, \sigma_r^2), \tag{10}$$

where σ_r^2 is a variance which adjusts an acceptable error against the reference trajectory.

To solve the filtering problem defined by Eqs. (9) and (10), we apply the SMC method with given observation data $\{r_\tau\}_{\tau=t}^{t+T_H}$. Here, to initialize the extended particles $\{\zeta_t^{(i)}, w_t^{(i)}\}_{i=1}^N$, we use the state particles $\{(x_t^{(i)}, w_t^{(i)})\}_{i=1}^N$ obtained via the state estimation. After applying the SMC method in the horizon, we obtain the filtered extended particles $\{\zeta_{t+T_H}^{(i)}, w_{t+T_H}^{(i)}\}_{i=1}^N$. By extracting the dummy input particles inheriting initial inputs and their weights $\{\tilde{I}_{t+T_H}^{(i)}, w_{t+T_H}^{(i)}\}_{i=1}^N$, we obtain the point estimation of this filtering distribution, e.g.

$$I_t^* = \frac{1}{N} \sum_{i=1}^{N} \tilde{I}_{t+T_H}^{(i)}. \tag{11}$$

Finally, we adopt $I_t = I_t^*$ as the actual control input at time t.

We have proposed an automatic control framework, even if the state and the dynamics are unknown. Although the conventional model predictive control (MPC) enables us to handle even a nonlinear model, MPC requires the state and the model to be known. The SMC-MPC [11] resolves one side of this problem by combining the SMC method with MPC effectively. However, they assumed that the model dynamics to be known. The proposed method constitutes the feedback control law by combining the SMC-MPC with the state and the parameter estimators.

3. Experiment

3.1. Setting

We verify the effectiveness of our proposal method using simulation environments. The true neuronal dynamics is assumed to be the Morris-Lecar neuron model [12]. Here, we assume multi-dimensional latent variables v and n, and the conductances of ion channels g_L, g_{Ca}, g_K are unknown. The reference membrane potentials are generated by using the true model in advance. In practice, the constant current influenced by other neurons exists. We consider that the net input current I for the neuron consists of the constant current I_0 and the controllable input I_{in}, i.e. $I = I_0 + I_{\text{in}}$.

3.2. Results

Fig. 2 shows the simulation result. Here, we verify whether our proposal framework can estimate the neural hidden state and its parameters from only noisy membrane potentials, but also simultaneously control the membrane potentials towards the desired ones. The upper left graph shows the true membrane potential v_{true} (blue line), the estimated one v_{est} (orange dashed line), and the reference one v_{ref} (green dotted line). We can see the estimated membrane potential tracks the true value as time passes. Here, the gray dash-dotted line represents the simulation result if no input current is applied. By applying the proposed method, the membrane potentials well tracks the time-varying reference membrane potentials. The middle left graph shows the true channel variable n_{true} (blue line) and the estimated value n_{est} (orange dashed line). Note that n is unobservable. We also find that the estimated value of the channel variable tracks the true value.

Next, we clarify whether our method can estimate and control the neuronal dynamics using feedback input current with a limited range of strength. The lower left graph of Fig. 2 shows the applied input current I_{in} (blue line), the streaming current I_0 (orange dotted line), and the limit I_{lim} (gray dash-dotted line). Here, I_{in} is automatically calculated by the proposed framework. We can see the sum of the control and streaming current (green dashed line) satisfy $|I_{\text{in}} + I_0| \leq I_{\text{lim}}$.

Finally, we clarify whether our method can estimate the unknown parameters. The graphs on the right side in Fig. 2 show the estimated parameters and their true values. We can see each estimated conductance value converges to the true value as the online-adapted EM algorithm is applied.

Note that our method can successfully estimate and control other types of neuronal dynamics such as Hopf and homoclinic bifurcations (data not shown), although only the result for the SNLC parameter set [12] is shown here.

The result indicates that our proposed framework can simultaneously estimate the latent state and the parameters and control the membrane potential towards the desired ones, even if the conductances are unknown.

4. Conclusion

In this paper, we propose a statistical machine learning based framework for estimating and controlling the neuronal state and dynamics, and simultaneously controlling the neuronal dynamics. We have shown that the proposal method not only well estimates the multi-dimensional hidden state and the parameters of the neuron, but also successfully control the membrane potentials towards the desired ones under a partial observation situation. In the future work, it would be important to consider the case where the internal fluctuation is non-Gaussian or chaotic noise.

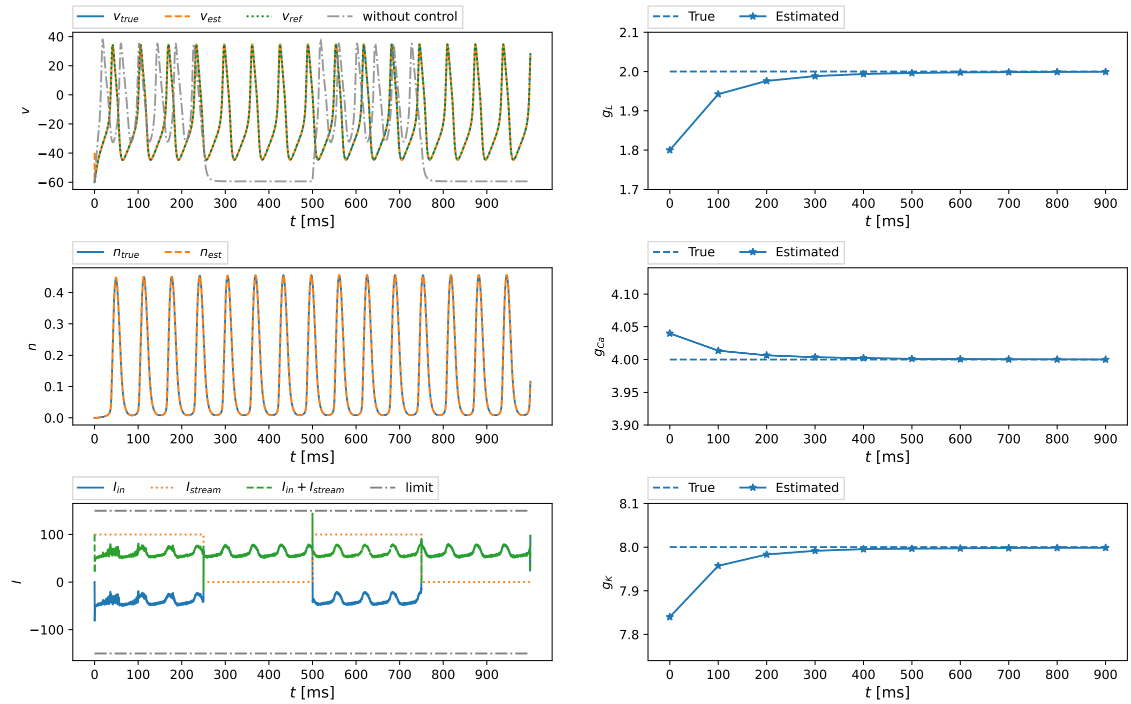

Figure 2: Result for estimating and controlling neuronal dynamics by the proposed method. Left: membrane potential v, channel variable n, and currents including control current. Right: membrane conductances g_L, g_{Ca}, and g_K.

Acknowledgments

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